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(*b*. Bologna, Italy, January 1526; *d*. 1572)

algebra.

Rafael Bombelli's family came from Borgo Panigale, a suburb three miles north of Bologana. The original family name was Mazzoli. The Mazzolis, who seem to have been small landowners, adopted the name Bombelli early in the sixteenth century. Some of them were supporters of the Bentivoglio faction. An unsuccessful conspiracy to restore the Bentivoglio *signoria* in 1508 resulted in the execution of seven men, among whom was Giovanni Mazzoli, Rafael Bombelli's great-grandfather. Giovanni Mazzoli's property was confiscated but was later restored to his grandchildren. One of them was Antonio Mazzoli, alias Bombelli, who later became a wool merchant and moved to Bologna. There he married Diamante Scudieri, the daughter of a tailor. Six children were born to this marriage, of whom the eldest son was Rafael Bombelli.

All that is known about Bombelli's education is that his teacher (*Precettore*) was Pier Francesco Clementi of Corinaldo, an engineer-architect. It has been suggested that Bombelli might have studied at the University of Bologna, but this seems unlikely when one considers his family background and the nature of his profession. He spent the greater part of his working life as an engineer-architect in the service of his patron, Monsignor Alessandro Rufini, a Roman nobleman. Rufini was *cameriere* and favorite of Pope Paul III, and later was bishop of Melfi. The major engineering project on which Bombelli was employed was the reclamation of the marshes of the Val du Chiana. It was at a time when the reclamation work had been suspended that he wrote his treatise on algebra in the peaceful atmosphere of his patron's villa in Rome. His professional engagements seem to have delayed the completion of the book, but the more important part of it was published in 1572. His death soon afterward prevented the publishing of the remainder of the work. It was not published until 1929.

Bombelli's teacher, Pier Francesco Clementi, was employed by the Apostolic Camera (*ca*. 1548) in draining the marches of the Topino River at Foligno (100 miles from Rome). It is not known whether Bombelli himself worked in Foligno; but by 1551 he had began to work for Rufini in the reclamation of the Val di Chiana marshes. Rufini began to take an interest in this project in 1549, when the rights to take an interest in this project in 1549, when the rights of reclamation of that part of the marches which belonged to the Papal States were obtained by his nominee. Evidence of Bombelli' activity is found in the record relating to the marking out and settlement of the boundaries of the reclaimed land. The work of reclamation was interrupted sometime between 1555 and 1560. By 1560 Bombelli had returned to the Val di Chiana, and his work there ended in that year. In 1561 he was in Rome, where he took part in the unsuccessful attempt to repair the Ponte Santa Maria, one of the bridges over the Tiber.

Bombelli's work in the Val di Chiana earned him a reputation as an engineer, and led to his being one of the consultants on a proposed project for draining a part of the Pontine Marshes during the reign of Pius IV (1559–1565). The historian Nicolai, in his *De' bonificamenti delle Terre Pontine* (1800), says that the work was to have been directed by Rafael Bombelli, "famous among hydraulic engineers for having successfully drained the marches of the Val di Chiana." The Project was not realized, however.

Rafael Bombelli grew up in an Italy that was active in the production of works on practical arithmetic. Luca Pacioli, author of the *Summa di arithmetica, geometria,…* (1491), had lectured at Bologna at the beginning of the century. So had Scipione dal Ferro, a citizen of Bologna and one of the foremost mathematicians of the time. Their successors, Cardano, Tartaglia, and Ferrari, who were attempting the solution of the cubic and biquadratic equations, lived and worked in the neighboring cities of northern Italy. Cardano's *Practica arithmeticae* was published in 1539 and was followed in 1545 by his great treatise on algebra, the *Ars magna*, which gave the methods of dal Ferro and Ferrari for solving the cubic and biquadratic equations, respectively. In 1546 the controversy between Cardano and Tartaglia became public with the appearance of the latter's *Quesiti et inventioni diverse*. Copies of the *Cartelli di mathematicadisfida* (1547–1548), exchanged between Ferrari and Tartaglia, were circulated in the principal cities of Italy. Such was the climate in which Bombelli conceived the idea of writing a treatise on algebra. He felt that none of his predecessors except Cardano had explored the subject in depth; but Cardano, he thought, had not been clear in his exposition. He therefore decided to write a book that would enable anyone to master the subject without the aid of any other text. The work, written between 1557 and 1560, was a systematic and logical exposition of the subject in five parts, or books. In Book I, Bombelli dealt with the definitions of the elementary concepts(powers, roots, binomials, trinomials) and applications of the fundamental operations. In Book II he introduced algebraic powers and notation, and then went on to deal with the solution of equations of the first, second, third, and fourth degrees. Bombelli considered only equations with positive coefficients, thus adhering to the practice of his contemporaries. He was therefore obliged to deal with a large number of cases: five types of quadratic equations, seven cubic, and fortytwo biquadratic. For each type of equation, he gave the rule for solution and illustrated the rule with examples. Bombelli feared that the examples given in Book II would not be sufficient for a beginner who wished to master the subject, so he decided to include in Book III a series of problems by which the student would be taken, in stages, through the various operation of algebra. For this purpose he chose problems that were common to books on practical arithmetic of his day. Many of them were "applied problems"—that is, problems that had denominate numbers—and not mere exercises in manipulating symbols. They were often woven into incidents that could have occurred in the marketplace or tavern. Books IV and V formed the geometrical portion of the work. Book IV contained the application of geometrical methods to algebra, *algebra linearia*, and Book V was devoted to the application of algebraic methods to the solution of geometrical problems. Unfortunately, Bombelli was unable to complete the work as he had originally planned, in particular Books IV and V.

He had the opportunity, however, of studying a codex of Diophantus' *Arithmetic* in the Vatican Library during a visit to Rome. It was shown to him by Antonio Maria Pazzi, *lector ad mathematicam* at the University of Rome. They set out to translate the manuscript, but circumstances prevented them from completing the work. The changes that Bombelli made in the first three books of his *Algebra* show evidence of the influence of Diophantus. At the end of Book III, Bombelli said that the geometrical part, Books IV and V, was not yet ready for the publisher, but that it would follow shortly. His death prevented his keeping the promise. It was only in 1923 that the manuscript of the *Algebra* was rediscovered by Ettore Bortolotti in the Biblioteca Comunale dell' Archiginnasio in Bologna.

In his *Algebra*, Bombelli gave a comprehensive account of the existing knowledge of the subject, enriching it with his own contributions. Cardano had observed that the general rule given by dal Ferro could not be applied in solving the so-called irreducible case of the cubic equation, but Bombelli's skill in operating with "imaginary numbers" enabled him to demonstrate the applicability the rule even in this case of the rule even in this case. Because of the special nature and importance of these imaginary quantities, he took great care to make the reader familiar with them by introducing them early in his work—at the end of Book I. He said he had found "un altra sorte di radice cuba legata"("another kind of cube root of an aggregate") different from the others. This was the cube root of a complex number occurring in the solution of the irreducible case of the cubic equation. He called the square roots of a negative quantity *più di meno* and *meno di meno* (that is, *p. di m.* 10, *m. di m.* 10 ,). Having pointed out that the complex root is always accompanied by its conjugate, he set out the rules for operating with complex numbers and gave examples showing their application. Here he showed himself to be far ahead of his time, for his treatment was almost that followed today. Bombelli also pointed out that the problem of trisecting an angle could be reduced to that of solving the irreducible case of the cubic equation(this was illustrated in Book V). Although he made no significant contribution to the solution of the biquadratic equation, he showed the application of Ferrari' rule to every possible case.

In Book III of the printed version of the *Algebra* one finds no trace of the influence that practical arithmetics originally had on Bombelli. He said in the preface that he had deviated from the custom of those authors of arithmetics who stated their problems in the guise of human actions; his intention was to teach the "higher arithmetic." The problems of applied arithmetic that were originally included in Book III were left out of the published work; by doing so, Bombelli helped to raise algebra to the status of an independent discipline. In place of these applied problems he introduced a number of these applied problems he introduced a number of abstract problems, of which 143 were taken from the *Arithmetic* of Diophantus. Although Bombelli did not distinguish Diophantus's problems from his own, he acknowledged that he had borrowed freely from the *Arithmetic*. He was in fact the first to popularize the work of Diophantus in the West.

Apart from the solution of the irreducible case of the cubic equation, the most significant contribution Bombelli made to algebra was in the notation he adopted. He represented the powers of the unknown quantity by a semicircle inside which the exponent was placed: for the modern for x^2, and or for 5x. In the printed work the semicircle was reduced to an arc: . The zero exponent, , was used in the manuscript, for 48, but was omitted from the published work. The notation was used in the manuscript in applying the radical to the aggregate of two or more terms: for . He even used the radical sign as a double bracket: for . In the printed work the horizontal line was broken, and R, R_3 became *Rq*. *Rc*: for example,

Although incomplete, Books IV and V of the *Algebra* reveal Bombelli's versatility as a geometer. He had reduced some of the arithmetical problems of Book III to an abstract from and had interpreted them geometrically. He did not feel obliged to give geometrical proofs for the correctness of the results that he had obtained by algebraic methods. In doing so, he had broken away from a long-established tradition. The linear representation of powers, the use of the unit segment, and the representation of a point by "orthogonal coordinates" are some of the noteworthy features of this part of the work.

Bombelli was the last of the algebraists of Renaissance Italy. The influence that his *Algebra* had in the Low Countries is attested to by Simon Stevin and Adrien Romain. In the course of a short historical survey of the solution of equations, in his *Arithmetique*, Stevin referred to Bombelli as "great arithmetician of our time" He used a slightly modified form of Bombelli's notation for the powers of the unknown. While giving Bombelli due credit, he stressed the superiority of his notation to that of the Cossists. About a century later Leibniz, while teaching himself mathematics, used Bombelli's *Algebra* as a guide to the study of cubic equations. His correspondence with Huygens shows the keen interst these two men took in the work of the Italian mathematicians of the Renaissance. In the words of Leibniz, Bombelli was an "Outstanding master of the analytical art"

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