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(*b*. Paris, France, 21 January 1874; *d*. Chambéry, France, 5 July 1932)

mathematics.

Baire was one of three children in a modest artisan's family. His parents had to sacrifice in order to send him through high school. Having won a scholarship competition for the city of Paris in 1886, he entered the Lyée Lakanal as a boarding student; there he completed his advanced classes in 1890 after having won two honorable mentions in the Concours Général de Mathématiques.

In 1891 Baire entered the section for special mathematics at the Lycée Henri IV, and in 1892 was accepted at both the École Polytechnique and the École Normale Supérieure. He chose the latter, and during his three years there attracted attention by his intellectual maturity. He was a quiet young man who kept to himself and was profoundly introspective. During this period he was found to be in delicate health.

Although he placed first in the written part of the 1895 *agrégation* in mathematics, Baire was ranked third because of a mistake in his oral presentation on exponential functions, which the board of examiners judged severely. In the course of his presentation Baire realized that his demonstration of continuity, which he had learned at the Lycée Henri IV, was purely an artifice, since it did not refer sufficiently to the definition of the function. This disappointment should be kept in mind, because it caused the young lecturer to revise completely the basis of his course in analysis and to direct his research to continuity and the general idea of functions. While studying on a scholarship in Italy, Baire was strengthened in his decisive reorientation by Vito Volterra, with whom he soon found himself in agreement and who recognized the originality and force of his mind.

On 24 March 1899 Baire defended his doctoral thesis. on the theory of the functions of real variables, before a board of examiners composed of Appell, Darboux, and Picard. The few objections, which Baire fully appreciated, proved that he had embarked on a new road and would not find it easy to convince his listeners.

Baire began his teaching career in the *lycée* of Troyes, Bar-le-Duc, and Nancy, but he could not long endure the rigors of teaching the young. In 1902, as lecturer at the Faculty of Sciences of Montpellier, he wrote a paper on irrational numbers and limits. In 1904 he was awarded a Peccot Foundation fellowship to teach for a semester at the Collège de France. At that time this award went to young teachers to enable them to spend several months, free of routine duties, in developing their own specialties. Baire chose to work on a course in discontinuous functions, later edited by his pupil A. Denjoy and published in the Collection Borel, a series of monographs on the theory of functions.

Upon his return to Montpellier, Baire experienced the first violent attack of a serious illness that became progressively worse, manifesting itself in constrictions of the esophagus. After the crisis passed, he began drafting his paper "Sur la représentation des fonctions discontinues." Appointed professor of analysis at the Faculty of Science in Dijon in 1905, to replace Méray, he devoted himself to writing an important treatise on analysis (1907-1908). This work revivified the teaching of mathematical analysis. His health continued to deteriorate, and Baire was scarcely able to continue his teaching from 1909 to 1914. In the spring of 1914 he decided to ask for a leave of absence. He went first to Alésia and then to Lausanne. War broke out while he was there, and he had to remain—in difficult financial circumstances—until the war ended.

Baire was never able to resume his work, for his illness had undermined his physical and mental health. He now devoted himself exclusively to calendar reform, on which he wrote an article that appeared in the *Revue rose* (1921.) While still at his retreat on the shores of Lake Geneva, Baire received the ribbon of the Legion of Honor, and on 3 April 1922 was elected corresponding member of the Academy of Sciences. A pension granted him in 1925 enabled him to live in comparative ease, but the devaluation of the franc soon brought money worries. His last years were a struggle against pain and worry.

Thus Baire was able to devote only a few periods, distributed over a dozen years, to mathematical research. In addition to the already mentioned works, of particular importance are "Sur les séies à termes continus et tous de même signe" and "Sur la nonapplicabilité de deux continus à *n et n* + *p* dimensions." His writings, although few, are of great value.

Baire's doctoral thesis solved the general problem of the characteristic property of limit functions of continuous functions, i.e., the pointwise discontinuity on any perfect aggregate. In order to imagine this characteristic, one needed very rare gifts of observation and analysis concerning the way in which the question of limits and continuity had been treated until then. In developing the concept of semicontinuity—to the right or to the left—Baire took a decisive step toward eliminating the suggestion of intuitive results from the definition of a function over a compact aggregate. But in order to obtain the best possible results, one needed a clear understanding of the importance of the concepts of the theory stemming from aggregates.

Generally speaking, in the framework of ideas that here concerns us, every problem in the theory of functions leads to certain questions in the theory of sets, and it is to the degree that these latter questions are resolved, or capable of being resolved, that it is possible to solve the given problem more or less completely [*Surles fonctions*].

In this respect, Baire knew how to use the transfinite in profoundly changing a method of reasoning that had been applied only once before—and this in a different field.

From the point of view of derivative sets that interests us here, it may be said that if α is a number of the first type, then P^{α} is the set deried from $P^{\alpha-1,}$ and if α is of the second type, P^{α} is by definition the set of points that belongs to all P^{α} where α is any number smaller than α. Independently of any abstract considerations arising from Cantor's symbolism, P^{α} represents a fully determined object. Nothing more than a convenient language is contained in the use we shall be making of the term "transfinite number" [*Ibid., p. 36*].

Until the arrival of Bourbaki, his success greatly influenced the orientation of the French school of mathematics.

In line with his first results, Baire was led to approach the problem of integrating equations with partial derivatives at a time when their solution was not subjected to any particular condition of continuity. But it is in another, less specialized field that Baire's name is associated with lasting results.

Assigning to limit functions of continuous functions the name of Class 1 functions, Baire first endeavored to integrate functions of several variables into this class. Thus he considered those functions that are separately continuous in relation to each of the variables of which they are a function. Then he defined as Class 2 functions the limits of Class 1 functions, and as Class 3 functions the limits of Class 2 functions. Having established basic solutions for these three classes, he obtained a characteristic common to the functions of all classes.

Since the beginning of the nineteenth century, mathematicians had been interested in the development of the theory of functions of real variables only incidentally and in relation to complex variables. Baire's work completely changed this situation by furnishing the framework for independent research and by defining the subject to be studied.

If Baire did not succeed, even in France and despite intensive efforts, in overcoming the mistrust of the transfinite, he nevertheless put an end to the privileged status of continuity and gave the field to aggregate-oriented considerations for the definition and study of functions. Baire's work, held in high esteem by Émile Borel and Henri Lebesgue, exerted considerable influence in France and abroad while its author found himself incapable of continuing or finishing the task he had set himself.

There can be no doubt that the progress of modern mathematics soon made obsolete Baire's work on the concept of limit and the consequences of its analysis. But this work, written in beautiful French, has an incomparable flavor and merits inclusion in an anthology of mathematical thought. It marks a turning point in the ciritism of commonplace ideas. Moreover, the class of Baire's functions, according to the definition adopted by Charles de la Vallée Poussin, remains unattainable as far as the evoluation of modes of expression is concerned. This model of a brief and compact work is part of the history of the most profound mathemtics.

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Baire's works are now being collected.

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