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(*b*. Būzjān [now in Iran], 10 June 940; *d*. Baghdad [now in Iraq], 997 or July 998)

mathematics, astronomy.

Abū'l-Wafāʾ was apparently of Persian descent. In 959 he moved to Baghdad, which was then the capital of the Eastern Caliphate. There he became the last great representative of the mathematics-astronomy school that arose around the beginning of the ninth century, shortly after the founding of Baghdad. With his colleagues, Abū'l-Wafāʾ conducted astronomical observations at the Baghdad observatory. He continued the tradition of his predecessors, combining original scientific work with commentary on the classics—the works of Euclid and Diophantus. He also wrote a commentary to the algebra of al-Khwārizmī. None of these commentaries has yet been found.

Abū'l-Wafāʾ's textbook on practical arithmetic, *Kitāb fī mā yaḥtaj ilayh al-kuttāb wa'l-ʿummāl min ʾilm al-ḥisāb* ("Book on What Is Necessary From the Science of Arithmetic for Scribes and Businessmen"), written between 961 and 976, enjoyed widespread fame. It consists of seven sections (*manāzil*), each of which has seven chapters (*abwāb*). The first three sections are purely mathematical (ratio, multiplication and division, estimation of areas); the last four contain the solutions of practical problems concerning payment for work, construction estimates, the exchange and sale of various grains, etc.

Abū'l-Wafāʾ systematically sets forth the methods of calculation used in the Arabic East by merchants, by clerks in the departments of finance, and by land surveyors in their daily work; he also introduces refinements of commonly used methods, criticizing some for being incorrect. For example, after indicating that surveyors found the area of all sorts of quadrangles by multiplying half the sums of the opposite sides, he remarks, "This is also an obvious mistake and clearly incorrect and rarely corresponds to the truth." Abū'l-Wafāʾ does not introduce the proofs here "in order not to lengthen the book or to hamper comprehension," but in a series of examples he defines basic concepts and terms, and also defines the operations of multiplication and division of both whole numbers and fractions.

Abū'l-Wafāʾ's book indicates that the Indian decimal positional system of numeration with the use of numerals—which Baghdad scholars, acquainted with it by the eighth century, were quick to appreciate—did not find application in business circles and among the population of the Eastern Caliphate for a long time. Considering the habits of the readers for whom the textbook was written, Abū'l-Wafāʾ completely avoided the use of numerals. All numbers and computations, often quite complex, he described only with words.

The calculation of fractions is quite distinctive. Operation with common fractions of the type m/n, where m, n are whole numbers and $m > 1$, was uncommon outside the circle of specialists. Merchants and other businessmen had long used as their basic fractions—called *ra's* ("principal fractions") by Abū'l-Wafāʾ—those parts of a unit from 1/2 to 1/10, and a small number of *murakkab* ("compound fractions") of the type m/n, with numerators, m, from 2 to 9 and denominators, n, from 3 to 10, with the fraction 2/3 occupying a privileged position. The distinction of principal fractions was connected with peculiarities in the formation of numerical adjectives in the Arabic language of that time. All other fractions m/n were represented as sums and products of basic fractions; businessmen preferred to express the "compound" fractions, other than 2/3, with the help of principal fractions, in the following manner;

Any fraction m/n, the denominator of which is a product of the sort $2^P\ 3^Q\ 5^R\ 7^S$, can be expanded into basic fractions in the above form. In the first section of his book, Abū'l-Wafāʾ explains in detail how to produce such expansions with the aid of special rules and auxiliary tables. Important roles in this operation are played by the expansion of fractions of the type $a/60$ and the preliminary representations of the given fraction m/n in the form $m \cdot 60/n \div 60$ (see below). Since usually for one and the same fraction one can obtain several different expansions into sums and products of basic fractions, Abū'l-Wafāʾ explains which expansions are more generally used or, as he wrote, more "beautiful."

If the denominator of a fraction (after cancellation of the fraction) contains prime factors that are more than seven, it is impossible to obtain a finite expansion into basic fractions. In this case approximate expansions of the type $^3/_{17} \approx (3 + 1) \div (17 + 1) = {}^2/_9$ or $^3/_{17} \approx 3^1/_2 \div 17^1/_2 = {}^1/_5$—or still better, $^3/_{17} \approx 3^1/_7 \div 17^1/_7 = {}^1/_6 + {}^1/_6 \cdot {}^1/_{10}$—were used.

Instead of such a method, which required the skillful selection of a number to be added to the numerator and denominator of a given fraction, Abū'l-Wafāʾ recommended the regular method, which enables one to obtain a good approximation with reasonable speed. This method is clear from the expansion

Analogously, one can obtain

or

The error of this last result as Abū'l-Wafāʾ demonstrates, equals

The calculation described somewhat resembles the Egyption method, but, in contrast with that, in (1) is limited to those parts of a unity $1/q$, for which $2 \leq q \leq 10$; (2) uses products of the fractions $1/q_1 \cdot 1/q_2$ and $2/3 \cdot 1/q$; and (3) does not renounce the use of compound fractions m/n, $1 < m < n \leq 10$. Opinions differ regarding the origin of such a calculation; many think that its core derives from ancient Egypt; M. I. Medovoy suggests that it arose independently among the peoples living within the territory of the Eastern Caliphate.

In the second section is a description of operations with whole numbers and fractions, the mechanics of the operations with fractions being closely connected with their expansions into basic fractions. In this sections there is the only instance of the use of negative numbers in Arabic literature. Abū'l-Wafāʾ verbally explains the rule of multiplication of numbers with the same ten's digit:

$$(10a + b)\,(10a + c) = [10a + \mathrm{b} - \{10(a + 1) - (10a + c)\}]10(a + 1) + [10(a + 1) - (10a + b)] \cdot [10(a + 1) - (10a + c)].$$

He then applies it where the tens digit is zero and $b = 3$ and $c = 5$. In this case the rule gives.

$$3 \cdot 5 = [3 - (10 - 5)] \cdot 10 + [10 - 3] \cdot [10 - 5] = (-2) \cdot 10 + 35 = 35 - 20.$$

Abū'l-Wafāʾ termed the result of the subtraction of the number 10–5 from 3 a "debt [*dayn*] of 2." This probably reflects the influence of Indian mathematics, in which negative numbers were also interpreted as a debt (*kśaya*).

Some historians such as M. Cantor and H. Zeuthen, explain the lack of positional numberation and "Indian" numerals in Abūl-Wafāʾ's textbook, as well as in many other Arabic arithmetic courses, by stating that two opposing schools existed among Arabic mathematicians: one followed Greek models the other Indian models; M.I. Medovoy, however, shows that such a hypothesis is not supported by fact. It is more probable that the use of the Positional "Indian" arithmetic simply spread very slowly among businessmen and the general population of the Arabic East, Who for a long time preferred the customary methods of verbal expression of whole numbers and fractions and of operations dealing with them. Many authors considered the needs of these people; and, after Abū'l-Wafāʾ, the above computation of fractions, for example, is found in a book by al-Karajī at the beginning of the eleventh century and in works by other authors.

In the third section Abū'l-Wafāʾ gives rules for the measurement of more common planar and three-dimensional figures—from triangles various types of quadrangles. regular polygons, and a circle and its parts, to a sphere and sectors of a sphere, inclusive. There is table of Chords corresponding to the the arcs of a semicircle of radius 7, which consists of $m/22$ of the semicircumference ($m = 1,2,..., 22$), and the expression for the diameter, d, of the a circle sup–rscribed around a regular n–sided polygon with side a:

Abū'l-Wafāʾ thought this rule was obtained from India; it is correct for $n = 3, 4, 6$, and for other values of n gives a good approximation, especially for small n. At the end of the third section, problems involving the determination of the distance to inaccessible objects and their heights are solved on the basis of similar triangles.

Another practical textbook by Abū'Wafāʾ is *Kitāb fī mā yaḥtaj ilayh al-ṣāniʿ min al-aʿmāl al-handasiyya* ("Book on what is necessary from Geometric Construction for the Artisan"), written after 990. Many of the two-dimensional and three-dimensional constructions set forth by Abū'l-Wafāʾ were borrowed mostly from the writings of Euclid, Archimedes, Hero of Alexandria, Theodosius, and Pappus. Some of the examples, however, are original. The range of problems is very wide, from the simplest planar constructiontions (the division of a segment into equal parts, the constructions of tangent to a circle from a point on or outside the circle, etc.) to the construction of regular and serniregular polyhedrons inscribed in a given sphere. Most of the constructions can be drawn with a compass and straightedge. In several instances, when these means are insufficient, intercalations is use (for the trisection of an angle or the duplication of a cube)or only an approximate construction is given for the side of a regular heptagon inscribed in a given circle, using half of one side of an equilateral triangle inscribed in the same circle, the error is very small).

A group of problems that are solved using a straightedge and compass with an invariable opening deserves mention. Such constructions are found in the writings of the ancient Indians and Greeks, but Ab''Wafā' was the first to solve a large number of problems using a compass with an invariable opening. Interest in these constructions was probably aroused by the fact that in practice they give more exact results than can be obtianed by changing the compass opening. These constructions were widely circulated in Renaissance Europe; and Lorenzo Mascheroni, Jean Victor Poncelet, and Jakob Steiner developed the general theory of these and analogous construtions.

Also in this work by Abū'l-Wafāʾ are problems concerning the division of a figure into parts that satisfy certain conditions, and problems on the transformation of squares (for example, the construction of a square whose area is equal to the sum of the areas of three given squares). In proposing his original and elegant constructions, Abū'l-Wafāʾ simultaneously proved the inaccuracy of some methods used by "artisans."

Abū'l-Wafāʾs large astronomical work, *al-majisṭī*, or *Kitāb al-kāmil* ("Complete Book"), closely follows Ptolemy's *Almagest*. It is possible that this work, available only in part, is the same as, or is included in, his *Zīj al-Wāḍiḥ*, based on observations that he and his colleagues conducted. The *Zīj* seems not to be extant. Abū'l-Wafāʾ apparently did not introduce anything essentially new into theoretical astronomy. In particular, there is no basis for crediting him with the discovery of the so-called variation of the moon (this was proved by Carra de Vaux, in opposition to the opinion expressed by L.A. Sédillot).E. S. Kennedy established that the data from Abū'l-Wafāʾs observations were used by many later astronomers.

Abū'l-Wafāʾs achievements in the development of trigonometry, specifically in the improvement of tables and in the means of solving problems of spherical trigonometry, are undoubted. For the tabulation of new sine tables he computed sin 30 more precisely, applying his own method of interpretation. This method, based on one theorem of Theon of Alexandria, gives an approximation that can be stated in modern terms by the inequalities

The values sin 15°/32 and sin 18°/32 are found by using the known values of sin 60° and sin 72°, respectively, with the aid of rational operations and the extraction of a square root, which is needed for the calculation of the sine of half a given angle; the value sin 12°/32 is found as the sine of the difference 72°/32–60°/32. Setting sin 30' equal to half the sum of the quantities bounding it above and below, with the radius of the circle equal to 60, Abül-Wafāʾ found, in sexagesimal fractions, sin 30′ = $31^{I}\ 24^{II}\ 55^{III}\ 54^{IV}\ 55^{V}$. This value is correct to the fourth place, the value correct to five places being sin 30' = $31^{I}\ 24^{II}\ 55^{III}\ 54^{IV}\ 0^{V}$.

In comparison, Ptolemy's method of interpolation, which was used before Abū'l-Wafāʾ, showed error in the third place. If one expresses Abū'l-Wafāʾ's approximation in decimal fractions and lets r = 1 (which he did not do), then sin 30' = 0.0087265373 is obtained instead of 0.008725355–that is, the result is correct to 10–8;. Abuū'Wafaʾ also compiled tables for tangent and cotangent.

In spherical trigonometry before Abu'l-Wafāʾ, the basic means of solving triangles was Menelaus' theorem on complete quadrilaterals, which in Arabic literature is called the "rule of six quantities." The application of this theorem in various cases is quite cumbersome. Abū'l-Wafāʾ enriched the apparatus of spherical trigonometry, simplifying the solution of its problems. He applied the theorem of tangents to the solution of spherical right triangles, priority in the proof of which was later ascribed to him by al-Bīrūnī. One of the first proofs of the general theorem of sines applied to the solution of oblique triangles also was originated by Abū'l-Wafāʾ. In Arabic literature this theorem was called "theorem which makes superfluous" the study of complete quadrilaterals and Menelaus' theorem. To honor Abū'l-Wafāʾ, a crater on the moon was named after him.

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