Al-Umawi, Abu ‘Abdallah Ya‘ish Ibn ibraHim Ibn Yusuf Ibn Simak Al-Andalusi | Encyclopedia.com

---

fl. Damascus, fourteenth century),

arithmetic.

Al-Umawī was a Spanish Arab who lived in Damascus, where he taught arithmetic. On the single authority of Ḥājjī Khalīfa, the year of his death is usually given as a.h. 895 (a.d. 1489/1490). But a marginal note on the ninth folio of his arithmetic (MS 1509, 1°, Carullah), written by him to give license to a copyist to teach his work, is dated 17 Dhu’1-Hijja 774 (9 June 1373). The copyist is Abd al-Qādir ibn Muhammad ibn ‘Abd al-Qādir, al-Hanbalĩ, al-Maqdisĩ. He states that he finished copying the text at Mount Qāsyūn in Damascus on 8 Dhu’1-Hijja 774.

The text referred to is *Marāsim al-intisāb fi’ilm al-hisāb fi’ilm al-ḥisāb*. A small work in eighteen folios, it is significant in being written by a western Muslim for Easterners, a circumstance that should not discredit the common belief that arithmetic flourished more in eastern than in western Islam. The work represents a trend of Arabic arithmetic in which, as early as the tenth century, the Indian “dust board” calculations had begun to be modified to suit paper and ink; and arithmetic was enriched by concepts from the traditional finger reckoning and the Pythagorean theory of numbers. The trend seems to have started in Damascus; the earliest extant text that shows it is al-Uqlĩdisĩ’s *al-Fusṭl fi’l-hisāb al-hindĩ,* written in a.h. 341 (a. d. 952/3). But there are reasons to believe that the trend had greater influence in the West than in the East.

The forms of the numerals used in the West differed from those in the East, but al-Umawĩ avoids using numerals except in a table of sequences, in which the western forms appear. The attempts to modify the Indian schemes resulted in several methods, especially of multiplication. Al-Umawĩ, however, says little about these methods and describes the principal operations briefly, as if his aim is to show what in western arithmetic is unknown, or not widely known, in the East. Thus he insists that the common fraction should be written as , whereas the easterners continued to write it as , like the Indians, or as .

He also insists that the numbers operated upon, say, in multiplication, must be separated from the steps of the operation by placing a straight line under them. Such lines appear in the works of Ibn al-Bannā of Morocco (*d*. 1321) but not in the East until late in the Middle Ages.

Like the classical Indian authors, in treating addition al-Umawī dispenses with the operation in a few words and moves on to the summation of sequences. Those he discusses are the following:

1. The arithmetical progression in general and the sum of natural numbers, natural odd numbers, and natural even numbers in particular

2. The geometrical progression in general and 2^r and in particular

3. The sequences and series of polygonal numbers, namely {1 + (r - 1 d} and

4. The sequences and series of pyramidal numbers, namely {S_r and

5. Summations of r^3, $(2r + 1)^3$, $(2r)^3$ from $r = 1$ to $r = n$

6. Summations of $r(r + 1)$. $(2r + 1)$ $(2r + 3)$, $2r(2r + 2)$ from $r = 1$ to $r = n$.

The sequences of polygonal and pyramidal numbers were transmitted to the Arabs in Thābit ibn Qurra’s translation of Nicomachus’ *Introduction to Arithmetic*. Also, al-Karajĩ had given geometrical proofs of $\Sigma\, r^3$, $(2r + 1)^3$, $(2r)^3$ in al-Fakhrī (see T. Heath, *Manual of Greek Mathematics* [Oxford, 1931], 68).

Without symbolism, al-Umawī often takes the sum of ten terms as an example, a practice started by the Babylonians and adopted by Diophantus and Arabic authors.

In subtraction al-Umawī considers casting out sevens, eights, nines, and elevens. All Hindu-Arabic arithmetic books consider casting out nines; and some add casting out other numbers. Some also treat casting out elevens in the way used today for testing divisibility by 11, which is attributed to Pierre Forcadel (1556). Al-Umawī adds casting out eights and sevens, in a way that leads directly to the following general rule:

Take any integer N in the decimal scale. Clearly $N = a_0 + a_1 \cdot 10 + a_2 \cdot 10^2 + \ldots = \Sigma\, a_8 \cdot 10^8$ It is required to find the remainder after casting out p's from N, where p is any other integer. Let r_8 be the remainder of 10^8, that is $10^8 \cong r_8 \pmod{p}$; it follows that if $\Sigma\, a_8 \cdot r_8$ is divisible by p so is N. This is a theorem that is attributed to Blaise Pascal (1664); see L. E. Dickson, *Theory of Numbers*, **I** (New York, 1952), p. 337.

In the text al-Umawā states that the sequence r_8, in the cases he considers, is finite and recurring. Thus for $p = 7$, $r_8 = (1, 3, 2, 6, 4, 5)$.

In dealing with square and cube roots, al-Umawī states rules of approximation that are not as well developed as those of the arithmeticians of the East, who had already developed the following rules of approximation.

where a^2 is the greatest integral square in n, and

where a^3 is the greatest integral cube in n. These rules do not appear in al-Umawī's test. Instead, we find

or

or

Again, al-Umawī does not consider the method of extracting roots of higher order, which had been known in the East since the eleventh century.

For finding perfect squares and cubes, however, he gives the following rules, most of which have not been found in other texts.

If n is a perfect square:

1. It must end with an even number of zeros, or have 1, 4, 5, 6, or 9 in the units' place.

2. If the units' place is 6, the tens' place must be odd: in all other cases it is even.

3. If the units' place is 1, the hundreds' place and half the tens' place must be both even or both odd.

4. If the units' place is 5, the tens' place is 2.

5. $n \cong 0, 1, 2, 4 \pmod{7}$

$\cong 0, 1, 4 \pmod{8}$

$\cong 0, 1, 4, 7 \pmod{9}$

If n is a perfect cube:

1. If it ends with 0, 1, 4, 5, 6, or 9, its cube root ends with 000, 1, 4, 5, 6, or 9, respectively. If it ends with 3, 7, 2, or 8, the root ends with 7, 3, 8, or 2, respectively.

2. $n \cong 0, 1, 6, \pmod{7}$

$\cong 0, 1, 3, 5, 7, \pmod{8}$

$\cong 0, 1, 8 \pmod{9}$

Evidently al-Umawī's *Marāsim al-intisāb fi'ilm al-hisāb* is worthy of scholarly interest, especially in connection with the early history of number theory.

Another work by the same author is preserved in MS 5174 h in Alexandria under the name of *Raf' al-ishkāl fi misāhat al-ashkāl* (removal of doubts concerning the mensuration of figures); it is a small treatise of seventeen folios in which we find nothing on mensuration that the arithmeticians of the East did not know.

BIBLIOGRAPHY

On al-Umawī and his work, see C. Brockelmann. *Geschichte der arabischen Literatur*, supp. 2 (Leiden. 1938). p. 379, and **II** (Leiden, 1949), p. 344; L. E. Dickson. *History of the Theory of Numbers*, 3 vols. (New York, 1952); Hājjī Khalīfa, Kashf alzunţn . . . , 2 vols. (Constantinople, 1941); T. L. Heath, *A History of Greek Mathematics*, 2 vols. (Oxford, 1921); Ibn al-Nadīm, *Al-Fihrist* (Cairo); Nicomachus, *Al-Madkhal* ilā ʻ*ilm al-ʻadad*, Thābit ibn Qurra, trans. , W. Kutch, ed. (Beirut, 1958); and H. Suter, *Die Mathematiker und Astronomen der Araber und ihre Werke* (Leipzig, 1950), no. 453, p. 187.

A. S. Saidan