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(*b*. Berdyansk, Russia, 24 January 1891; *d*. Cambridge, England, 2 November 1970)

mathematics.

Besicovitch was the fourth child in the family of four sons and two daughters of Samuel and Eva Besicovitch. The family had to live frugally. All the children were talented and studied at the University of St. Petersburg, the older ones in turn earning money to help support the younger. From an early age Besicovitch showed a remarkable aptitude for solving mathematical problems.

Besicovitch graduated in 1912 from the University of St. Petersburg, where one of his teachers was Andrei A. Markov. When, in 1917, he became professor in the School of Mathematics at the newly established University of Perm, his intention was to work in mathematical logic. He abandoned this idea because the library was inadequate and as a result continued to work on fundamental problems in analysis. In order to obtain a counterexample to a plausible conjecture about repeated Riemann integrals in IR^2 he was led to construct a compact plane set F of zero Lebesque measure but containing unit line segments in every direction (1919–1920). Using a suggestion of J. Pál, Besicovitch later (1928) used F to show that zero is the lower bound of the area of plane sets in which a unit segment can be continuously turned through two right angles. The solution of this “Kakeya problem” was the subject of a lecture filmed by the Mathematical Association in 1958. Theset F has turned out to be useful in many contexts. For example, R.O. Davies applied the Besicovitch method to a construction of Otton Nikodym to produce a plane set of full measure with continuum many lines of accessibility through each point. Charles Fefferman used F to obtain a negative solution to the multiplier problem for a ball, and C. R. Putnam obtained a characterization of the spectra of hyponormal operators using Davies’ work.

In 1916 Besicovitch married Valentina Vietalievna, a mathematician older than himself. Three years later, during the civil war, the University of Perm was destroyed and partially reestablished at Tomsk. Besicovitch locked books in cellars and preserved much of the valuable properly of the Faculty of Mathematics, then worked with A. A. Fridman to reestablish the university after the liberation of Perm. In 1920 he returned to Leningrad as professor in the Pedagogical Institute and lecturer in the university. The political powers forced him to teach classes of workers who lacked the background to understand the lectures, so there was little time for research.

Besicovitch was offered a Rockefeller fellowship to work abroad, but repeated requests for permission to leave the country were refused. In 1924 he escaped and made his way to Copenhagen, where the Rockefeller fellowship enabled him to work for a year with Harald Bohr, who was then developing the theory of almost periodic functions. This contact resulted in several papers, in the most important of which (“On Generalized Almost Periodic Functions”) he showed that the analogue of the Riesz-Fischer theorem is false for Bohr’s almost periodic functions and developed a new definition of “almost periodic” for which Riesz-Fischer is valid. His *Almost Periodic Functions* became the standard work on the classical theory of that subject.

In 1925 Besicovitch visited Oxford, Godfrey H. Hardy quickly recognized his analytical abilities, securing for him a position as lecturer at the University of Liverpool in 1926–1927, He moved to Cambridge in 1927 as a university lecturer and was elected a fellow of Trinity College in 1930. Besicovitch’s wife remained in Russia, and the marriage was dissolved in 1926. While in Perm he had befriended a woman named Maria Denisova and her children. He brought them to England and in 1928 married the elder daughter. Valentina Alexandrovna, then aged sixteen.

At about the time of his arrival in England, Besicovitch started his deep analysis of plane sets of dimension I that has become the foundation of modern geometric measure theory as developed by Herbert Federer and his school. He obtained the fundamental structure theorems for linearly measurable plane sets—the regular sets are a subset of a countable union of rectifiable arcs and have a tangent almost everywhere (“On the Fundamental Geometrical Properties of Linearly Measurable Plane Sets of Points”), while the irregular sets intersect no rectifiable are in a set of positive length, have a tangent almost nowhere, and project in almost all directions onto a set of zero linear Lebesque measure. Besicovitch also considered sets with non-integer dimension (“On Linear Sets of Points of Fractional Dimension,” “On Lipschitz Numbers”) and showed that these could not have nice geometric properties of regularity. These sets occur naturally in many physical situations where there is an element of randomness (for an extensive recent account see Mandelbrot, who calls such sets “fractals”). The basic Besicovitch contribution to geometric measure theory is carefully developed by Federer, and a simpler version is given by Falconer.

Besicovitch had shown that the study of local density was fundamental to an understanding of the geometric properties of small sets. John Manstrand, 1954, showed that a strict density cannot exist for any set of non-integer dimension. Claude Tricot developed a new packing measure and showed by considering density that only for surfacelike sets with integer dimension can Hausdorff and packing measures be equal and finite positive.

In 1950, on his fifty-ninth birthday, Besicovitch was elected to the House Ball chair of mathematics at Cambridge. Although he retired in 1958, he remained active in teaching and research and spent eight successive years as visiting professor at various universities in the United States, Besicovitch developed a new interest in the definition of area for a parametric surface in IR^3 starting about 1942. He produced a beautiful example of a topological disk with arbitrarily small Lebesque-Fréhet area (defined by approximating polyhedral) but arbitrarily large (three-dimensional) Lebesque measure. He concluded that the only satisfactory concept of area is a two-dimensional Hausdorff measure and undertook a program of solving anew many of the classical problems of surface area.

Besicovitch exhibited an open mind in all of his work. When solving a problem, most mathematicians make a commitment to the nature of the solution before that solution has been found, and this commitment interposes a barrier to the consideration of other possibilities. Besicovitch seems never to have been troubled in this way. He therefore obtained results that astounded his contemporaries and remain surprising today.

In recognition of his outstanding talent, Besicovitch received the Adams Prize of the University of Cambridge in 1930, was elected a fellow of the Royal Society in 1934, was awarded the Morgan Medal by the London Mathematics Society in 1950, and received the Sylvester Medal of the Royal Society in 1952. His many mathematical contributions remain a stimulus to research activity.

In the decade 1980–1989 there was an explosion of interest in fractals as a tool for modeling phenomena from a wide variety of different contexts. Dynamical systems involving the interation of a transformation produce fractals for a critical set— see the book by Heinz-Otto Peitgen and P. H. Richter *The Beauty of Fractals*. In physics there are many critical phenomena where fractals provide a helpful insight, and each year has seen international symposia focusing on this area. It is fair to say that the geometrical insights of Besicovitch's work laid the foundation for this development.

[The material in this biography is condensed from an obituary notice published in the Bulletin of the London Mathematical Society, with some comments about recent developments.]

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