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(*b*. Baghdad, Iraq [?]; *fl*. *ca*. 900–905; *d*. Cairo, Egypt, 912/913 [?])

mathematics.

Aḥma ibn Yūsuf ibn Ibrāhīm ibn al-Dāya al Miṣrī was the son of an Arab scholar, Yūsuf ibn Ibrāhīm. Yūsuf's home was in Baghdad, but in 839/840 he moved to Damascus, and later to Cairo; hence his son was known as an Egyptian. Aḥmad's birth date is not known, although it seems probable that he was born before the move to Damascus. His death date is likewise in doubt, although the most probable date is 912/913.

Aḥmad's father, sometimes referred to as *al-ḥāsib* ("the reckoner"), was one of a group of learned and influential men. A work on the history of medicine, another on the history of astronomy, and a collection of astronomical tables are attributed to him, although no written work of his surives today.

In Egypt, Aḥmad ibn Yūsuf was a private secretary to the Ṭūlūn family, which ruled Egypt from 868 to 905. In his writing, Aḥmad made several references to one Hudā ibn Aḥmad ibn Ṭūlūn. This was probably Abu'l-Baqāʾ Hudā, the thirteenth son of Aḥmad ibn Ṭūlūn, and probably Aḥmad ibn Yūsuf's employer.

Aḥmad ibn Yūsuf wrote a treatise on ratio and proportion, a work on similar ares, a commentary on Ptolemy's *Centiloquium*, and a work on the astrolabe. All the works survive in Arabic manuscript, and all but the work on the astrolbe exist in Latin translation. While it is impossible to distinguish absolutely the work of the father from that of the son, there seems to be little doubt of Aḥmad's authorship of the above four works. A number of other works are attributed to him, but these cannot be authenticated.

Aḥmad's most significant work is the treatise on ratio and proportion. This was translated from the Arabic into Latin by Gerard of Cremona and then extensively copied. Manuscript copies of the Latin version exist today in at least eleven libraries in England, Spain, Austria, France, and Italy, thus testifying to the wide interest in the treatise in medieval times. Arabic versions of the work are in manuscript form in Cairo and Algiers libraries. The work is largely an expansion of and commentary on Book V of Euclid's *Elements*. Aḥmad developed and expanded Euclid's definitions of ratio and proportion in a long dialectic argument. Having clarified the meaning of these terms, he went on to show in great detail various methods for finding unknown quantities from given known quantities when the knowns and unknowns existed in certain proportional relationships.

By applying the Euclidean definitions of composition, separation, alternation, equality, and repetition to the given proportional relationships, Aḥmad found eighteen different cases: six when there are three different quantities in the proportion, eight when there are four quantities, and four when there are six. The discussion and geometrical interpretation of these eighteen cases form the nucleus of the treatise. Since many of his proofs referred to variations on a single triangular figure, later authors have referred to his work as the eighteen cases of the divided figure.

Besides his obvious dependence on Euclid, Aḥmad acknowledged his indebtedness to Ptolemy. The latter part of the treatise on ratio and proportion is actually an extension of two lemmas from Book I, chapter 13, of Ptolemy's *Almagest*. Aḥmad also made reference to, and quoted from, Archimedes, Hero, Plato, Empedocles, and Apollonius, indicating that he was acquainted with at least some of their works.

Writing as he did at the beginning of the tenth century, not only was Aḥmad ibn Yūsuf profoundly influenced by his Greek predecessors, but also in his turn he exerted an influence on the works of several medieval mathematicians. Leonardo Fibonacci, in his *Liber abacci*, mentioned the work of Aḥmad (Ametus in the Latin form) in the eighteen cases of proportion, and he used Aḥmad's methods in the solution of tax problems. Some traces of Aḥmad's influence have been seen in the work of Jordanus de Nemore, *Arithmetica in decem libris demonstrata*. Aḥmad was cited as an authority by Thomas Bradwardine in his differentiating between continuous and discontinuous proportions. Pacioli listed Aḥmad (Ametus), along with such well-known scholars as Euclid, Boethius, Jordanus, and Bradwardine, as one of those whose work on proportions was of major significance.

On the somewhat negative side, Aḥmad was guilty of a grave logical error. Campanus of Novara, in his commentary on the definitions of Book V of Euclid's *Elements*, devoted considerable attention to Aḥmad's method of proof and pointed out a subtle but real bit of circular reasoning. In his eagerness to establish definitions and postulates. Aḥmad did, at one point in his treatise, accept as a postulate a principle that he later was to prove as a theorem. This logical error does not detract from the

value of his careful classification and solution of the various cases of proportional quantities. In fact, it is for this that he is remembered : his eighteen cases of the divided figure.

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