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(*d*. Tabrīz [?], Iran, 1320)

*optics, mathematics*.

Kamāl al-Dīn was the disciple of the famous Qutb al-Dīn al-Shīrāzī, mathematician, astronomer, and commentator on Ibn Sīnā.[1] Scholars since Wiedemann and Sarton have linked the names of the two, and some questions of priority have arise, as will be seen below. Although Kamāl al-Dīn produced a number of writings in different branches of mathematics—particularly arithmetic and geometry—his essential contribution was in optics. It was in response to a question addressed to him on the principles of refraction that al-Shīrāzī recommended to Kamāl al-Dīn that he study the *Kitāb al-manāẓir* ("Book of Optics") of Ibn al-Haytham. Once Kamāl al-Dīn had undertaken this study, al-Shīrāzī, who was at this time occupied in commenting on the *Canon* of Ibn Sīnā, suggested further that Kamāl al-Dīn write his own commentary of Ibn al-Haytham's book.

Kamāl al-Dīn chose to extend the task set him to other works of Ibn al-Haytham as well, so that his *Tanqīḥ al-manāẓir li-dhawi ʾl-abṣār wa'l-basāʾir* contains, in addition to the originally planned study of the *Kitāb al-manāẓir*, essays on Ibn al-Haytham's *The Burning Sphere, The Halo and the Rainbow, Shadows, The Shape of Eclipse*, and the *Discourse on Light*. He was also led, in the course of this work, to study Ibnal-Haytham's *The Solar Rays*, although he did not comment upon it. Kamāl al-Dīn was thus dealing with the essential optical works of Ibn al-Haytham, and with this group we must also consider his own work on optics, *Al-baṣāʾir fi ʿilm al-manāẓir* ("Insights Into the Science of Optics"). This is basically a textbook for students of optics, presenting the conclusions of the *Tanqīḥ* without the proofs or experiments.

In order to grasp the meaning and scope of Kamāl al-Dīn's contribution, it must first be understood that his work was more properly a revision (*tanqīḥ*) than a commentary (*sharḥ*), as the title itself indicates. To Kamāl al-Dīn "to comment" meant a reconsideration and reinterpretation, rather than the medieval notion of a return to the original sources for a more faithful reading. In the course of his revision, Kamāl al-Dīn did not hesitate to refute certain of Ibn al-Haytham's theories, such as the analogy between impact and the propagation of light, an essential element of the explanation of reflection and refraction. He further had no reluctance in developing other of Ibn al-Haytham's ideas, notably the example of the camera obscura, refraction in two transparent spheres, and the numerical tabulation of refraction (air to glass); indeed, from time to time he simply set aside Ibn al-Haytham's doctrine to substitute one of his own. An important instance is the theory of the rainbow.

This profound change in the notion of a commentary is directly attributable to the new stage reached by Ibn al-Haytham in his optics, which may be briefly characterized as the systematic introduction of new norms—mathematical and experimental—to treat traditional problems in which light and vision are united. Until then light had been considered to be the instrumentality of the eye and to see an object was to illuminate it. In order to construct a theory of light, it was necessary to begin with a theory of vision; but to establish a theory of vision required taking a position on the propagation of light. Each task immediately involved the other and each theory borrowed the language of the other. The optics of Aristotle, like that of Euclid and even that of Ptolemy, comprised both factors. In order to introduce the new norms systematically, a better differentiation forced itself on Ibn al-Haytham. But the distinction between seeing and illuminating had to allow the transfer of the notions of a physical doctrine to an experimental situation and thus to bring about a realization of the initial project.

The essential and most representative part of Kamāl al-Dīn's work, however, is his study of the rainbow. The question of Kamāl al-Dīn's originality here has been raised; recalling that Kamāl al-Dīn had borrowed the idea of studying the rainbow from his teacher, Carl Boyer writes, "Hence the discovery of the theory presumably should be ascribed to the latter [al-Shīrāzī], its elaboration to the former [Kamāl al-Dīn]."[2] Although the same notion is supported by Crombie and many subsequent authors, it remains unconvincing, despite a manuscript text on the rainbow attributed to al-Sahīrāzī (at the end of his commentary on Ibn Sīnās *Canon*, in a manuscript Kept at Paris). The manuscript, written before 1518, is incomplete, and the next dealing with the rainbow occurs after several pages on alchemy that are irrelevant to the rest of the book, and in a different hand. The next on the rainbow itself is in yet another hand; after examining this manuscript and comparing it with one of the same book in the National Library at Cairo, Naẓīf suggested that the passage is an interpolation.[3] The Cairo manuscript has in turn been compared with a complete version fo the dating from 1785.[4] In confirmation of Naẓīf's theory, this last altogether lacks the passage on the rainbow.

Even were this text on the rainbow to be accepted as being by al-Shīrāzī, no doubt would be cast on Kamāl al-Dīn's originality, since we have seen that Kamāl al-Dīn drew upon a new upon a new interpretation of Ibn al-Haytham's optics. The theory of the rainbow elucidated in the text in question deals with the reflection of light on droplets of water dispersed in the atmosphere, a traditional conception that does not agree with Kamāl al-Dīn's (although it is not too unlike al-Shīrāzī's, since the latter, following in the path of such geometers as al-Ṭūsī, was still concerned with visual rays). The disputed manuscript reveals a further fundamental difference from the work of Kamāl al-Dīn in its optical terminology.

Ibn al-Haytham, on the other hand, had in his discussion of the rainbow dealt specifically with the problem of reflection; that is, in order to explain the form of the arc, he had proposed that the light from the sun is reflected on the could before reaching the eye. He sought the condition under which a ray emanating from a source of light—the sun—and reflected on a concave spherical surface, outside the axis, passes through the eye after its reflection. Admitting, as did the Aristotelian tradition before him, the possibility of a direct study of the arc, Ibn al-Haytham did not attempt to construct an experimental situation in order to verify the geometrical hypotheses. But the direct study of the rainbow did not lend itself to this sort of proof, even though Ibn al-Haytham called for it.

Kamāl al-Dīn took up Ibn al-Haytham's project at this point. Despite Ibn al-Haytham's authority, Kamāl al-Dīn began by submitting his predecessor's attempt to a severe criticism that, essentially, showed the need of a better physics which, when joined with geometry, would allow him to reach the goal formulated but unattained by Ibn al-Haytham.

Thus Kamāl al-Dīn returned to the doctrine of the rainbow proposed by Ibn Sīnā, who conceived of the are as being produced by reflection from a totality of the water droplets dispersed in the atmosphere at the moment when the coulds dissolve into rain. Ibn Sīnȧ's improvement justified an analogy—important for the explanation of the rainbow—between a drop of water and a transparent sphere filled with water.

Having stated the analogy, Kamāl al-Dīn wished to introduce two refractions between which one or several reflections occur. He benefited here from the results obtained by Ibn al-Haytham in *The Burning Sphere*, in which the latter showed that the paths followed by the light propagated between the two refractions are a function of the relationships of the increase in the angles of incidence and those of the increase in the angles of deviation.

Ibn al-Haytham established that for two rays to intersect inside the circle—that is, for the points of the second refraction to approach *O*'s instead of moving away from each other—it is necessary that $D' - D > 1/2\ (i' - i)$ (compare Kamāl al-Dīn's diagram, Figure 1). While it is true that this relationship is valid for the passage from air to glass, it can be easily demonstrated that it is independent of *n*. Drawing upon this relationship, however, Ibn al-Haytham was able to show by a simple geometric demonstration that the angle beginning with which this intersection occurs is 50° for the case in which $n = 3/2$ (from air to glass). This can be verified by the relation . It should be noted that Ibn al-Haytham thought that with the incident ray at 90°, the second point of refraction was on the same side of the axis as the point of the first refraction; this was not verified in the air-to-glass case that he was considering. In the water-to-air case that Kamāl al-Dīn studied, on the other hand, this was easily verifiable, so that in taking up Ibn al-Haytham's results, Kamāl al-Dīn did not encounter the same difficulty.

Kamāl al-Dīn thus considered the incident rays to be parallel to the axis *OO*'. These rays intersect the sphere at points increasingly removed from *O* and are refracted in it at points distant from *O*' on the opposite portion of the sphere up to the angle of incidence of 50°. For an angle of incidence greater than 50°. the points of the second refraction successively approach *O*'. Concerning the propagation of rays at their exit from the sphere, Ibn al-Haytham had already demonstrated spherical aberration.

With these results Kamāl al-Dīn attempted to show how, following double refraction in the sphere and depending on whether rays near to or distant from the axis are considered, one or several images of a Iuminous object as well as different from can be obtained—an arc or a ring in the case of a circular object. Before treating in detail double refraction in the sphere, however, Kamāl al-Dīn eliminated a difficulty resulting from the fact that, unlike the sphere, the drop does not have a glass envelope and that there are therefore four refractions, not two, in the sphere. In order to guarantee the correspondence between the manufactured object—the sphere—and the natural object—the drop of water—Kamāl

al-Dīn employed an approximation furnished by the study of refraction and justified by the consideration that the indexes of the two mediums are quite close, which allowed him, finally, to disregard the glass envelope.

Kāmal al-Dīn considered the circle of center $\gamma$ and the rays that form angles of incidence of 10°, 20°,..., 90° with it. He divided the rays into two groups. The first five form angles of incidence of less than 50°; the four others, of more than 50°. (See Figure 2.) He divided the are *DE* into two equal parts at *O*' and took *F* and *G* equidistant from *O*'. Let *SJ* be the ray with the angle of incidence 50° and *SJ*' its symmetric counterpart in relation to the axis *OO*'. These two rays are refracted along the lines *JE* and *J*' *D* and meet after the second refraction at point *A*, exterior to the sphere on the axis. Following the first refraction, all the rays of incidence of less than 50° are contained in the interior of the trunk of the cone generated by *JE* and *J*' *D*, called the "central cone" by Kamāl al-Diīn. Following the second refraction, these same rays are contained within the cone generated by *EA* and *DA*, the "burning cone." The rays that constitute the second group—with angle of incidence greater than 50°—are refracted, some between *JE* and *LG* and others symmetrically between *J*' *D* and *L*' *F*, which generate the two exterior cones, or "hollow cones." These rays are refracted a second time, some between *GB* and *EA* and some between *FC* and *DA*; they generate the exterior refracted cones or "hollow opposites." These rays intersect on the axis at points *H* and *A*.

At this stage, Kamāl al-Dīn's problem was to produce, under certain conditions, several possible images of the same object placed before the sphere. He could then vary their respective positions, causing them to become more distant from each other or superimposing them. Kamāl al-Dīn sought, in fact, to place himself outside what are today called Gauss's approximation conditions in order to produce this multiplicity of images.

He then returned to his model and complicated it with new, precise details. He examined the propagation of rays inside the sphere between two refractions and also treated the different types of reflection. Kamāl al-Dīn believed that a bundle of parallel rays falling on the drop of water is transformed, following a certain number of reflections in the sphere, into a divergent bundle. He knew, moreover, that the rays refracted in the drop of water after one or several reflections in its interior are not sent equally in all directions but produce a mass of rays in certain regions of space. This mass—and Kamāl al-Dīn's text allows no doubt on this point—is in the vicinity of the point of emergence of the ray which corresponds to the maximum (actually maximum or minimum) of deviation.[6] He stated, in addition, that the intensities of the lights join together, producing a greater illumination. He expressed these ideas in the complicated language of "cones" of rays that have been refracted after having undergone one or two reflections in the interior of the sphere and also in the concept of a greater illumination at the edges of the "cones." In the case of one reflection between two refractions, he distinguished two bundles of rays coming from the exterior cones and the central cone (see Figure 2); in the case of two reflections, he obtained two groups of rays that were more divergent than in the case of one reflection and that also gave one or two images. If the eye receives the rays coming from the central cone, Kamāl al-Dīn stated, a single image will be seen in a single position; and if the eye is placed in the region where the rays issuing from the central cone and the exterior cone intersect, two images will be seen in two positions.

In order to test the completed model, Kamāl al-Dīn employed an experimental procedure that was independently rediscovered by Descartes. He constructed a dark chamber with one opening, and placed inside it a transparent sphere illuminated by the rays of the sun. He masked half of the sphere with a dense white body and observed the face on the side toward the sphere: on it he saw an arc whose center was on the axis leading from the center of the sphere to the sun. This arc was formed from light rays that had undergone a refraction, a reflection, and another refraction. The inside of the arc was brighter than the outside because it contained rays emitted by both the central cone and the exterior cone. Kamāl al-Dīn next placed another white body, less dense than the first, before the sphere and again observed the face turned toward the sphere. This time he saw a complete ring that always displayed the colors of the rainbow. This ring was formed from the rays refracted a second time after having been reflected in the sphere. He noted the variation in the intensity of the colors according to the position of the screen, then employed the same dark chamber to consider the case of two reflections between two refractions.

This introduced into the study an important possibility that had not been considered then: the transfer through geometry of a physical doctrine of this phenomenon—essentially that of Ibn Sīnā—into the realm of experiment. It was in fact a question of restoration, contrary to Ibn al-Haytham, of the latter's own style of optics. The new optics promised to respect the norms of the combination of geometry and physics. But to follow the new norms with some

prospect of success necessarily led, in the case of a phenomenon as complicated as the rainbow, to the abandonment of direct study. This abandonment led to research on phenomena better mastered by the contemporary optical knowledge and more accessible to experimental verification—to the use of practical analogy. The analogue could be subjected to objective observation, and the resulting data applied to the study of the proposed natural object. Thus, Kamāl al-Dīn's spherical glass vial filled with water served to demonstrate the natural phenomenon of refraction.

On the problem of color, Kamāl al-Dīn turned to a commentary by al-Shīrāzī on the text of Ibn Sīnā's *Canon*[5]. His work soon began to diverge from its older model, however. In particular, Kamāl al-Dīn chose to treat four colors instead of three and to treat the problem of color by a reformulation of al-Shīrāzī's method. Kamāl al-Dīn set forth the doctrine of color, then limited its scope so as to consider only the colors formed on the screen in front of the sphere after the combination of reflections and refractions. He wrote:

The colors of the arc are different but related, between the blue, the green, the yellow, and the dark red, and come from a strong luminous source reaching the eye by a reflection or a refraction or a combination of the two [*Tanqīḥ* …, p. 337].

Thus varying the respective positions of the images in the different cones formed by the refracted rays, Kamāl al-Dīn declared that he perceived the different colors gradually as the two images were superposed. The bright blue was produced by the approach, without superposition, of two images; the bright yellow resulted from the superposition of two images; and the darkish red appeared at the edge of the bundle of rays. It was no longer, therefore— as in a traditional doctrine of color—the mixture of light and darkness that produced color, but the bringing together or the superposition of two or more images— or, still better, "forms"—of light on a background of darkness that explained the formation and diversity of colors.

Kamāl al-Dīn thought that he finally could explain how the rainbow should be observed. He showed that when the sphere was moved up and down along the perpendicular to the axis from the eye to the center of the sun (see Figure 3), then according to the position

of the sphere the image of the sun could be produced by simple reflection between two refractions. In other words, depending on the angle formed by the rays of sun meeting the sphere, the well-placed observer will perceive either the rays refracted after one reflection or the rays refracted after two reflections. Then the colors of the first arc and those of the second are obtained successively. It must be noted that Kamāl al-Dīn employed here—as elsewhere—the principle of reversibility. Thus he imagined the cones of the rays refracted after one or two reflections, by putting, in the first step, the light source where the eye had been. In the second step he reversed the situation in order to consider the displacement of the sun in relation to these cones of rays, the eye being returned to its initial position. He wrote:

Let us suppose that *B,* the center of the eye, is between *A*, the center of the sun, and *C*, the center of a polished transparent sphere. *ABC* is a straight line. Draw a perpendicular, *CD*, from *C* and suppose that the sphere is moved away from the line *ABC* in such a manner that its center remains on the perpendicular. If its center is moved away from *ABC*, the cone of rays refracted after one reflection will incline toward the sun while the latter, proportionally to the displacement of the sphere from *ABC*, will continue to approach the edge of the cone in the direction of the movement of the sphere and will appear in two images, at two positions on the sphere. … To the extent that the sphere is displaced, the two images draw closer until they become tangent. It is then that the light becomes stronger and produces an *isfanjānī* blue if it blends with the darkness or with the green. If the images then interpenetrate, the light is again intensified and produces a bright yellow. Next, the blended image diminishes and becomes a darker and darker red until it disappears when the sun is outside the cone of rays refracted after one reflection.

If the sphere continues to become more distant from the line *ABC*, the cone of rays refracted after two reflections approaches closer and closer to the sun until the sun is contained within this cone, and then what had disappeared in the beginning reappears in inverse order, beginning with the purple red, then the bright yellow, then the pure blue, and finally a light that is not really perceived because of the disappearance of one of the images or because of their mutual separation. If there are a great many drops of water massed in the air, these, arranged in a circle—each drop giving one of the images mentioned according to its size—produce the image of two arcs, as one may see: the small one is red on its exterior circumference, then yellow, then blue. The same colors appear in inverse order on the superior arc, hiding what is behind it by the colors and lights that appear in it. The air between the two arcs is darker than the air above and below them, because the portions between the two arcs are screened from the light of the sun [*Tanqīḥ* …, pp. 340–342].

In order to bring the combination of geometry and physics as in Ibn al-Haytham's optics to the study of the rainbow—that is, to arrive at a valid proof through geometrical deduction and experimental verification— Kamāl al-Dīn was led to reject as a starting point the notion of direct study, used by Ibn al-Haytham and by a whole tradition. He therefore elaborated a mode of explanation by reduction by establishing a group of correspondences between a natural object and a synthetic object, which he then systematically reduced by the geometry of the propagation of light in the first object to its propagation in the second.

Appearing in the wake of Ibn al-Haytham's reform, this achievement was a means of extending that reform to an area where it was not yet operative. It is in this way that the importance of Kamāl al-Dīn's contribution is to be understood.

It remains for us to consider Kamāl al-Dīn's work on the rainbow in conjunction with that of Dietrich von Freiberg. Dietrich's *De iride et radialibus impressionibus* was written between 1304 and 1311; [6] Krebs found the direct influence of Ibn al-Haytham in this work: "However, it seem very likely," he wrote, "that Dietrich used fully the great work of the Arabic father of modern optics. …"[7] Würschmidt, too, stated, "that Dietrich, by his own testimony, used in the treatment of this problem of the rainbow … the optics of Ibn al-Haytham."[8]

Wiedemann concluded that Kamāl al-Dīn completed the definitive version of his work between 1302 and 1311,[9] during Dietrich's lifetime. In support of this thesis he offered the arguments that the book was written during al-Shīrāaī's lifetime (that is, before 1311), and that in it Kamāl al-Dīn refers to a lunar eclipse that, according to Wiedemann, occurred in 1302. This evidence has been accepted by other historians; Naẓīf, however, took exception to it.

In his research on the rainbow included in the appendix to the *Tanqiḥ, al-Fārisī* [Kamāl al-Dīn] borrowed from al-Shīrāzī's commentary to the *Canon* the latter's conception of the manner in which colors originate; the passage containing al-Shīrāzī's remarks clearly indicates that the commentary had not been completed. This is tantamount to saying that al-Fārisī had completed the *Tanqiḥ* before al-Shīrāzī finished the commentary to the *Canon*. As for the lunar eclipse that Wiedemann emphasizes, if the year 1304 is accepted for its occurrence (Wiedemann gives 1302), the fact remains that the eclipse is not mentioned either in the main portion of the *Tanqiḥ*, in its conclusion, or in the appendix. The eclipse is referred to only in al-Fārisī's commentary on one of Ibn al-Haytham's treatises that al-Fārisī appended to his own book. This is *Shadows*, and it is conceivable that these treatises were added to the book after publication or that the reference to the eclipse was added at a later date.

At least one can speculate; I do not believe that it is mistaken to say that al-Fārisī had completed the research on which he would base his two theories of the rainbow before al-Shirazi had finished his commentary. This is not to generalize and include the entire *Tanqiḥ*— corpus, conclusion, appendixes, and excursus—in this chronology. Thus, I am not suggesting what is probable but, rather, what is certain, in alleging that al-Fārisī had completed the research on the rainbow that is included in the appendix to the *Tanqiḥ* at least ten years before Theodoricus [Dietrich] wrote his treatise between the years 1304 and 1311 [M. Naẓīf, "Kamāl al-Dīn al-Fārisī …," p. 94].

Naẓīf went on to posit the possibility of Kamāl al-Dīn's influence upon Dietrich. Such influence would seem tenuous at best, however; no trace has been found of Kamāl al-Dīn's work in Latin, and Dietrich himself did not cite him. The influence of Ibn al-Haytham on Dietrich is another matter. As Würschmidt wrote:

… a comparison of these works [those of Kamāl al-Dīn] with those of Master Dietrich indicates that the latter definitely did not know Kamāl al-Dīn's commentary; Kamāl al-Dīn avoided a succession of errors which occur with Dietrich as well as with earlier Arab scholars, and saw clearly especially the returned rays so important later in Descartes's rainbow theory.[10]

It may thus be seen that Kamāl al-Dīn's priority in no way implies his influence upon Dietrich, but, rather, that both Kamāl al-Dīn and Dietrich were disciples of Ibn al-Haytham and, relying upon the same source for their essential ideas, independently arrived at the model of the transparent sphere to explain the rainbow.

# NOTES

1. See R. Rashed, "Le modèle de la sphère …" p. 114.

2. *The Rainbow: From Myth to Mathematics. P*. 125.

3. The Paris MS is Bib. Nat., Fonds arabe, MS 2517; That at Cairo, written in 1340 at Mossoul, is National Libray MS 7797.

4. paris, Bib. Nat., Fonds arabe, MS 2518.

5. *Tanqīḥ*, p, 331. *et seq*.

6. See E. Krebs, "Meister Dietrich (Theodoricus Teutonicus de Vriberg), sein Leben, seine Werke, seine Wissenschaft," in *BEiträge zur Geschichte der Philosophie und Theologie des Mittelalters,* V, pts. 5–6 (Münster in Westfalen, 1906), 105 ff.; and P. Duhem, *Le système du monde*, III (paris, 1915), 383 ff.

7. See Krebs, *op. cit*., p. 40.

8. "Dietrich con Freiberg; Über den Regenbogen und die durch Strahlen erzeugten Eindrücke," in *Beiträge zur Geschichte der Philosophie und Theologie des Mittelalters,* XII, pts. 5–6 (Münster in Westfalen, 1906), p. 1.

9. "Zu Ibn al-Haitams Optik," in *Archiv Für die Geschichte der Naturwissenschaften und der Technik,* no. 3 (1912), pp .3-4.

10. *Op. cit*., p. 2, note 8.

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Works still in MS are *Al-Basāir fī 'ilm al-manāẓir fi'lhikma; Asās al-gawā'id fi uṣūl al'fawa'id; Taḍhirat al-aḥhāb fi bayān al-tahābb*; and "Treatise on a Geometrical Proposition of Nasir al-Din al-Tūsi." See C. Brockelmann, *Geschichte der arabischen Literature*, supp. II (Leiden, 1938), p. 295; and H. Suter, *Die Mathematiker und Astronomen der Araber und ihre Werke* (Leipzig, 1900), p. 159.

II. Secondary Literature. On Kamāl al-Dīn or his work, see Carl Boyer, *The Rainbow: From Myth to Mathematics* (New York, 1959), pp. 127–129; M. Schramm, "Steps Towards the Idea of Function: A Comparison Between Eastern and Western Science of the Middle-Ages," in *History of Science*, **4** (1956), 70–103, esp. 81–85; M. Naẓif, "Kamāl al-Dīn al-Fārisī wa ba'ḍ Buḥūṭuhu fi 'ilm al-ḍaw', in *La sociètè ègyptienne et histoire des sciences*, no. 2 (Dec. 1958), 63–100 (in Arabic); R. Rashed, "Le Modèle de la sphère transparente et l'explication de l'arc-en-ciel: Ibn al-Haytham—al-Fārisī" in *Revue d'histoire des sciences*, **22** (1970), 109–140; and J. Würschmidt, "Über die Brennkugel," in *Monatshefte für den naturwissenschaftlichen Unterricht*, **4** (1911), 98–113

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