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(*b*. Vienna, Austria, 3 March 1898; *d*. Hamburg, Germany, 20 December 1962)

mathematics.

Artin was the son of the art dealer Emil Artin and the opera singer Emma Laura-Artin. He grew up in Reichenberg, Bohemia (now Liberec, Czechoslovakia), where he passed his school certificate examination in 1916. After one semester at the University of Vienna he was called to military service. In January 1919 he resumed his studies at the University of Leipzig, where he worked primarily with Gustav Herglotz, and in June 1921 he was awarded the Ph.D.

Following this, he spent one year at the University of Göttingen, and then went to the University of Hamburg, where he was appointed lecturer in 1923, extraordinary professor in 1925, and ordinary professor in 1926. He lectured on mathematics, mechanics, and the theory of relativity. In 1929 he married Natalie Jasny. Eight years later they and their two children emigrated to the United States, where their third child was born. Artin taught for a year at the University of Notre Dame, then from 1938 to 1946 at Indiana University in Bloomington, and from 1946 to 1958 at Princeton. He returned to the University of Hamburg in 1958, and taught there until his death. He was divorced in 1959. His avocations were astronomy and biology; he was also a connoisseur of old music and played the flute, the harpsichord, and the clavichord.

In 1962, on the three-hundredth anniversary of the death of Blaise Pascal, the University of Clermont-Ferrand, France, conferred an honorary doctorate upon Artin.

In 1921, in his thesis, Artin applied the arithmetical and analytical theory of quadratic number fields over the field of rational numbers to study the quadratic extensions of the field of rational functions of one variable over finite constant fields. For the zeta function of these fields he formulated the analogue of the Riemann hypothesis about the zeros of the classical zeta function. In 1934 Helmut Hasse proved this hypothesis of Artin’s for function fields of genus 1, and in 1948 André Weil proved the analogue of the Riemann hypothesis for the general case.

In 1923 Artin began the investigations that occupied him for the rest of his life. He assigned to each algebraic number field k a new type of L-series. The functions

$L(s, X) = \Sigma X(n)(Nn)^{-s}$

—generalizations of the Dirichlet L-series—in which X is the character of a certain ideal class group and n traverses certain ideals of k were already known. These functions play an important role in Teiji Takagi’s investigations (1920) of Abelian fields K over k. Artin started his L-series from a random Galois field K over k with the Galois group G; he utilized representations of the Frobenius character X by matrices. Further, he made use of the fact that, according to Frobenius, to each unbranched prime ideal, p, in K, a class of conjugated substitutions σ from G, having the character value X (σ), can be assigned in a certain manner. Artin made $X(p^h) = X(\sigma^h)$ and formulated $X(p^h)$ for prime ideals p branched in K; he also defined his L-series by the formula

Artin assumed, and in 1923 proved for special cases, the identity of his L-series formed of simple character and the functions $L(s, X)$ for Abelian groups, if at the same time X were regarded as a certain ideal class character. The proof of this assumption led him to the general law of reciprocity, a phrase he coined. Artin proved this in 1927, using a method developed by Nikolai Chebotaryov (1924). This law includes all previously known laws of reciprocity, going back to Gauss’s. It has become the main theorem of class field theory.

With the aid of the theorem, Artin traced Hilbert’s assumption, according to which each ideal of a field becomes a principal ideal of its absolute class field, to a theory on groups that had been proved in 1930 by Philip Furtwaengler.

Artin had often pointed to a supposition of Furtwaengler’s according to which a series k_i (i = 1,2,....) is necessarily infinite if k_{i+1} is an absolute class field over k_i, This was disproved in 1964 by I. R. Safarevic and E. S. Gold.

In 1923 Artin derived a functional equation for his L-series that was completed in 1947 by Richard Brauer. Since then it has been found that the Artin L-series define functions that are meromorphic in the whole plane. Artin’s conjecture—that these are integral if X is not the main character—still remains unproved.

Artin had a major role in the further development of the class field theory, and he stated his results in *Class Field Theory*, written with John T. Tate (1961).

In 1926 Artin achieved a major advance in abstract algebra (as it was then called) in collaboration with Otto Schreier. They succeeded in treating real algebra in an abstract manner by defining a field as real—today we say formal-real—if in it – 1 is not representable as a sum of square numbers. They defined a field as real-closed if the field itself was real but none of the algebraic extensions were. They then demonstrated that a real-closed field could be ordered in one exact manner and that in it typical laws of algebra, as it had been known until then, were valid.

With the help of the theory of formal-real fields, Artin in 1927 solved the Hilbert problem of definite functions. This problem, expressed by Hilbert in 1900 in his Paris lecture "Mathematical Problems," is related to the solution of geometrical constructions with ruler and measuring standard, an instrument that permits the marking off of a single defined distance.

In his work on hypercomplex numbers in 1927, Artin expanded the theory of algebras of associative rings, established in 1908 by J. H. Maclagan Wedderburn, in which the double-chain law for right ideals is assumed; in 1944 he postulated rings with minimum conditions for right ideals (Artin rings). In 1927 he further presented a new foundation for, and extension of, the arithmetic of semisimple algebras over the field of rational numbers. The analytical theory of these systems was treated by his student Käte Hey, in her thesis in 1927.

Artin contributed to the study of nodes in threedimensional space with his theory of braids in 1925. His definition of a braid as a tissue made up of fibers comes from topology, but the method of treatment belongs to group theory.

Artin's scientific achievements are only partially set forth in his papers and textbooks and in the drafts of his lectures, which often contained new insights. They are also to be seen in his influence on many mathematicians of his period, especially his Ph.D. candidates (eleven in Hamburg, two in Bloomington, eighteen in Princeton). His assistance is acknowledged in several works of other mathematicians. His influence on the work of Nicholas Bourbaki is obvious.

BIBLIOGRAPHY

1. Original Works. Artin's works are in *The Collected Papers of Emil Artin*, Serge Lang and John T. Tate, eds. (Reading, Mass., 1965), and in the books and lecture notes that are listed there, including "Einführung in die Theorie der Gammafunktion" (1931); "Galois Theory" (1942); "Rings With Minimum Condition," written with C. J. Nesbitt and R. M. Thrall (1944); *Geometric Algebra* (1957); and *Class Field Theory*, written with J. T. Tate (1961). Missing from the list is "Vorlesungen über algebraische Topologie," a mathematical seminar given with Hel Braun at the University of Hamburg (1964).

II. Secondary Literature. Works on Artin are R. Brauer, "Emil Artin," in *Bulletin of the American Mathematical Society*, **73** (1967), 27–43; H. Cartan, "Emil Artin," in *Abhandlungen aus dem Mathematischen Seminar der Hamburgischen Universität*, **28** (1965), 1–6; C. Chevalley, "Emil Artin," in *Bulletin de la Société mathématique de France*, **92** (1964), 1–10; B. Schoeneberg, "Emil Artin zum Gedächtnis," in *Mathematisch-physikalische semesterberichte*, **10** (1963), 1–10; and H. Zassenhaus, "Emil Artin and His Work," in *Notre Dame Journal of Formal Logic*, **5** (1964), 1–9, which contains a list of Artin's doctoral candidates.

Bruno Schoeneberg