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(*b*. Cracow, Austria-Hungary [now Poland]). 20 August 1899: *d*. Houston, Texas. 2 May 1982)

mathematics.

Bochner, a mathematician noted for the breadth and originality of his work in analysis, was born in a highly orthodox Jewish community. His father, Joseph, a small businessman, and his mother. Rude Haber, were self-educated beyond grade school hut were both assiduous readers—she especially of Shakespeare and Ibsen, and he especially of works of Hebrew scholarship. Bochner had a younger sister. Fanny.

When a Russian invasion threatened in 1914. the family lied to Berlin. Although they were poorer after this move, young Salomon had much greater educational opportunities in the more liberal atmosphere of Berlin. He attended an outstanding gymnasium, brilliantly passing his entrance examinations only a few months after arriving in the city. He was strongly drawn to history and the humanities, but his mathematical talents had long been evident and the limited financial resources of his family impelled him to pursue a surer career in mathematics. He earned a Ph.D. from the University of Berlin in 1921 with a dissertation on orthogonal systems of complex analytic functions.

Inflation was then devastating Germany, so to help his family Bochner abandoned mathematics for a few years and went into the import-export business, in which he was successful but quite unhappy. His real interest was still mathematics; and at the urging and with the full support of his family, he returned to that field.

Boehner's dissertation topic had been treated independently by S. Bergman, so Bochner moved on to other topics. At that time Harald Bohr was developing the theory of almost periodic functions, which he described in short notes published in 1923. Bochner read these and was inspired to work out much of the theory on his own, in the process developing a highly original method of summation quite different from, and in some ways better than, that used by Bohr. This so impressed Bohr that he promptly invited the younger mathematician to visit Copenhagen. Bochner also discovered an alternative characterization of almost periodic functions in terms of the compactness of the sets of translates of the functions; this was the basis for future generalizations of the notion of almost periodicity by Bochner and others. From 1924 to 1926 Bochner was a fellow of the International Education Board, and in addition to Copenhagen he visited Oxford to work with G. H. Hardy and Cambridge to work with J. K. Littlewood. That led to an interest in yet another area, the theory of zeta functions and their functional equations, to which he would return later.

Bochner spent the years from 1926 to 1933 as a lecturer at the University of Munich. This was a very productive period, during which he began the research on Fourier analysis that was perhaps his greatest achievement. The culmination of this work at Munich was the publication in 1932 of *Vorlesungen über Fouriersche Integrale*, an influential book that has become a mathematical classic. Among much else the book contains Bochner's most famous theorem, characterizing the Fourier-Stielties transforms of positive measures as positive-definite functions; this result was the cornerstone of the subsequent development of abstract harmonic analysis. The hook contained the seeds of what developed in other hands into the theory of distributions, another major mathematical tool.

In 1937 Bochner published his generalization of the Lebesgue integral to functions with values in an infinite-dimensional normed linear space, the Bochner integral. In a quite different area, during his earlier years at Munich he studied the continuation of Riemann surfaces; his paper on that subject (1928) contained, incidentally, a result about mathematical logic that was discovered independently some seven years later In Max Zorn and is usually called Zorn's lemma. Bochner was an influential figure in Munich, not just among the mathematicians but also among the physicists He published papers on X-ray crystallography with H. Seylarth.

Therise of Nazism in Germany impelled Bochner to move in 1933, when he accepted a position at Princeton University. He became a naturalized citizen of the United States in 1938. Except for a visiting professorship at Harvard in the spring of 1947 and one at the University of California at Berkeley in the spring of 1953, he was away from Princeton only for brief trips and summers during the thirty-five years before his retirement. Until World War El broke out, he returned to Europe every summer to visit his family, to whom he was devoted. He urged them to move to England; and when they did so, he helped to arrange for his sister Fanny's children to attend good schools there. During one of these trips he met Naomi Weinberg, whose father was a New York real estate entrepreneur and founder (and for fifty years publisher) of the Jewish newspaper *The Day*. He and Naomi were married on I November 1938, with John von Neumann as best man. They had a daughter, Deborah.

Bochner was a major figure at Princeton and influential in the world of mathematics. During the 1930's a number of National Research Council fellows came to Princeton to work with him: Ralph Boas, R, H. Cameron, Norman Levinsom William T. Martin, and Angus Taylor, among others: K, Chandrasekharan and Kentaro Yano followed later. He had thirty-live doctoral students in a wide variety of fields, almost a quarter of the Ph.D.s in mathematics produced during his years on the Princeton faculty. When he was awarded the Leroy P. Steele Prize of the American Mathematical Society in 1979, he was cited for the cumulative impact of his mathematical work and for his influence on mathematics through his students. He was elected to the National Academy of Sciences in 1950 and served as vice-president of the American Mathematical Society from 1957 to 1958.

Bochner's research continued unabated at Princeton, in part as an extension of the work on almost periodic functions and Fourier analysis he had begun in Germany. He undertook the first profound investigation of the summation of Fourier series in several variables, discovering that Riemann's localization theorem, a basic result for Fourier series in one variable, fails in the case of several variables. John von Neumann had extended the theory of almost periodic functions to functions on arbitrary groups, and he and Bochner jointly extended the theory still further to functions with values in complete normed linear spaces. Bochner also introduced various generalized notions of almost periodicity.

With characteristic originality and facility Bochner extended his research in a number of new directions. In the late 1930's he began investigations in the theory of functions of several complex variables by determining the envelopes of holomorphy of tube domains. He continued in the 1940's by developing extensions of the Cauchy integral formula, including the Bochner-Martinelli formula that has been basic in the subject, and by using these formulas to characterize the boundary values of complex analytic functions. In this work, in the guise of conglomerate functions and their saltuses, there appeared the germ of the theory of cohomology groups with coefficients in holomorphic vector bundles, a theory that was extensively developed by others over the next quarter of a century as a basic tool in complex analysis. Bochner's outlook in this field was summarized in *Several Complex Variables* (1948), written with W. T. Martin, Also in the late 1930s Bochner began to work in differential geometry, showing that Riemann surfaces admit real-analytic embeddings into Euclidean spaces.

This research continued in the 1940's in joint work with Deane Montgomery in which they proved (hat the group of holomorphic automorphisms of a compact complex manifold is a complex Lie group. At the Princeton Bicentennial Conference in 1946, Bochner presented a proof showing that the additive Cousin problem always has a solution on a compact Köhlcr manifold with positive Ricci curvature, thus inaugurating the theory of curvature and Betti numbers that he developed further in the 1940's and 1950's This theory has been of fundamental importance, with a wide variety of applications—for instance, in the magisterial work of Kunihiko Kodaira characterizing algebraic manifolds as those complex manifolds that admit a Hodge metric. During the 1940's and 1950's Bochner also embarked on investigations in mathematical probability, summarized in his 1955 book *Harmonic Analysis and the Theory of Probability*. In addition he wrote a number of papers on partial differential equations. on zeta junctions, and on gamma factors, among other topics.

In the mid 1960's Bochner turned in an altogether new direction, devoting himself u wholeheartedly to the history and philosophy of science, particularly of mathematics. His books *The Role of Mathematics in the Rise of Science* (1966) and *Eclosion and Synthesis* (1969). as well as papers of that period, contain fascinating, idiosyncratic, and often provocative observations. They fuse his lifelong interest in history, literature, and philosophy, along with his undoubted gift for languages, with an understanding of mathematical creativity. He participated in the seminars of the history of science program at Princeton during this period, and was the only scientist member of the editorial board that supervised the publication of the five-volume *Dictionary of the History of Ideas*.

In 1968 Bochner retired from the Henry Burchard Fine professorship of mathematics al Princeton, which he had held since 1959, and moved to Rice University as the first Edgar Odell Lovett professor of mathematics. He was as influential at Rice as he had been at Princeton, if not more so. for he also served as chairman of the mathematics department there from 1969 to 1976 and took a great interest in developing the department. In addition to teaching mathematics he participated in the history of science program, was responsible for the foundation of an interdisciplinary institute for the history of ideas, and gave university-wide public lectures on the history of science. He died after a brief illness.

BIBLIOGRAPHY

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II. Secondary Literature. On Bochner's role in the "prehistory" of the theory of distributions, see J. Lutzen. *The Prehistory of the Theory of Distributions* (New York. 1982).

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