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(*b*. Basel, Switzerland, 6 August 1667; *d*. Basel, 1 January 1748)

mathematics.

The tenth child in the family, Johann proved unsuited for a business career, much to his father's sorrow. He therefore received permission in 1683 to enroll at his native city's university, where his brother Jakob (or Jacques), who was twelve years older and who had recently returned from the Netherlands, lectured as *magister artium* on experimental physics. In 1685 Johann, respondent to his brother in a logical disputation, was promoted to *magister artium* and began the study of medicine. He temporarily halted his studies at the licentiate level in 1690, when his first publication appeared, a paper on fermentation processes.[1] (His doctoral dissertation of 1694[2] is a mathematical work despite its medical subject, and reflects the influence of the iatromathematician Borelli.)

Bernoulli privately studied mathematics with the gifted Jakob, who in 1687 had succeeded to the vacant chair of mathematics at the University of Basel. From about this time, both brothers were engrossed in infinitesimal mathematics and were the first to achieve a full understanding of Leibniz abbreviated presentation of differential calculus.[3] The extraordinary solution[4] of the problem of *catenaria* posed by Jakob Bernoulli (*Acta eruditorum*, June 1691) was Johann's first independently published work, and placed him in the front rank with Huygens, Leibniz, and Newton. Johann spent the greater part of 1691 in Geneva. There he taught differential calculus to J. C. Fatio-de-Duillier (whose brother Nicolas later played a not very praiseworthy role in the Leibniz Newton priority dispute) and worked on the deepening of his own mathematical knowledge.

In the autumn of 1691 Bernoulli was in Pairs, where he won a good place in Malebranche's mathematical circle as a representative of the new Leibnizian calculus, and did so by virtue of a "golden theorem" (stemming actually from Jakob)—the spectacular determination of the radius of curvature of a curve by means of the equation $p = dx/ds$: d^2y/ds^2. During this period he also met L'Hospital, then probably France's most gifted mathematician. "Grandseigneur of the science of mathematics"—he corresponded also with Huygens—L'Hospital engaged Bernoulli to initiate him into the secrets of the new infinitesimal calculus. The lessons were given in Paris and sometimes in L'Hospital's country seat at Oucques, and Bernoulli was generously compensated. L'Hospital even induced Bernoulli to continue, for a considerable fee, these lessons by correspondence after the latter's return to Basel. This correspondence[5] subsequently became the basis for the first textbook in differential calculus,[6] which assured L'Hospital's place in the history of mathematics. (Bernoulli's authorship of this work, which was still doubted by Cantor,[7] has been substantiated by the Basel manuscript of the *Differential Calculus* discovered in 1921 by Schafheitlin,[8] as well as by Bernoulli's correspondence with L'Hospital.[9])

In 1692 Bernoulli met Pierre de Varignon, who later became his disciple and close friend. This tie also resulted in a voluminous correspondence.[10] In 1693 Bernoulli began his exchange of letters With Leibniz, which was to grow into the most extensive correspondence ever conducted by the latter.

Bernoulli's most significant results during these years were published in the form of numerous memoirs in *Acta eruditorum (AE)* and shorter papers in the *Journal des Sçavans (JS)*. Bernoulli's two most important achievements were the investigations concerning the function $y = x^x$ and the discovery, in 1694, of a general development in series by means of repeated integration by parts, the series subsequently named after him:

(cf. *Addidamentum AE*, 1694, letter to Leibniz of 2 September 1694). This series—whose utility, incidentally, Jakob Bernoulli failed to recognize—is based on the general Leibnizian principle for the differentiation of a product:

This formalism is characteristic of a large part of the Bernoulli-Leibniz correspondence between 1694 and 1696.

Integration being viewed as the inverse operation of differentiation, Bernoulli worked a great deal on the integration of differential equations. This view was generally accepted in the Leibniz circle. In Paris he had already demonstrated the efficacy of Leibniz' calculus by an anonymous solution of "Debeaune'sproblem" (*JS*, 1692), which had been put to Descartes as the first inverse tangent problem. Five years later he demonstrated that with the aid of the calculus much more complex differential equations could be solved. In connection with Debeaune's problem, Jakob Bernoulli had proposed the general differential equation since called by his name,

$y' + P(x)y + Q(x)y^n = 0$,

and had solved it in a rather cumbersome way. Johann, more flexible with regard to formalism, solved this equation by considering the desired finalfunction as the product of two functions, $M(x)$ and $N(x)$. In the resulting equation,

the arbitrariness of the functions M and N makes it possible to subject one of them (e.g., M) to the secondary condition

resulting in $M = \exp[- \int P(x)\, dx]$. This substitution promptly leads to a linear differential equation in N.

Bernoulli's "exponential calculus" is nothing other than the infinitesimal calculus of exponential functions. Nieuwentijt, in a paper criticizing the lack of logical foundations in Leibniz' calculus,[11] pointed out the inapplicability of Leibniz' published differentiation methods to the exponential function x^y. There upon Bernoulli developed, in "Principia calculi exponentialium seu percurrentium" (*AE*, 1697), the "exponential calculus," which is based on the equation

$d\,(x^y) = x^y \log x\, dy + yx^{y-1}\, dx.$

Also in 1695 came Bernoulli's summation of the infinite harmonic series

from the difference scheme, the development of the addition theorems of trigonometric and hyperbolic functions from their differential equations, and the geometric generation of pairs of curves, wherein the sum or difference of the arc lengths can be represented by circular arcs. Neither Johann nor Jacob Bernoulli succeeded in mastering the problem, originated by Mengoli, of the summation of reciprocal squares This problem was solved only by Johann's greatest pupil, Leonhard Euler.[12]

In 1695 Bernoulli was offered both a professorship at Halle, and, through the intervention of Huygens, the chair of mathematics at Groningen. He eagerly accepted the latter offer, particularly since his hopes of obtaining a chair in Basel were nil as long as his brother Jakob was alive. On 1 September 1695 he departed for Holland with his wife (the former Dorothea Falkner) and seven-month-old Nikolaus, his first son, not without resentment against Jakob, who had begun to retaliate for Johann's earlier boastulness when he solved the differential equation of the velaria (*JS*, 1692): Jakob termed Johann his pupil, who after all could only repeat what he had learned from his teacher. This cutting injustice was promptly paid back by Johann, now his equal in rank.

In June 1696, Johann posed (in *AE*) the problem of the brachistochrone, i.e., the problem of determining the "curve of quickest descent." Since no solution could be expected before the end of the year, Bernoulli, at Leibniz' request, republished the problem in the form of a leaflet dedicated to *acutissimisqui into to orbe florent mathematicis* ("the shrewdest mathematicians of all the world") and fixed a six month limit for its solution. Leibniz solved the problem on the day he received Bernoulli's letter, and correctly predicted a total of only five solution: from the two Bernoullis, Newton, Leibniz, and L'Hospital. (It should be noted that it was only through Johann's assistance—by correspondence—that L'Hospital had arrived at his solution.)

This problem publicly demonstrated the difference in the talents of the two brothers. Johann solved the problem by ingenious intuition, which enabled him to reduce the mechanical problem to the optical problem already resolved by means of Fermat's principle of least time. He deduced the differential equation of the cycloid from the law of refraction. Jakob, on the other hand, furnished a detailed but cumbersome analysis, and came upon the roots of a new mathematical discipline, the calculus of variations. Unlike Jakob, Johann failed to perceive that such extreme-value problems differed from the customary ones in that it was no longer the unknown extreme values of a function that were to be determined, but functions that made a certain integral an extreme.[13]

In connection with his solution of the brachistochrone problem, Jakob (*AE*, May 1697) posed a new variational problem, the isoperimetric problem.[14] Johann underestimated the complexity of this problem by failing to perceive its variational character; and he furnished an incomplete solution (wherein the resulting differential equation is one order too low) in *Histoire des ouvrages des savants* (VI, 1697), and thereby brought on himself the merciless criticism of his brother.[15] This was the beginning of alienation and open discord between the brothers—and also the birth of the calculus of variations. A comparison of Jakob's solution (Basel, 1701; *AE*, May 1701) with Johann's analysis of the problem (which he presented through Varignon to the Paris Academy on 1 February 1701) clearly shows Johann's to be inferior. Nevertheless, Jakob was not able to enjoy his triumph, since—for reasons that remain mysterious—the sealed envelope containing Johann's solution was not opened by the Academy until 17 April 1706, the year following Jakob's death.

Soon after publication of Jakob's *Analysis magni problematis isoperimetrici* (1701), Johann must have felt that his brother's judgment was valid, although he never said so. Only after having been stimulated by Taylor's *Methodus incrementorum* (1715) did he produce a precise and formally elegant solution of the isoperimetric problem along the lines of Jakob'sideas (*Mémoires de l'Académie des sciences*, 1718). The concepts set forth in this paper contain the nucleus of modern methods of the calculus of variations. Also in this connection Bernoulli made a discovery pertaining to the variational problem of geodetic lines on convex surfaces: in a letter addressed to Leibniz, dated 26 August 1698, he perceived the characteristic property of geodetic lines, i.e., three consecutive points determine a normal plane of the surface.

Bernoulli's studies on the determination of all rationally quadrable segments of the common cycloid—the "fateful curve of the seventeenth century" (*AE*, July 1699)—in connection with the cyclotomic equation (*AE*, April 1701; more detailed in his correspondence with Moivre[16])—resulted in a systematic treatment of the integrals of rational functions by means of resolution into partial fractions. The general advance in algebraic analysis under Bernoulli's influence is evident in the typical case of the relation

Nevertheless, Bernoulli had not yet perceived that such logarithmic expressions may take on infinitely many values.

Immediately after Jakob's death, Johann succeeded him in Basel, although he would undoubtedly have preferred to accept the repeated invitations extended to him by the universities of Utrecht and Leiden (see correspondence of the rector of Utrecht University, Pieter Burman, with Bernoulli's father-in-law, Falkner[17]). Family circumstances, however, caused him to settle in Basel.

Bernoulli's criticism of Taylor's *Methodus incrementorum* was simultaneously an attack upon the method of fluxions, for in 1713 Bernoulli had become involved in the priority dispute between Leibniz and Newton. Following publication of the Royal Society's *Commercium epistolicum* in 1712, Leibniz had no choice but to present his case in public. He released—without naming names—a letter by Bernoulli (dated 7 June 1713) in which Newton was charged with errors stemming from a misinterpretation of the higher differential. Thereupon Newton's followers raised complicated analytical problems, such as the determination of trajectories and the problem of finding the ballistic curve, which Newton had solved only for the law of resistance $R = av$ (R = resistance, a = constant, v = velocity). Bernoulli solved this problem (*AE*, 1719) for the general case ($R = av^n$), thus demonstrating the superiority of Leibniz' differential calculus.

After Newton's death in 1727, Bernoulli was unchallenged as the leading mathematical preceptor to all Europe. Since his return to Basel in 1705, he had devoted himself—in the field of applied mathematics—to theoretical and applied mechanics. In 1714 he published his only book, *Théoriede la manoeuvredes vaisseaux*. Here Bernoulli (as Huygens had done before him) criticizes the navigational theories advanced in 1679 by the French naval officer Bernard Renau d'Eliçagaray (1652–1719), a friend of Varignon's. In this book Bernoulli exposed the confusion in Cartesian mechanics between force and *visviva* (now kinetic energy). On 26 February 1715—and not 1717, as stated in the literature because of a printing error in Varignon's *Nouvelle mécanique* (1725)—Bernoulli communicated to Varignon the principle of virtual velocities for the first time in analytical form. In modern notation it is

Since this principle can be derived from the energy principle $A + mv^2/2$ = const., which Bernoulli applied several times to conservative mechanical systems of central forces, he considered it a second general principle of mechanics—which, however, he had demonstrated only for the statical case. For central forces, Bernoulli applied the *vis viva* equation to the inverse two-body problem, which he for the first time expressed in the form used today for the equation of the orbit (*Mémoires de l'Académie des sciences*, 1710):

For the corresponding problem of centrally accelerated motion in a resisting medium (*ibid*., 1711), he solved the differential equation

(ϱ = radius of curvature of the orbit) on the premise that $v = M(r)N(r)$, and determined the central force, in accordance with Huygens' formula, from

Newton severely criticized the Cartesian vortex theory in Book II of the *Principia*. Bernoulli's advocacy of the theory delayed the acceptance of Newtonian physics on the Continent. In three prizewinning papers, Bernoulli treated the transmission of momentum (1727), the motions of the planets in aphelion (1730), and the cause of the inclination of the planetary orbits relative to the solar equator (1735). Bernoulli's 1732 work on hydraulics (*Opera*, IV) was generally considered a piece of plagiarism from the hydrodynamics of his son Daniel. Nevertheless, Bernoulli did try to manage without Daniel's formulation of the principle of *vis viva*.

Bernoulli also worked in experimental physics. In several papers (*Mémoires de l'Académie des science*, 1701; Basel, 1719), he investigated the phenomenon of the luminous barometer within the framework of contemporary Cartesian physics, although he was unable to furnish a sufficient explanation for the electrical phenomenon of triboluminescence discovered by Picard.

Bernoulli was a member of the royal Académies of Paris and Berlin, of the Royal Society, of the St. Petersburg Academy, and the Institute of Bologna. As son-in-law of Alderman Falkner, he not only enjoyed social status in Basel, but also held honorarycivic offices there. He became especially well known as a member of the school board through his efforts to reform the humanistic Gymnasium. His temperament might well have led him to a career in politics, but instead it only involved him in scientific polemics with his brother Jakob and in the Leibinz-Newtonpriority dispute. Even abroad he was unable to curbhis "Flemish pugnacity." In 1702, as professor in Groningen, he became involved in quarrels with the theologians, who in turn, because of his views in natural philosophy, accused him of what was then the worst or heresies, Spinozism.

Bernoulli's quarrelsomeness was matched by his passion for communicating. His scientific correspondence comprised about 2,500 letters, exchanged with some 110 scholars.

NOTES

1.*De effervescentia et fermentatione*.

2.*De motu musculorum*.

3. Leibniz, *Nova methodus de maximis et minimis*.

4. J. E. Hofmann, "Vom öffentlicnen Bekanntwerden der Leibniz'schen Infinitesimalmathematik."

5. O. Spiess, ed., *Der Briefwechsel von Johann Bernoulli*.

6. L'Hospital, *Analyse des infiniment petits*.

7. Cantor, *Vorlesungen über, Geschichte der Mathematik*.

8.*Lectiones de calculo differentialium*, MS Universitätsbibliothek, Basel.

9. O. J. Rebel, *Der Briefwechsel zwischen Johann Bernoulli un dem Marquis de l'Hôpital*.

10. E. J. Fedel, *Johann Bernoullis Briefwechsel mit Varignon ausden Jahren 1692–1702*.

11.*Considerationes secundae circa calculi differentialis principia*.

12. O. Spiess, *Die Summe der reziproken Quadratzahlen*.

13. P. Dietz, *Die Ursprünge der Variations rechnung bei Jakob Bernoulli:* J. E. Hofmann, *Ueber Jakob Bemoullis Beiträge zur Infinitesimalmathematik*.

14. J. O. Fleckenstein, *Johann und Jakob Bernoulli*.

15. Hofmann, *Ueber Jakob Bernoullis Beiträge zur Infinitesimalmathematik*.

16. K. Wollenschlaeger, *Der mathematische Briefwechsel zwischen Johann I Bernoulli und Abraham de Moivre*.

17. O. Spiess, ed., *Der Briefwechsel von Johann Bernoulli*.

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