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(*b*. Angerburg, Germany [now Węgorzewo, Poland], 16 July 1819; *d*. Berlin, Germany, 13 March 1884)

mathematics.

Aronhold attended the Angerburg elementary school and the Gymnasium in Rastenburg (now Kętrzyn, Poland). Following the death of his father, his mother moved to Königsberg, where the boy attended a Gymnasium. He graduated in 1841 and then studied mathematics and natural sciences at the University of Königsberg from 1841 to 1845. Among his teachers were Bessel, Jacobi, Richelot, Hesse, and Franz Neumann. When Jacobi went to Berlin, Aronhold followed him and continued his studies under Dirichlet, Steiner, Gustav Magnus, and Dove. He did not take the state examinations, but in 1851 the University of Königsberg awarded him the *Doctor honoris causa* for his treatise “Über ein neues algebraisches Prinzip” and other studies.

From 1852 to 1854 Aronhold taught at the Artillery and Engineers’ School in Berlin and, from 1851, at the Royal Academy of Architecture in Berlin, where he was appointed professor in 1863. In 1860 he joined the Royal Academy for Arts and Crafts, where, when Weierstrass became ill in 1862, he took over the entire teaching schedule. He was appointed professor in 1864. In 1869 Aronhold became a corresponding member of the Academy of Sciences in Göttingen. He was considered an enthusiastic and inspiring teacher, and was held in high esteem everywhere.

Aronhold was particularly attracted by the theory of invariants, which was then the center of mathematical interest, and was the first German to do research in this area. The theory of invariants is not, however, connected with Aronhold alone—others who worked on it were Sylvester, Cayley, and Hesse—but he developed a special method that proved to be extremely successful. In 1863 he collected his ideas in a treatise entitled “Über eine fundamentale Begründung der Invariantentheorie.”

In this treatise, Aronhold offers solid proof of his theory, which he had welded into an organic entity. His method refers to functions that remain unchanged under linear substitutions. He stresses the importance of the logical development of a few basic principles so that the reader may find his way through other papers. Aronhold establishes his theory in general and does not derive any specific equations. He derives the concept of invariants from the concept of equivalency for the general linear theory of invariants. Special difficulties arise, of course, if not only general but also special cases are to be considered. His efforts to obtain equations independent of substitution coefficients led to linear partial differential equations of the first order, which also have linear coefficients. These equations, which are characteristic for the theory of invariants, are known as Aronhold’s differential equations.

With these equations “Aronhold’s process” can be carried out. This process permits the derivation of additional concomitants from one given concomitant. Aronhold investigates the characteristics of these partial differential equations and expands the theory to include the transformation of a system of homogeneous functions, furnishes laws for simultaneous invariants, and investigates contravariants (relevant forms), covariants, functional invariants, and divariants (intermediate forms).

Aronhold stresses that he arrived at his principles as early as 1851, citing his doctoral dissertation and the treatise “Theorie der homogenen Funktionen dritten Grades …” (1858). Since the subsequent theory and terminology did not yet exist, he claimed priority.

Before Aronhold developed his theory, he had worked on plane curves. The problem of the nine points of inflection of the third-order plane curve, which had been discovered by Plücker, was brought to completion by Hesse and Aronhold. Aronhold explicitly established the required fourth-degree equation and formulated a theorem on plane curves of the fourth order. Seven straight lines in a plane always determine one, and only one, algebraic curve of the fourth order, in that they are part of their double tangents and that among them there are no three lines whose six tangential points lie on a conic section.

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