Al-Tusi, Sharaf Al-Din Al-Muzaffar Ibn Muhammad Ibn Al-Muzaffar | Encyclopedia.com

(*b*. Tus [?], Iran;

d. Iran, c. 1213–1214), *astronomy, mathematics*. For the original article on al-Tūsī, see *DSB*, vol. 13.

Al-Tusi is best known for his mathematically impressive study of the conditions under which cubic equations have a positive real root and of numerical methods for finding a solution of such equations. He was, according to the thirteenth-century biographer Ibn Abī Usaibi'a, "outstanding in geometry and the mathematical sciences, having no equal in his time." In 1986 Roshdi Rashed published *Oeuvres Mathématiques* (2 vols.), containing the extant mathematical works of Sharaf al-Dīn with an edited Arabic text and French translation and commentary. These works are his *Equations*, his *On the Construction of a Geometric Problem*, and his *On Two Lines that Approach Each Other but Do Not Meet* (a treatise on the asymptotes of the hyperbola).

Al-Tūsī Solutions of Equations. Sharaf al-Din organized his treatment of the twenty-five possible cubic equations around whether the equation has a positive real root. (Since any cubic equation with real coefficients must have at least one real root, the discussion hinges on whether that root is positive.) Accordingly, the five cases in which there may be no root form the last group he discussed.

Sharaf al-Din brought new ideas even to the solution of quadratic equations. For example, in discussing the equation $ax + b = x^2$, he noted that making the substitution x = X + a changes the equation to one of the form he has previously explained, that is, $x^2 + ax = b$. This use of such substitutions served him well in a number of places, especially in the case of cubic equations.

From his description of the algorithms for solving the equations, it is clear that Sharaf al-Din assumed the reader would be doing the work on a *takht* (dust board), where one can conveniently write only a very few rows of numbers. The algorithms themselves are based on the idea that if $f(x) = c$ is a polynomial equation with root $a_n 10^n + b$, where b is a real number whose integer part is of order less than 10^n, then $f(a_n 10^n + x)$ has root b. The procedures then consist of finding $a_n 10^n$ and using the algorithms to calculate the coefficients of the latter polynomial.

Sharaf al-Din's method for finding $a_n 10^n$ succeeds (in most cases) because he was working with cubic equations. (The solutions in all cases are the same number, 321.) And, following his description of the algorithm, Sharaf alDin justified in each case the positions of the coefficients as well as the algorithm he followed. These efforts to justify a wide class of numerical procedures show a new side of mathematics in medieval Islam. Sharaf al-Din, in his analysis of the equation $x^3 + c = ax^2$, showed first the obvious fact that if x is a real root then *x*<#60;*a*, and he then showed that $x^2(a - x) = c$. He thought of c as representing a solid and showed that it is a rectangular parallelopiped with base x^{2xs} and height $(a - c)$. He then showed that if , then the solid represented by $x^2(a - x)$ is as large as possible. Hence, he knew that if the equation has a solution, then c cannot exceed the maximum, In the case where c is equal to the maximum, he knew, of course, that is the unique root of the equation,

and when there are two roots, one in the interval and one in the interval (, a).His proof that the maximum value is attained when $x = 2a/3$ is an exercise in manipulation of volumes and areas that would be straightforward for a mathematician of Sharaf al-Din's caliber who had studied *Elements*, II.

How Sharaf al-Dīn discovered such conditions, however, is a subject on which historians have disagreed. The present writer inclines toward J. P. Hogendijk's suggestion that Sharaf al-Dīin arrived at the defining condition for the point where $f(x)$ obtained its maximum by assuming that x was such a point and then studying $f(x) - f(y)$ for y on either side of x to find a sufficient condition so that this difference is positive in both cases.

As it turns out, these two conditions force x to satisfy a quadratic equation that is equivalent (in modern terms) to setting the derivative, $f'(x)$, equal to 0. Other restorations of Sharaf al-Dīn's analysis of the equations appear in Rashed's *Oeuvres Mathématiques* and in Ali A. Daffa and John J. Stroyls's *Studies in the Exact Sciences in Medieval Islam*(1984).

Al-Tūsī Linear Astrolabe. Sharaf al-Din was also the inventor of a linear astrolabe, a single rod with markings on it (sometimes called "the rod of al-Tūusī"). The rod represents the meridian of the planispheric astrolabe, and two threads attached to it, with movable beads on them, can be positioned at various points along the rod to serve in place of the rete (the top plate in the usual planispheric astrolabe, whose pointers indicate the position of certain prominent stars). The rod has a number of scales, one of which represents the intersections of the altitude circles with the meridian. Another represents the intersections with the meridian of concentric circles that are the stereographic projections of the circles containing the beginnings of the zodiacal signs. (See Figure 1.)

Sharaf al-Dīn's student, Kamāl al-Dīin Ibn Yūnus, improved his teacher's linear astrolabe, and AbūAlī alMarrākūshī wrote directions on how to use it. Although the instrument was known in al-Andalus (the Arab-controlled areas of the Iberian Peninsula), it was not, apparently, transmitted to the rest of Europe. Nor was it very popular in medieval Islam; Najm al-Dīn al-Misrī gave a very confused discussion of it in his *Treatise on Astronomical Instruments*.

Al-Tūsī on Geometrical Dissection. Sharaf al-Dīn's connection with the practical arts in Islam is witnessed by a problem that may have originated in his experience teaching the carpenter, Abūal-Fadl, in Damascus, who, according to Ibn Abū Usaibi'a, had become a devoted student of the writings of Euclid and Ptolemy. (Although the latter did not say so, the fact that Abu al-Fadl had studied the works at the beginning and end of the standard curriculum of medieval Islamic instruction in the exact sciences

indicates that he would also have studied the intermediate works as well, such as Euclid's *Phenomena* and Theodosius's *Spherics*.)

The problem Sharaf al-Dīn addressed is that of dissecting a square ABGD of given side AB into one rectangle and three trapezia so that the areas have given ratios to each other. And he solved the problem for the case when the trapezea (beginning from the top and working clockwise in Figure 1) have the ratios 3, 5, and 2 to the rectangle.

The relevance of this kind of problem to problems that a craftsman would encounter in doing medieval Islamic tilings of plane surfaces is clear. That the solution would be of purely theoretical interest as well, however, is shown by Sharaf al-Din's requirement in the construction that a certain line segment be 3,024 times another segment.

In another short work, titled "On Two Lines that Never Meet," al-Tusi demonstrates that a hyperbola and its asymptotes, however far they are extended, never meet. This property was well-known to medieval Islamic geometers and al-Tusi's reasons for writing the treatise are unclear.

SUPPLEMENTARY BIBLIOGRAPHY

All three of Sharaf al-Dīn al-Tūsī's extant mathematical works can be found in the Oeuvres Mathématiques, *edited and translated by Roshdi Rashed. No archive of his work exists; virtually no original manuscripts are extant, and the available medieval copies are spread across a number of libraries in the Islamic and western worlds.*

WORKS BY SHARAF AL-DīN AL-TūSī'S

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