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(*b*. Warsaw, Poland, 8 May 1905; *d*. Warsaw, Poland, 24 January 1982)

mathematics.

Borsuk was the son of Marian Borsuk and Zofia Muciejewska. His father was a well-known surgeon in Warsaw. After receiving a master’s degree in 1927 and a doctorate in 1930, both from the University of Warsaw, he became *Privatdozent* there in 1934. Borsuk married Zofia Paczkowska on 26 April 1936; they had two daughters.

After joining the faculty at Warsaw in 1929. Borsuk advanced to professor of mathematics in 1946 and director of the Mathematical Institute from 1952 to 1964. He was at the Institute for Advanced Study at Princeton from 1946 to 1947, and later visiting professor at the University of California at Berkeley (1959–1960) and at the University of Wisconsin at Madison (1963–1964).

Borsuk was vice director of the Institute of Mathematics of the Polish Academy of Sciences in 1956. He was corresponding member of the Polish and Bulgarian academies of sciences.

Borsuk worked primarily in the area of geometric topology. Although he is known for widespread contributions in topology, a particularly important discovery was the distillation of a central topological feature of polyhedra and its generalization to a larger class of spaces. This concept, that of absolute neighborhood retract, was introduced in Borsuk’s doctoral dissertation at the University of Warsaw under S. Mazurkiewicz, “Sur les rétractes” (published in 1931). and permeated a great deal of his work. It has greatly influenced the direction of research in topology throughout the world.

It will be helpful to define an absolute neighbor hood retract. Let X be a metric space and A a subset of *X*. A continuous function *r* from *X* to *A* is said to be a retraction provided it has the property that $r(x) = x$ for all *x* in *A*. If *X* has the property that whenever *X* is embedded as a closed subset of a metric space *Y*, there is a retraction of *Y* onto *X*, then *X* is said to be an absolute retract (AR). The unit interval and the real line are examples of AR’s. If whenever *X* is embedded in Las a closed subset. there is a retraction of a neighborhood of *X* in *Y* onto *X*, then *X* is said to be an absolute neighborhood retract (ANR). Prime examples of ANR’s are metric polyhedra and manifolds. If the polyhedron or man ifold is contractible. then it is in fact an AR. The abstracting of this property of polyhedra is one of the most remarkable accomplishments of geometric topology. Although the concept was due to Borsuk. the entire community of topologists contributed to its full development.

John H. C. Whitehead showed in 1950 that an ANR is homotopy equivalent to a polyhedron. Thus this property virtually characterizes spaces that are homotopy equivalent to polyhedra. Borsuk asked in 1954 whether a compact ANR is homotopy equivalent to a compact polyhedron. This query was answered in the affirmative by J. E. West. In 1974 Robert Edwards was able to bring together the results of many researchers in topology to show that the product of a compact ANR with the Hilbert cube is a Hilbert cube manifold and the product of a compact AR with the Hilbert cube is the Hilbert cube. Complete proofs of these results, together with references, are given in Thomas Chapman’s *Lectures on Hilbert Cube Manifolds* (1976). Borsuk had been intrigued by the Hilbert cube at an earls stage in his career. In *The Scottish Book* in 1938 he posed the following questions; Is it true that the product of a triod with the Hilbert cube is homeomorphic to the Hilbert cube? Is it also true that the infinite product of triods is homeomorphie to the Hilbert cube? An affirmative answer to these questions was given by Richard D. Anderson in 1964. A published proof of this result, together with the fact that the product of any compact polyhedron with the Hilbert cube is a Hilbert cube manifold, was given by Andersons student J. E. West. Theremarkable theorem by Kdwards would not have been possible without a thorough investigation into infinite dimensional topologv. of which the AndersonWest result was the preliminary essential step. Borsuk showed exceptional insight in conceiving of this conjecture so early in his career.

For most of his careei Morsuk was connected with the Universitv of Waisaw. During the Nazi occupation of Poland, he labored at keeping intellectual life in Poland alive through the “underground university.” This and other “illegal” activities led to his imprisonment, escape, and hiding until the end of the war.

When Poland began to rebuild, Borsuk and Kazimierz Kuratowski began the work of restoring mathematics in Warsaw, and Borsuk’s disrupted career came back into focus. He continued his studies in topology. In the mid 1960’s he came across another fundamental concept in topology, the notion of shape. An ANR is a very nice space with many convenient local properties. Unfortunately, there are many mathematical spaces that du not have nice local properties. It was Borsuk’s idea that such spaces could be “smoothed” by embedding them in AR’s, in particular by embedding them in the Hilbert cube. One could then study the ougmal space by studying the system of neighborhoods of the space in the Hilbert cube. Since these neighborhoods are ANR's, one is thus studying an arbitrary compact metric space by approximating it by a system of ANR's. Borsuk's first publication in shape theory was in 1968 in *Fundamenta mathematicae*. At about the same time he gave several talks in Europe and the United States disseminating his ideas.

Shape theory has had tremendous influence in topology. There are. however, complications in giving credit to Borsuk for its discovery because it has been shown that it is equivalent to several earlier theories and constructions. In particular. etale homotopy theory and the Kan and Cech extension of the homotopy functor on the category of polyhedra are equivalent to Borsuk's theory of shape. Several mathematicians discovered these equivalences independently at about the same time. Although these constructions are in a technical sense the same, it can certainly be said that Borsuk was the first to use these ideas as he did. His motivation was always to understand the geometry of separable metric spaces, and he showed how shape theory could be an effective tool for this purpose.

In recent times many different areas of topology have begun to merge. The theory of manifolds, the theory of CW-complexes and polyhedra, the theory of ANR's, combinatorial topology, homotopy theory, algebraic topology, shape theory, infinite-dimensional topology, and geometric topology have had considerable interaction. Borsuk played a significant role in developing several of these areas and in making them fit coherently into the whole.

Borsuk was much honored by Poland, receiving many decorations to honor his contributions to mathematics, education, and political life. He was also widely honored in the mathematical community. He participated in some twenty conferences and delivered major addresses at many of them. He gave numerous talks on his work at centers of learning. In 1978 Borsuk organized the International Conference on Geometric Topology, held in Warsaw. This conference demonstrated his widespread and profound influence on topology and the high regard in which he was held.

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