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Three brothers—**Muḥammad**, **Aḥmad**, and **al-Ḥasan**—always known under the one name, which means “sons of Mūsā” (*b.* Baghdad, Iraq, beginning of ninth century; *d.* Baghdad. Muḥammad the eldest, *d.* January or February a.d. 873)

*mathematics, astronomy.*

Their father, Mūsā ibn Shākir, was a robber in his youth but later became a proficient astrologer. He died during the reign of Calif al-Ma'mūn (813–833), while his children were still young. Al-Ma'mūn recognized the mental ability of the brothers and enrolled them in the House of Wisdom—the first scientific institution in the Abbasid Empire and quite similar to the modern academy—which he himself had founded. Soon the Banū Mūsā excelled in mathematics, astronomy, and mechanics and became the most active members of the House of Wisdom. With Muḥammad ibn Mūsā al-Khwārizmī they led its scientific research. Al-Khwārizmī was the founder of the Arabic school of algebra, while the Banū Mūsā were especially interested in geometry. They also led the astronomical observations in Baghdad and organized a school of translators who rendered many Greek scientific manuscripts into Arabic. These translations were very useful in the development of science. Some important Greek works are now known only in their Arabic translations.

The most famous translators of that time worked under the guidance of the Banū Mūsā. Among them were Ḥunayn ibn Ishāq, who became the foremost translator of medical works, and Thābit ibn Qurra, the famous scientist and translator of the ninth century, to whom are ascribed many works besides the translations of such Greek works as Euclid's *Elements* and three books of Apollonius' *Conics*. The Banū Mūsā were among the first Arabic scientists to study the Greek mathematical works and to lay the foundation of the Arabic school of mathematics. They may be called disciples of Greek mathematicians, yet they deviated from classical Greek mathematics in ways that were very important to the development of some mathematical concepts.

It is difficult to distinguish the role played by each of the brothers in their common works, but it seems that ja'far Muḥammad was the most important. Muḥammad and al-Ḥasan were especially interested in geometry; Aḥmad was interested in mechanics. Muḥammad also did work in astronomy.

Of the many works ascribed to the Banū Mūsā the most important was the geometrical treatise called *Book on the Measurement of Plane and Spherical Figures*. Manuscripts of this treatise are in Oxford, Paris, Berlin, Istanbul, and Rampur, India. One of these manuscripts, with a recension by the thirteenth century mathematician Naṣīr al-Dīn al-Ṭūsī, has been published in Arabic. It was well-known in the [Middle Ages](#) in both Islam and Europe. The best evidence for this is the twelfth-century Latin translation by [Gerard of Cremona](#), entitled *Liber trium fratrum de geometria*. Manuscripts of this translation are in Paris, Madrid, Basel, Toruń, and Oxford. The main purpose of the treatise—as stated in the introduction—was to demonstrate the most important part of the Greek method of determining area and volume. In the treatise the method was applied to the measurement of the circle and the sphere.

In *Measurement of the Circle* and *On the Sphere and Cylinder*, Archimedes found the area of the circle and the surface and volume of the sphere by means of the method of Eudoxus, which was later called the “method of exhaustion.” This method was based on the same ideas that underlie the limit theory of modern mathematics. After Archimedes, this method was followed without further development. In fact, there is no evidence of work on the measurement of areas and volumes until the ninth century.

The Banū Mūsā found the area of the circle by a method different from that of Archimedes but based on his ideas of infinitesimals. They used the “method of exhaustion” but omitted the main part of it, inscribing in the circle a sequence of right polygons with  $2^k$  sides ( $k = 2, 3, \dots, n$ ) and finding their areas. Then they used the method of the “rule of contraries” to find the desired result. They omitted the transition to the limit condition, however; that is, they did not find the area of such a polygon when  $k \rightarrow \infty$ . Instead, they depended upon a proposition whose proof included the transition. This is the sixteenth proposition of the twelfth book of the *Elements*.

Using this theorem the Banū Mūsā proved the following: If we have a circle of circumference  $C$  and a line of length  $L$ , and if  $L < C$ , then we can inscribe in this circle a right polygon of perimeter  $P_n$  ( $n$  is the number of sides) such that  $P_n > L$ . This means that we can find an integer,  $N$ , such that  $C - p_n < C - L$  for every  $n > N$ . In the second part of this proposition the Banū Mūsā proved that if  $L > C$ , then we can circumscribe a right polygon of perimeter  $Q_n$ , such that  $Q_n < L$ . After this the proof of  $A = r \cdot 1/2 C$  (where  $A$  is the area of the circle and  $r$  its radius) becomes easy.

It should be noted that the Banū Mūsā defined the areas and volumes as equal to the products of certain values, while in Greek geometry they were expressed as comparisons with other areas and volumes. For example, Archimedes defined the volume of the sphere as four times the volume of the cone with the radius of the sphere as its height and the [great circle](#) of the sphere as its base. The Banū Mūsā found that the volume is equal to the radius of the sphere multiplied by one third of its surface. In other words, they used arithmetical operations for determining geometrical values. It was an important step to extend the [number system](#) and make it include irrational as well as integers and rationals. In the sixth proposition the Banū Mūsā demonstrated the Method of Archimedes for the approximate determination of the value of  $\pi$ . By means of inscription and circumscription of right polygons of ninety-six sides, Archimedes proved that  $\pi$  must lie between the values  $3 \frac{1}{7}$  and  $3 \frac{10}{71}$ . The Banū Mūsā wrote that this method can be continued to get nearer to the boundaries of the value of  $\pi$ . This means that  $\pi = \lim P_n$  (where  $P_n$  is the perimeter of the inscribed or circumscribed right polygon).

Like Archimedes, the Banū Mūsā determined that the surface of the sphere is four times its [great circle](#), but their proof is different. Archimedes' proof is equivalent to the calculation of the definite integral

where  $r$  is the radius of the sphere. This cannot be said for the Banū Mūsā's proof, for they calculated only a finite sum of the sine series proving that

they did not extend this formula to the limit condition. Instead, they used the following fact without proving it: for any two concentric spheres we can inscribe in the larger a solid generated by rotating a right polygon about the diameter of the sphere that passes through two vertices of the polygon, such that the surface of this solid does not touch or intersect the smaller sphere. This was proved by Euclid in the seventeenth theorem of the twelfth book of the *Elements*. The Banū Mūsā calculated the volume of the solid; then, using Euclid's theorem and the rule of contraries, they proved that  $A = 4C$  (Where  $A$  is the surface of the sphere and  $C$  is its great circle).

In addition to the measurement of the circle and the sphere, three classical Greek problems were solved in the treatise:

(1) In the seventh proposition of the treatise the Banū Mūsā proved the following theorem: If  $a$ ,  $b$ , and  $c$  are sides of any triangle and  $A$  its area, then

where  $p = (a+b+c)/2$ . This theorem is often called Hero's theorem because Europeans met it for the first time in Hero's *Metrics*, but it existed in a lost book of Archimedes, which was known to the Arabs. The Banū Mūsā's proof however, is different from that of Hero.

(2) The determination of two mean proportionals. This problem concerns the determination of two unknowns,  $x$  and  $y$ , from the formula  $a/x = x/y = y/b$ , where  $a$  and  $b$  are given. This problem was solved for the first time by Archytas. The Banū Mūsā included this solution but stated that they had borrowed it from a geometrical treatise by Menelaos. Archytas found  $x$  and  $y$  through three intersecting curved surfaces: right cylinder  $x^2 + y^2 = ax$ , right cone  $b^2(x^2 + y^2 + z^2) = a^2x^2$ , and torus  $x^2 + y^2 + z^2 = x_0^2$ , where  $x_0, y_0$ , and  $z_0$  are the coordinates of the point of intersection of these surfaces, then it is clear that

Therefore and are the required two mean proportionals between  $a$  and  $b$ . The Banū Mūsā gave a practical method for solving this problem by means of an instrument constructed from hinged rules. This instrument is very much like that devised by Plato for the same purpose.

(3) The trisection of the angle. Their solution to this problem, like all those given previously, is kinematic.

Thus, the contents of the Banū Mūsā's treatise are really within the boundaries of the ancient knowledge of geometry. This treatise, however, is not merely an exposition of Greek geometrical works, for it contains new proofs for the main theorems of the measurement of the circle and the sphere. Having studied the works of Greek mathematicians, the Banū Mūsā assimilated many of their methods. But in using the Greek infinitesimal method—the "method of exhaustion"—they omitted the transition to the limit conditions.

In the tenth and eleventh centuries a number of Arabic mathematical works on the measurement of figures were influenced by the Banū Mūsā's treatise, *On the Measurement of Plane and Spherical Figures*. The most important of these works were Thābit Ibn Qurra's *On the Measurement of the Conic Section Named Parabola* and *On the Measurement of the Parabolic Solids*, and [Ibn al-Haytham](#)'s *On the Measurement of Parabolic Solids* and *On the Measurement of the Sphere*. In the [Middle Ages](#) the treatise played a great role in spreading the tradition of Euclid and Archimedes in the Arabic countries and in Europe. Its influence upon European scientists in the Middle Ages can easily be seen in the *Practica geometrica* of [Leonardo Fibonacci](#). In this book we can see some theorems of the Banū Mūsā that did not exist in the Greek books—for example, the theorem that says that the plane section of a right cone parallel to the base of the cone is a circle.

In addition to the treatise *On the Measurement of Plane and Spherical Figures*, the Banū Mūsā are credited with a number of other works that have been studied either insufficiently or not at all. Following is a list of the most important of these works.

(1) *Premises of the Book of Conics*. This is a recension of Apollonius' *Conics*, which was translated into Arabic by Hilāl al-ḥimṣī (Bks. I–IV) and Thābit ibn Qurra (Bks. V–VII). The recension was probably prepared by Muḥammad. Manuscripts of it are in Oxford, Istanbul, and Leiden.

(2) *Book of the Lengthened Circle*. This treatise, written by al-Ḥasan, seems to be on the “gardener’s construction of the ellipse,” that is, the construction of an ellipse by means of a string attached to the foci.

(3) *Qarastūn*. This is a treatise on the balance theory and its instruments.

(4) *On Mechanical Devices* (or *On Mechanics*). This treatise on pneumatic devices was written by Aḥmad. Manuscripts of it are in Berlin and the Vatican.

(5) *Book on the Description of the Instrument Which Sounds by Itself*. This work is on musical theory. A manuscript is in Beirut.

Some of these works deserve to be carefully studied, especially *Qarastūn* and *On Mechanical Devices*.

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