Borel, Émile (F | Encyclopedia.com

(*b*. Saint-Affrique, Aveyron, France, 7 January 1871; *d*. Paris, France, 3 February 1956)

mathematics.

Borel's father, Honoré, son of an artisan, was a Protestant village pastor. His mother, Émilie Teissié-Solier, came of a local merchant family. In 1882, already known as a prodigy, he left his father's school for the *lycée* at nearby Montauban. In Paris as a scholarship student preparing for the university, he entered the family circle of G. Darboux through friendship with his son, saw the "good life" of a leading mathematician, and set his heart on it. In 1889, after winning first place in the École Polytechnique, the École Normale Supérieure, and the general competitions, Borel chose the gateway to teaching and research, in spite of the blandishments of a special representative of the École Polytechnique.

Fifty years later Borel's colleagues celebrated the jubilee of his entrance to the École Normale, rightly considering that as the beginning of his scientific career. Indeed, he published two papers during his first year and appears to have established there his lifetime pattern of intensely serious and well-organized activity. He embraced an agnostic, scientific, and rational outlook that implied a responsible interest in all aspects of human affairs, and the extensive friendships of his undergraduate days helped make possible his broad cultural and political influence in later life. First in the class of 1893, he was promptly invited to teach at the University of Lille, where he wrote his thesis and twenty-two papers in three years before being called back to the École Normale, where publications, honors, and responsibilities piled up rapidly.

In 1901 Borel married Paul Appell's eldest daughter, Marguerite, who had interested him for some time but had only then turned seventeen. She wrote more than thirty novels (as Camille Marbo), was president of the Société des Gens de Lettres, and both assisted and complemented her husband's many-sided activity. They had no children but adopted Fernand Lebeau, son of the older of Borel's two sisters, after the early death of his parents. In 1906 they used money from one of Émile's prizes to launch *La revue du mois*, which appealed successfully to a very broad circle until the war and economic crisis killed it in 1920. During this period Borel's publications and activities showed a progressive broadening of interest from pure mathematics to applications and public affairs. Without seeming to diminish his mathematical creativity, he wrote texts and popularizations, edited several distinguished series of books, contributed to popular magazines and the daily press, played leading roles in professional and university affairs, and maintained acquaintances ranging from poets to industrialists.

Such an implausible level of activity was possible because Borel's uncommon intelligence and vigor were accompanied by efficient organization and self-discipline. He could be kind and generous of his time and energy in meeting his official or self-imposed obligations. He was even ready to risk his status for a good cause. But he had no time for "small talk" or trivial activity, seemed formidable and even rude to outsiders, and with increasing age grew more impatient with would-be wasters of his time. His lectures displayed his mind at work rather than a finished exposition, and his teaching consisted primarily in directing his students' efforts.

In 1909 Borel occupied the chair of theory of fonctions, newly created for him at the Sorbonne, and began thirty-two years on the University Council, representing the Faculty of Science. In 1910 he entered what he called the happiest time of his life as vice-director of the École Normale in charge of science students, but World War I cut it short. His service in sound location at the front (while Marguerite headed a hospital), and in organizing research and development in the War Office under his old friend Paul Painlevé, turned his interests more than ever toward applications. After the war he could not be happy again at the École Normale. There were "too many ghosts in the hallways." including that of his adopted son. At his request he moved to the chair of probability and mathematical physics at the Sorbonne and maintained only honorary connections with the cole Normale. The era was closed by the longest of his many trips abroad, including five months in China with Painlevé, and by his election to the Academy in 1921.

While continuing his flow of publications and his lectures in mathematics, Borel now moved rapidly into politics as mayor of Saint-Affrique (with Marguerite presiding over the Jury Femina), councillor of the Aveyron district, Radical and Radical-Socialist member of the Chamber of Deputies (1924–1936), and minister of the navy (1925). Important scientific legislation, the founding of the Centre National de la Recherche Scientifique, and several ships named after mathematicians are traceable to his initiative. He helped plan and raise funds for the Institut Henri Poincaré and served as director from its founding in 1928 until his death.

Retired from politics in 1936 and from the Sorbonne in 1940, Borel still had the vigor to produce more than fifty additional books and papers, to participate in the Resistance in his native village, to which he returned after a brief imprisonment by the Germans in 1940, and to travel extensively. The breadth of his services was recognized by such honors as the presidency of the

Academy (1934), the Grand Cross of the Legion of Honor (1950), the first gold medal of the Centre Nationale de la Recherche Scientifique (1955), the Croix de Guerre (1918), and the Resistance Medal (1945). A fall on the boat while returning from giving a paper at a meeting of the International Institute of Statistics in Brazil in 1955 hastened his death the following year at eighty-five.

Borel's undergraduate publications showed virtuosity in solving his elders' problems rather than great originality, but a "big idea" was incubating. Already in 1891 he was "extrêmement seduit" by Georg Cantor, whose "romantic spirit" mixed explosively with Borel's rigorous training in classical analysis and geometry. By sensing both the power and danger of set concepts, Borel anticipated the unifying themes of his lifework and of much mathematics in the twentieth century. In his thesis of 1894, which Collingwood rightly calls "an important mathematical event," can be found the ideas with which he initiated the modern theories of fonctions of a real variable, measure, divergent series, nonanalytic continuation, denumerable probability, Diophantine approximation, and the metrical distribution theory of values of analytical fonctions. All are related to Cantorian ideas, especially to the notion of a denumerable set. This is obvious for the two most famous results in the thesis, the Heine-Borel covering theorem (misnamed later by Schoenflies) and the proof that a denumerable set is of measure zero. The first asserted that if a denumerable set of open intervals covers a bounded set of points on a line, then a finite subset of the intervals is sufficient to cover. The second involves implicitly the extension of measure from finite sets of intervals to a very large class of point sets, now known as Borel-measurable sets.

Borel exploited his first insights in many directions. His *Leçons* of 1898 and other works laid the basis of measure theory so solidly that in that field the letter *B* means Borel. In 1905 he noticed that probability language was convenient for talking about measure of point sets, and in 1909 he introduced probability on a denumerable set of events, thus filling the gap between traditional finite and "geometrical" (continuous) probability. In the same paper he proved a special case of his strong law of large Borel remained skeptical of the of the actual infinite beyond the denumerable and of nonconstructive definitions. Much of his work was motivated by finitistic ideas, and his last book (1952) discussed his observation that most real numbers must be "inaccessible." since with a finite alphabet we can name at most a denumerable subset. By this caution he avoided some of the pitfalls into which others fell, but he also was barred from the fruits of greater daring. It was Lebesgue, Baire, Fréchet, and others who pushed set and measure theoretic ideas more boldly and so opened the way to the abstract analysis of the mid-twentieth century.

Other motivations are visible in Borel's work: the challenge of unsolved classical problems and visible gaps, an early and increasing admiration for Cauchy, an interest in physical and social problems, all tinged strongly with French patriotism. Often his solutions opened whole fields for exploitation by others. His "elementary" proof of Picard's theorem in 1896 not only created a sensation because the problem had resisted all attacks for eighteen years, but also established methods and posed problems that set the theme of complex function theory for a generation. Borel's work on divergent series in 1899 filled the gap between convergent and divergent series. His work on monogenic fonctions (summed up in his monograph of 1917) showed the primacy of Cauchy's idea of the existence of the derivative over the Weierstrassian notion of series expansion and filled the gap between analytic and very discontinuous fonctions.

Before World War I, Borel had worked out most of his original ideas, and thereafter his scientific publications were largely the development and application of earlier ideas and the solution of minor problems. A major exception is the series of papers on game theory (1921–1927) in which he was the first to define games of strategy and to consider best strategies, mixed strategies, symmetric games, infinite games, and applications to war and economics. He proved the minimax theorem for three players, aftersome doubts for five and seven, and finally (1927) conjectured its truth a year before John von Neumann independently first took up the subject and proved the general theorem. Although Borel's papers were overlooked until after von Neumann's work was well known, he must be considered the inventor, if not the founder, of game theory.

Borel's innovations are essential in twentieth-century analysis and probability, but his research methods belong rather to the nineteenth. He abjured generalization except when it was forced on him. He was motivated by specific problems and applications. He disliked formalism ("pure symbolism turning about itself"), logicism, and intuitionism (both too removed from the physical reality that he thought should guide mathematics). Borel was the most successful mathematician of his generation in using specific problems and results as scientific parables pointing the way to broad theories that still remain fertile.

BIBLIOGRAPHY

I. Original Works. A complete scientific bibliography to 1939 appears in *Selecta. Jubilé scientifique de M. Émile Borel* (Paris, 1940), and is extended to 1956 in the biographies by Collingwood and Fréchet, which also analyze Borel's work in detail. The papers in the *Selecta* are in part more representative of the commentators' interests than of Borel's most significant work, but a complete collected works is in preparation. His writings on philosophical questions, pedagogy, and social problems are well covered in *Émile Borel, philosophe et homme d'action, Pages choisies et presentées par Maurice Fréchet* (Paris, 1967). Borel's own analysis of his work appears in his *Notice sur les travaux scientifiques* (Paris, 1912) and his *Supplément (1921) à la Notice* (1912), in the *Selecta*. Very revealing also are his "Documents autobiographiques." in *Organon* (Warsaw). **1** (1936), 34–42, repr, in *Selecta*, and "Allocution," in *Notices et discours de l'Académie des Sciences*, **2** (1949), 350–359.

Of more than 300 scientific publications the most notable are his thesis, “Sur quclques points de la théorie des fonctions,” in *Annales de l’École Normale* 3rd ser., **12** (1895), 9–55: “Démonstration élémentaire d’un théorème de M. Picard sur les fonctions entières,” in *Comptes rendus de l’Académie des Sciences*, **122** (1896), 1045–1048: “Fondements de la théorie des séries divergentes sommables,” in *Journal de mathématique*, 5th ser. **2** (1896). 103–122: “Sur les zéros des fonctions entières,” in *Acta mathematica*, **20** (1897), 357–396: *Leçons sur la théorie des fonctions* (Paris, 1898: 4th ed., 1950), his most influential book: “Mémoire sur les séries divergentes.” in *Annales de l’École Normale*, 3rd ser., **16** (1899), 9–131, which won a grand prize of the Academy and led to over 200 papers by others during the following two decades; *Leçons sur les fonctions entières*(Paris, 1900; 2nd ed., 1921), an exposition of the work growing out of his paper on the Picard theorem; *Leçons sur les séries divergentes*(Paris, 1901; 2nd ed., 1928); *Leçons sur les fonctions de variables réeles et les développements en séries de polynomes* (Paris, 1905; 2nd ed., 1928); “Les probabilités dénombrables et leurs applications arithmétiques,” in *Rendiconti del Circolo Matematico di Palermo*, **27** (1909), 247–270; *Le hasard* (Paris, 1914), probably his best popularization: “I. Aggregates of Zero Measure. II. Monogenic Uniform Non-analytic fonctions,” in *Rice Institute Pamphlet*, 4th ser., **1** (1917). 1–52; *Leçons sur les fonctions monogénes uniformes d’un variable complexe* (Paris, 1917), the definitive exposition of his work in this area: “La théorie du jeu et les équational intégrales à noyau symétrique,” in *Comptes rendus de l’Académié des Sciences*, **173** (1921), 1302–1308—this and two later notes (1924, 1927) on game theory appear in translation with commentary by Fréchet and von Neumann in *Econometrica*, **21** (1953), 95–125; *Méthodes et problèmes de la théorie des fonctions* (Paris, 1922), a collection winding up his work in that area: *La politique républicaine* (Paris, 1924), his most substantial political work; *Principes et formules classiques du calcul des probabilités* (Paris, 1925), the first fascicle of the Traité; *Valeur pratique et philosophique des probabilités* (Paris, 1939), the last fascicle of the Traité; *Théorie mathématique du bridge à la portée de tous* (Paris, 1940), written with A. Cheron; *Le jeu, la chance et les théories scientifiques modernes* (Paris, 1941); “Sur l’emploie du théorème de Bernoulli pour faciliter le calcul d’une infinité de coefficients—Application au problème de l’attente è un quichet,” in *Comptes rendus de l’Académie des Sciences*, **214** (1942), 425–456, his last original contribution to probability theory; *Les probabilités* et la vie (Paris, 1943), another fine popularization with later editions and translations; *Éléments de la théorie des ensembles* (Paris, 1949), a summation containing some new results; and *Les nombres inaccessibles* (Paris, 1952), his last book.

Series that he edited (always contributing substantially also) include Collection de Monographies sur la Thèorie de Fonctions (Paris, 1898–1952)—sometimes called the Borel tracts, this totaled over fifty volumes, ten by Borel himself—and Cours Cours de Mathématiques (Paris, 1903–1912), a series of elementary texts designed to cover various curricula, usually written with collaborators. Other series include La Nouvelle Collection Scientifique (1910–1922), thirty-five popularizations: Bibliothèque d’Éducation par la Science (Paris, l924–1946), high-level popularizations for the educated layman: Traité de Calcul des Probabilités et de Ses Applications (Paris, 1925–1938), eighteen fascicles in four volumes, intended to cover the whole field as it had developed since 1875: Collection de Physique Mathématique (Paris, 1928–1950); and Collection de Monographies des Probahilités et de Leurs Applications (Paris, 1937–1950), seven volumes intended to supplement the Traité by current research.

II. Secondary Literature. Along with the material in the *Selecta* and Borel’s autobiographical writings cited above, the best sources are “Jubilé scientifique de M. Émile Borel… 14 janvier 1940” in *Notices et discours de l’Académie des Sciences*, **2** (1949), 324–359; L. de Broglie, *ibid*., **4** (1957), 1–24; E. F. Collingwood, in *Journal of the London Mathematical Society*, **34** (1959), 488–512, and **35** (1960), 384; M. Fréchet, “La vie et l’oeuvre d’Émile Borel,” in *Enseignement mathématique*. 2nd ser. **11** (1965), 1–95;M. Loève, “Integration and Measure,” in **Encyclopaedia Britannica** (1965); and P. Montel, in *l’Académie des Sciences*, **242** (1965), 848–850.

Kenneth O. May