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(*b. ca.* 850, *d. ca.* 930), mathematics.

Often called al-Ḥāsib al-Miṣrī ("the reckoner of Egypt") Abū Kāmil was one of Islam's greatest algebraists in the period following the earliest Muslim algebraist, al-Khwārizmī (*fl. ca.* 825). In the Arab world this was a period of intellectual ferment, paric ularl in mathematics and the sciences

There is virtually no biographical material available on Abū Kāmil. He is first mentioned by al-Nadīm in a bibliographical work, *The Fihrist* (987), where he listed with other mathematicians under "The New Reckoners and Arithmeticians," which refers to those mathematicians who concerned themselves with the practical algorisms, citizens' arithmetic, and practical geometry (see Bibliography). Ibn khaldūn (1322–1406) state that Abū Kāmil wrote his algebra after the first such work by al-Khwārizmī, and Ḥajjī Khalīfa (1608–1658) attributed to him a work supposedly concerned with algebraic solutions of inheritance problems.

Among the works of Abū Kāmil extant in manuscripts is the *Kitāb al-ṭarāʾif fiʾl-ḥisāb* ("Book of Rare Things in the Art of Calculation"). According to H. Suter[1] this text is concerned with integral solutions of some indeterminate equations; much earlier, Diophantus (*ca.* first century a.d.) had concerned himself with rational, not exclusively integral, solutions. Abū Kāmil's solutions are found by an ordered and very systematic procedure. Although indeterminate equations with integral solutions had been well known in ancient Mesopotamia, it was not until about 1150 that they appeared well developed in India. Aryabhata (*b.* a.d. 476) had used continued fractions in solutions, but there is uncertain evidence that this knowledge had been passed on in any ordered form to the Arabs by the time of Abū Kāmil.

A work of both geometric and algebraic interest is the *Kitāb … al-mukharnmas wa'al-muʿashshar* … ("On the Pentagon and Decagon"). The text is algebraic in treatment and contains solutions for a fourth-degree equation and for mixed quadratics with irrational coefficients. Much of the text was utilized by Leonardo Fibonacci (1175-*ca.* 1250) in his *Practica geometriae*.[2] Some of the equations solved by Abū Kāmil in this work read as follows:

with *s* the side of a regular polygon inscribed in a circle. Also

with *S* the side of a regular polygon (pentagon here) which has an inscribed circle.[3]

The outstanding advance of Abū Kāmil over al-Khwārizmī, as seen from these equations, is in the use of irrational coefficients.[4] Another manuscript, which is independent of the *Ṭarā'if*, mentioned above, is the most advanced work on indeterminate equations by Abū Kāmil. The solutions are not restricted to integers; in fact, most are in rational form. Four of the more mathematically interesting problems are given below in modern notation. It must be remembered that Abū Kāmil gave all his problems rhetorically; in this text, his only mathematical notation was of integers.

Many of the problems in *Kitāb fi'l-jabr wa'lmuqābala* had been previously solved by al-Khwārizmī. In Abū Kāmil's work, a solution[5] for x^2 was worked out directly instead of first solving for x. Euclid had taken account of the condition x less than $p/2$ in $x^2 + q = px$, whereas Abū Kāmil also solved the case of x greater than $p/2$ in this equation.

Abū Kāmil was the first Muslim to use powers greater than x^2 with ease. He used x^8 (called "square square square square"), x^6 (called "cube cube"), x^5 (called "square square root"), and x^3 (called "cube"), as well as x^2 (called "square"). From this, it appears that Abū Kāmil's nomenclature indicates that he added "exponents." In the Indian nomenclature a "square cube" is x^6, in contradistinction. Diophantus (*ca.* a.d. 86) also added "powers," but his work was probably unknown to the Arabs until Abu'l Wafā' (940–998) translated his work into Arabic (*ca.* 998).

Abū Kāmil, following al-Khwārizmī, when using *jadhr* ("root") as the side of a square, multiplied it by the square unit to get the area ($x \cdot 1^2$). This method is older than al-Khwārizmī's method and is to be found in the *Mishnat ha-Middot*, the oldest Hebrew geometry, which dates back to a.d. 150.[6] This idea of root is related to the Egyptian *khet* ("cubit strip").[7]

The Babylonians stressed the algebraic form of geometry as did al-Khwārizmī. However, Abū Kāmil not only drew heavily on the latter but he also derived much from Heron of Alexandria and Euclid. Thus he was in a position to put together a sophisticated algebra with an elaborated geometry. In actuality, the resulting work was more abstract than al-Khwārizmī's and more practical than Euclid's. Thus Abū Kāmil effected the integration of ancient Mesopotamian practice and Greek theory to yield a wider approach to algebra.

Some of the more interesting problems to be found in the *Algebra*, in modern notation, are:[8]

It is possible that Greek algebra was known to Abū Kāmil through Heron of Alexandria, although a direct connection is difficult to prove. The influence of Heron is, however, definite in Abraham bar Ḥiyy's work.[9] That Abū Kāmil influenced both al-Karajī and Leonardo Fibonacci may be demonstrated from the examples they copied from his work. Thus through Abū Kāmil, mathematical abstraction, elaborated together with a more practical mathematical methodology, impelled the formal development of algebra.

NOTES

1. "Das Buch der Seltenheiten."

2. See also Suter, "Die Abhandlung des Abū Kāmil,"

3.*Ibid*., p. 37. Levey will soon publish the Arabic text of "On the Pentagon and Decagon," discovered by him.

4. At least twenty of these problems from this text may be found in Leonardo Fibonacci, *Scritti*, Vol. 1, sect. 15: Vol. II.

5. Tropfke, *Geschichte der Elementar-Mathematik*, pp. 74–76; 80–82; Weinberg, "Die Algebra des abu Kamil."

6. S. Gandz, "On the Origin of the Term 'Root.'"

7. M. Levey, *The Algebra of Abū Kāmil*, pp. 19–20. P. Schub and M. Levey will soon publish Abū Kāmil's advanced work on indeterminate equations, newly discovered in Istanbul.

8.*Ibid*., pp. 178, 184, 186, 202.

9. M. Levey, "The Encyclopedia of Abraham Savasorda" and "Abraham Savasorda and His Algorism."

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