Al-Samaw'al, Ibn Yah?ya Al-Maghribi | Encyclopedia.com

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(*b*. Baghdad, Iraq; *d*. Marāgha, Iran, [?] *ca*. 1180)

mathematics, medicine.

Al-Samaw'al was the son of Yehuda ben Abun (or Abu'l-'Abbās Yaḥya al-Magribī), a Jew learned in religion and Hebrew literature who had emigrated from Fez (Morocco) and settled in Baghdad. His mother was Anna Isaac Levi, an educated woman who was originally from Bạra (Iraq). al-Samaw'al thus grew up in a cultivated milieu; a material uncle was a physician, and after studying Hebrew and the Torah the boy was encouraged, when he was thirteen, to take up the study of medicine and the exact sciences. He then began to study medicine with Abu'l-Barakāt, while taking the opportunity to observe his uncle's practice. At the same time he started to learn mathematics, beginning with Hindu computational methods, *zījes* (astronomical tables), arithmetic, and *misāḥa* (practical techniques for determining measure, for use in surveying), then progressing to algebra and geometry.

Since scientific study had declined in Baghdad, al-Samaw'al was unable to find a teacher to instruct him beyond the first books of Euclid's *Elements* and was therefore obliged to study independently. He finished Euclid, then went on to the *Algebra* of Abū Kāmil, the al-Badī' of al-Karajī, and the *Arithmetic* of al-Wasītī (most probably Maymūn ibn Najib al-Wasītī, who collaborated in making astronomical observations with Umar al-Khayyāmī between 1072 and 1092). By the time he was eighteen, al-Samaw'al had read for himself all of the works fundamental to the study of mathematics and had developed his own mode of mathematical thinking.

In science, this independence of thought led al-Samaw'al to point out deficiencies in the work of al-Karaj̈ (whom he admired) and to challenge the arrangement of the *Elements*; in religion he was similarly inclined to test the validity of the claims of the various creeds and came to accept those of Islam, although he postponed his conversion for a number of years to avoid distressing his father. His autobiography states that he reached his decision at Marāgha on 8 November 1163 as a result of a result of a dream: four years later he wrote to his father, setting out the reasons for which he had changed his religion, and his father immediately set out to Aleppo to see him, dying en route. al-Samaw'al himself spent the rest of his life as an itinerant physician in and around Marāgha. His earlier travels had taken him throughout Irag, Syria, Kūhistān. and Ādharbayjān.

His biographers record that al-Samw'al was a successful physician, and had emirs among his patients. In his autobiography al-Samaw'al recorded that he had compounded several new medicines, including an almost miraculous theriac. but no other account of them has survived. His only extant medical work is his *Nuzhat al-ashāb* (usually translated as "The Companions' Promenade in the Garden of Love"), which is essentially a treatise on sexology and a collection of erotic stories. The medical content of the first and longer section of the book lies chiefly in descriptions of diseases and sexual deficiencies; the second, more strictly medical, part includes a discussion of states of virile debility and an account of diseases of the uterus and their treatment. This part is marked by al-Samaw'al's acute observation and his interest in the psychological aspects of disease; he provides a detailed description of the condition of being in love without recognizing it, and gives a general prescription for the anguished and melancholic that comprises well-lighted houses, the sight of running water and verdure, warm baths, and music.

It is, however, chiefly as a mathematician that al-Samaw'al merits a place in the history of science. His extant book on algebra, Al-bāhir ("The Dazzling"), written when he was nineteen years old, represents a remarkable development of the work of his predecessors. In it al-Samaw'al brought together the algebraic rules formulated by, in particular, al-Karajī and, to a lesser extent, Ibn Aslam and other authors. including al-Sijzī, Ibn al-Haytham. Qustā ibn Lūqā, and aḷ-Harīrī. The work consists of four parts. of which the first provides an account of operations on polynomials in one in known with rational coefficients; the second deals essentially with second-degree equations, indeterminate analysis, and summations; the third concerns irrational quantities; and the fourth, and last, section presents the application of algebraic principles to a number of problems.

It is apparent in the first section of this work that al-Samaw'al was the first Arab algebraist to undertake the study of relative numbers. He chose to treat them as if they possessed an identity proper to themselves, although he did not recognize the significance of this choice. He was thus able, in a truly bold stroke. to subtract from zero, writing that

If we subtract and additive [positive] number from an empty power ($0\cdot x^n - a\cdot x^n$), the same subtractive [negative] number remains; if we subtract the subtractive number from an empty power, the same additive number remains ($0\cdot x^n-[-ax^n]=ax^n$)... . If we subtract and additive number from a subtractive number, the remainder is their subtractive sum: $(-ax^n)-(bx^n)= -(a+b)\,x^n$; if we subtract a subtractive number from a greater subtractive number, the results is their subtractive difference; if the number from which one subtracts is smaller than the number subtracted, the result is their additive difference.

These rules appear in the later European work of Chuquet (1484), Pacioli (1494), Stifel (1544), and Cardano (1545): it is likely that al-Samaw'al reached them by considering the extraction of the square root of a polynomial.

Al-Karajī had conceived the algorithm of extraction, but did not succeed in applying it to the case in which the coefficients of subtractive. His failure may have stimulated al-Samaw'al's abilities, since the problem in which these rules are stated in the *Al-bāhir* is that of the extraction of the square root of

$25x^6+9x^4+84x^2+64+100(1/x^2)+64(1/x^4)$
$-30x^5-40x^3-116x-48x^3(1/x)-96(1/x^3)$.

Since al-Karajīs algebra lacked symbols, so that the numbers had to be spelled out in letters, this operation would have presented an insurmountable obstacle. Al-Samaw'al was able to overcome this trial of memory and imagination by using a visualization in which he assigned to each power of x a place in a table in which a polynomial was represented by the sequence of its coefficients, written in Hindu numerals. This technique, a major step in the development of symbolism, was requisite to the progress of algebra because of the increasing complexity of mathematical computations.

Al-Samaw'al's rules of subtraction were also important in the division of polynomials; in the interest of obtaining better approximations he pursued division up to negative powers of x, and thereby approached the technique of development in series (although he overlooked the opportunity of unifying the various cases of the second-degree equation and of computation by double error). He computed the quotient of $20x^2+30x$ by $6x^2+12$, for example, to obtain the result $3\ 1/3+5(1/x)-6\ 2/3(1/x^2)-10(1/x^3)+13\ 1/3(1/x^4)+20(1/x^3-26\ 2/3(1/x^6)-40(1/x^7)$.He then recognized that he could apply the law of the formation of coefficients $a_{n+2}=-2a^n$, which allowed him to write out the terms of the quotient directly up to $54{,}613.\ 1/3\ (1/x^{28})$.

Al-Samaw'al further applied the rules of subtraction to the multiplication and division of the powers of x, which he placed in a single line of both sides of the number 1, to which he assigned the rank zero. The other powers and other constants are displayed on each side of zero, in ascending order:

The rules of multiplication and division that al-Samaw'al enunciated are, except for their notation, those still in use.

The second part of the *Al-bāhir* contains the six classical equations ($ax=b$, $ax^2+bx=c$, and so forth) that were set out by al-Khwāriznī. Interestingly, however, al-Samaw'al gave only geometrical demonstrations of the equations, although their algebraic solutions were known to his predecessor al-Karajī, who dedicated a monograph, *'llaḷ hisāb al-jabr wa 'l-muqābala*, , to them. Al-Samaw'al then presented a remarkable calculation of the coefficients of $(a+b)^n$, which al-Karajī discovered after 1007 (for the dating of this and other of al-Karajī's works, see A. Anbouba, ed., *L'salgèbre al-Badī' d'al-karagi* [Beirut, 1964], p. 12). Since al-Karajī's original computation has been lost, al-Samaw'al's work is of particular interest in having preserved it: these coefficients are arranged in the triangular table that much later became known in the west as Tartaglia's or Pascal's triangle.

A further, and equally important, part of the second section of the book deals with number theory. This chapter contains about forty propositions, including that among n consecutive integers there is one divisible by n; that

$1\cdot2+3\cdot4+\cdots+(2n-1)\ 2n=1+3+\cdots$
$+(2n-1)+1^2+3^2+\cdots+(2n-1)^2$;

and that $1^2+^2+\cdots+n^2=n(n+1)(2n+1)/6$. Al-Samaw'al was especially proud of having established the last, since neither Ibn Aslam nor al-Karajī had been successful in doing so.

The chief importance of the second part of the *Al-bāhir* lies, however, in al-Samaw'al's use of recursive reasoning, which appears in such equations as

The third part of Al-bāhir is chiefly concerned with the classification of irrationals found in Book X of the Elements. Al-Samaw'al's account is complete and claer, but contains nothing new or notworthy except his rationalization of which had eluded al-Karajī.

The final section of the work contains a classification of problems by the number of their known solutions, a device used by earlier writers. Al-Samaw'al was led by this procedure to solve a varied group of these problems and to master a prodigious system of 210 equations in ten unknowns, a result of his having undertaken the determination of ten numbers of which are given their sums, taken six at time. He further elucidated the 504 conditions necessary to the compatibility of the system. He overcame the lack of symbolic representation by the expedient of designating the unknown quantities 1,2,3,⋯,10, and was then able to draw up a table that started from

123456···65

123457···70

123458···75

123459···80.

Al-Samaw'al's intention in writing the Al-bāhir was to compensate for the deficiencies that he found in al-Karajī's work and to provide for algebra the same sort of systematization that the *Elements* gave to geometry. He wrote the book when he was young, then allowed it seems quite possible that he reworked it a number of times. It is difficult to ascertain the importance of the book to the development of algebra in the Arab world, but an indirect and restricted influence may be seen in the *Miftaḥ-al-ḥisab* ("Key of Arithmetic") of al-Kāshī, published in 1427. The book was apparently altogether unknown in the west.

The mathematical counterpart of the Al-bāhir, the *Al-zāhir* ("The Flourishing"), has been lost, and of al-Samaw'al's mathematical writings only two almost identical elementary treatises remain. These are the *Al-tabsira* ("Brief Survey") and *Al-mūjiz* ("Summary"). The influence of the Arabic language is seen in their classification of fractions as deaf. Arab, or genitive, and both contain sections on ratios that are clearly derived from the work of al-Karajī. The ratio 80:3·7·9·10, for example, is expressed as a sum of fractions with numerator 1 and the respective denominators 3·10, 3·7·9, and 3·9·10; this use of the sexagesimal system reflects the containing importance of the concerns of the commercial and administrative community of Baghdad, since this system was still favored by merchants and public servants. In this account of division, al-Samaw'al noted the periodicity of the quotient 5:11, also calculated in the sexagesimal system. The last section of *Al-mūjiz* exists only in mutilated form, but what remains would indicate that it contained interesting material on abacuses.

In an additional work related to his mathematics al-Samaw'al again demonstrated the independence of his thinking. In this, the Kashf 'uwār al-munajjimīn ("The Exposure of the Errors of the Astrologers"), he refuted the pronouncements of scientific astrology by pointing out the multiple contradictions in its interpretation of sidereal data, as well as the errors of measurement that he found in astrological observations. He then for the sake of argument, assumed astrology to be valid, but showed that the astrologer could scarcely hope to make a valid prediction since, by al-Samaw'al's count, he would have to take into simultaneous consideration 6,817 celestial indicators, a computation that would surely exceed his abilities.

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Al-Samaw'al's only surviving medical work, *Nuzhar al-asḥāb fi mu'āsharat al-aḥbāb* ("The Companions' Promenade in the Garden of Love"), is preserved in Berlin MS 6381, Paris MS 3054, Gotha MS 2045, Istanbul MS Aya Sofya 2121, and Escorial MS 1830. His other extant works are apologetics, and include *Ifhām tāifal al-yahūd* ("Confutation of the Jews"), Cairo MS Cat. F. Sayyid, I, p. 65, Cairo MS VI, ii. and Teheran MS I. 184, and II 593, of which the text and an English trans. with an intersting introduction by M. Perlmann are *Proceedings of the American Academy for Jewish Research*. **32** (1964); and *Ghāyat al-maqsūd fi 'l-radd ala 'l-nasārā wa 'l-yahūd* ("Decisive Refutation of the Christians and the Jews"), Istanbul MS As'ad 3153 and Asir 545. A last MS. *Badhl al-majhūd fi iquā al-yahūd* ("The Effort to Persuade the Jews"), formerly in Berlin, has been lost since World War II. See also Perlmann (above). pp. 25-28, 127).

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Adel Anbouba