

# Bienaymé, Irénée-Jules | Encyclopedia.com

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(b. Paris, France, 28 August 1796; d. Paris, 19 October 1878),

probability, mathematical statistics, demography, [social statistics](#).

Bienaymé's secondary education began at the *lycée* in Bruges and concluded at the Lycée Louis-le-Grand, Paris. He took part in the defense of Paris in 1814 and enrolled at the École Polytechnique the following year; the institution was closed briefly in his first year, however. Bienaymé became lecturer in mathematics at the military academy at St.—Cyr in 1818, leaving in 1820 to enter the Administration of Finances. He soon became inspector and, in 1834, inspector general. At about this time he became active in the affairs of the Société Philomatique de Paris. The revolution of 1848 led Bienaymé to retire from the [civil service](#), and he received a temporary appointment as professor of the calculus of probabilities at the Sorbonne. Despite his retirement, he had considerable influence as a statistical expert in the government of [Napoleon III](#); he was attached to the Ministry of Commerce for two years and was praised in a report to the Senate in 1864 for his actuarial work in connection with the creation of a retirement fund. At the Paris Academy, to which he was elected in 1852, he served for twenty-three years as a referee for the statistics prize of the Montyon Foundation, the highest French award in that field.

Bienaymé was a founding member of the Société Mathématique de France (president in 1875, life member thereafter), corresponding member of the [St. Petersburg](#) Academy of Sciences and of the Belgian Central Council of Statistics, and honorary member of the Association of Chemical Conferences of Naples. He became an officer of the Legion of Honor in 1844. Bienaymé had a considerable knowledge of languages; he translated a work by Chebyshev from Russian and at his death was preparing an annotated translation of Aristotle from classical Greek.

Laplace's *Théorie analytique des probabilités* (1812) was Bienaymé's guiding light: and some of his best work, particularly on least squares, elaborates, generalizes, or defends Laplacian positions. Bienaymé corresponded with Quetelet, was a friend of Cournot's, and was on cordial terms with Lamé and Chebyshev. His papers were descriptive, his mathematics laconic; and he had a penchant for controversies. No sooner was he elected to the Academy than he locked horns with Cauchy over linear least squares. In 1842 he had attempted to criticize Poisson's law of large numbers (pertaining to inhomogeneous trials): this invalid criticism was not published until 1855. He also sent a group of communications to the Academy criticizing the Metz Mutual Security Society. Bertrand was to take blistering exception to his style.

Bienaymé did not publish until he was in his thirties, and only twenty-two articles appeared. Almost half are in a now obscure source, the scientific newspaper-journal *L'Institut, Paris*, and were reprinted at the end of the year of their appearance in the compendium *Extraits des Procès-Verbaux de la Société philomatique de Paris*.

The early writings lean to demography; "De la durée de la vie en France" (1837) discusses the life tables of Antoine Deparcieux and E. E. Duvillard de Durand, both of which were widely used in France. Its major object is to present overwhelming evidence against the continued use of the Duvillard table, employment of which by insurance companies had been to their undoubted financial advantage, since the mortality rates that it predicted were much more rapid than was appropriate for France at that time. Apart from writings on the stability of insurance companies, Bienaymé's other direct contribution to the social sciences concerned the size of juries and the majority required for conviction of the accused. The jury system in France had been in a state of flux; its revision was based largely on interpretations of results obtained by Laplace, whose conclusions were later rejected by Poisson in *Recherches sur la probabilité des jugements...* (1837): Bienaymé naturally sided with Laplace. Some other papers by Bienaymé have demographic or sociological motivation but involve major methodological contributions to probability or mathematical statistics.

The stability and dispersion theory of statistical trials is concerned with independent binomial trials, the probability of success  $p_i$  in the  $i$ th trial in general depending on  $i$ . The subject forms the early essence of the "Continental direction" of statistics, which is typified by Lexis, Bortkiewicz, A. A. Chuprov, and Oskar Anderson. One of its major aims is to test for, and typify, any heterogeneity in the  $p_i$ 's (the homogeneous, or Bernoulli, case is that of all  $p_i$ 's being equal). Bienaymé introduced a physical principle of *durée des causes*, with which he showed that the proportion of successes exhibits more variability than in the homogeneous case (this fact might therefore explain such manifested variability). His reasoning here was not understood until much later. In the context of a set of Bernoulli trials in which there is a specified number of successes, divided into two successive blocks of trials, Bienaymé in 1840 showed an understanding of the important statistical concept of sufficiency, now attributed to Fisher (1920). These contributions established him, after Poisson, as a founder of the "Continental direction."

In linear least squares one is concerned with the estimation of (in modern notation) an  $r \times 1$  vector,  $\beta$ , of unknowns from a number  $N$  of observations  $Y$ , related linearly to  $\beta$  but subject to error  $\varepsilon$ :  $Y = A\beta + \varepsilon$ . Bienaymé extended Laplace's asymptotic treatment of the system as  $N \rightarrow \infty$  but the only essential originality is in a largely unsuccessful attempt to find a simultaneous confidence region for all the coefficients  $\beta_i, i = 1, \dots, r$ . Nevertheless, "Mémoire sur la probabilité des erreurs" contains a deduction of an almost final form of continuous chi-squared density, with  $n$  degrees of freedom, for the sum of squares of  $n$  independent and identically distributed  $N(0,1)$  random variables. The impassioned defense of least squares against Cauchy "Considérations à l'appui de la découverte de Laplace," 1853) contains three important incidental results, the most startling of which is the Bienaymé-Chebyshev inequality,

proved by the simple argument still used. Chebyshev obtained it by a more difficult means in 1867 and published his results simultaneously in Russian and French. This was juxtaposed with a reprint of Bienaymé's paper and in 1874 Chebyshev himself credited Bienaymé with having arrived at the inequality via the "method of moments", the discovery of which he ascribed to Bienaymé.

Perhaps the most startling of Bienaymé's contributions to probability is a completely correct statement of the criticality theorem for simple branching processes. His "De la loi de la multiplication et de la durée des familles" (1845) anticipated the partly correct statement of Galton and Watson by some thirty years, and predated the completely correct one of Haldane (until recently thought to be first) by over eighty years. This work may have been stimulated by L. F. Benoiston de Chateauneuf. Of only slightly less significance is a remarkably simple combinatorial test for randomness of observations on a continuously varying quantity. This method involves counting the number of local maxima and minima in the series; Bienaymé stated in 1874 that the number of intervals, complete and incomplete, between extrema in a sequence of  $N$  observations is (under assumption of randomness) approximately normally distributed about a mean of  $(2N-1)/3$  with variance  $(16N-29)/90$ . This result, which is describable in modern terms as both a nonparametric test and a limit theorem, is technically complex to prove even by modern methods.

A sophisticated limit theorem proved (but not rigorously) by Bienaymé is the following: If the random variables  $\Theta_i, i = 1, \dots, m$  satisfy  $0 \leq \Theta_i \leq 1, \sum \Theta_i = 1$ , and the joint probability density function of the first  $(m-1)$  is const.  $\Theta_1^{x_1} \Theta_2^{x_2} \dots$ . Where  $x_i \geq 0$  is an integer, then if  $\sum \gamma_i \Theta_i$ , as  $n \rightarrow \infty$

where  $\sum \gamma_i x_i / n, n = \sum x_i, r_i = x_i / n = \text{const.}, > 0$  With a more general distribution for the  $\Theta_i$ , this result was reobtained in 1919 by Von Mises, who regarded it as a *Fundamentalsatz*.

Finally, there is the algebraic result announced by Bienaymé in 1840: let  $\{a_i\}, \{C_i\}$  be sets of positive numbers. Then

is nondecreasing in  $m$ . In a probabilistic setting this result is credited to Lyapounov, the mathematical attribution being to O. Schlömilch (1858). The result contains the Cauchy inequality and the earlier inequality between the arithmetic and geometric means, which it also complements by another consequence:

Bienaymé was far ahead of his time in the depth of his statistical ideas. Because of this and the other characteristics of his work, and his being overshadowed by the greatest figures of his time, his name and contributions are little known today.

## BIBLIOGRAPHY

I. Original Works Bienaymé's writings include "De la durée de la vie en France depuis le commencement du XIX<sup>e</sup> siècle", in *Annales d'hygiène publique et de médecine légale*, **18** (1837), 177-218; "De la loi de multiplication et de la durée des familles", in *Société philomatique de Paris. Extraits des procès-verbaux des séances*, 5th ser., **10** (1845), 37-39, also in *L'Institut*, Paris, no. 589 (1845) 131-132, and reper. by Kendall in 1975 (see below); *De la mise l'alignement des maisons* (Paris, 1851), a satirical dialogue that demonstrates his abhorrence of the legal system; "Mémoire sur la probabilité des erreurs d'après la méthode des moindres carrés", in *Journal des mathématiques pures et appliquées*, **17** (1852), 33-78, repr. in *Mémoires présentés par savants étrangers à l'Académie des sciences*, **15** (1868), 615-663; *Notice sur les travaux scientifiques de M.I.-J. Bienaymé* (Paris, 1852), with a partial bibliography; and ("Considérations à l'appui de la découverte de Laplace sur la loi de probabilité dans la méthode des moindres carrés," in *Comptes rendus... de l'Académie des sciences*, **37** (1853), 309-324, repr. in *Journal des mathématiques pures et appliquées*. 2nd ser., **1** (1867), 158-176.

II. Secondary Literature. See A. Gatine, "Bienaymé", in *École polytechnique. Livre du centenaire 1794-1894*, III (Paris, 1897), 314-316; J. de la Gournerie, "Lecture de la note suivant, sur les travaux de M. Bienaymé", in *Comptes rendus... de l'Académie des sciences*, **87** (1878), 617-619; C. C. Heyde and E. Seneta, "The Simple Branching Process, a Turning

Point Test and a Fundamental Inequality: A Historical Note on I.-J. Bienaymé,” in *Biometrika*, **59**, no. **3** (1972), 680-683; “Bienaymé,” in *Proceedings of the 40th Session of the International Statistical Institute* (Warsaw, 1975); D. G. Kendall, “Branching Processes Since 1873,” in *Journal of the London Mathematical Society*, **41** (1966), 385-406; and “The Genealogy of Genealogy: Branching Processes Before (and After) 1873,” in *Bulletin of the London Mathematical Society*, **7** (1975), 225-253; M. G. Kendall and A. Doig, *Bibliography of Statistical Literature*, **III** (Edinburgh, 1968), the most extensive bibliography; H. O. Lancaster, “Forerunners of the Pearson  $X^2$ ,” in *Australian Journal of Statistics*, **8** (1966), 117-126; R. von Mises, *Mathematical Theory of Probability and Statistics*, edited and complemented by Hilda Geiringer ([New York](#), 1964), 352-357; and L. Sagnet, “Bienaymé (Irénée-Jules),” in *Grenade encyclopédie, inventaire raisonné des sciences, des lettres et des arts*, **VI** (Paris, n.d. [1888]), 752.

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