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(b. England, ca. 1290–1300; d. Lambeth, England, 26 August 1349)

*mathematics, natural philosophy, theology.*

Both the date and place of Bradwardine's birth are uncertain, although record of his early connection with Hartfield, Sussex, has often been taken as suggestive. His own reference (*De causa Dei*, p. 559) to his father's present residence in Chichester is too late to be relevant.

Our knowledge of Bradwardine's academic career begins with the notice of his inscription as fellow of Balliol College in August 1321. Two years later we find him as fellow of Merton College, a position he presumably held until 1335. We also have evidence of a number of other university positions during this period. The succession of his Oxford degrees would seem to be the following: B.A. by August 1321; M.A. by about 1323; B. Th. by 1333; D. Th. by 1348.

Bradwardine's ecclesiastical involvement appears to have begun with his papal appointment as canon of Lincoln in September 1333, although his less official entry about 1335 into the coterie of [Richard de Bury](#), then bishop of Durham, was probably of greater importance in determining the remainder of his career. For not only did this latter move place Bradwardine in more intimate contact with some of the more engaging theologians in England, but it also may well have proved to be of some effect in introducing him to the court of [Edward III](#). Indeed, shortly after his appointment as chancellor of [St. Paul's](#), London (19 September 1337), we find him as chaplain, and perhaps confessor, to the king (about 1338/1339). We know that he accompanied Edward's retinue, perhaps to Flanders, but certainly to France during the campaign of 1346. In point of fact, it was late in that year, in France, that Bradwardine delivered his (still extant) *Sermo epinicius* in the presence of the king, the occasion being the commemoration of the battles of Crécy and Neville's Cross. The closeness of his ties with Edward might also be inferred from the fact that the king annulled Bradwardine's first election to the archbishopric of Canterbury (31 August 1348). He was, however, elected a second time (4 June 1349), apparently without Edward's opposition, and consecrated at Avignon approximately a month later (10 July 1349). Bradwardine immediately returned to England, where, after scarcely more than a month as archbishop, he fell before the raging plague and died at the residence of the bishop of Rochester in Lambeth, 26 August 1349.

Although our evidence is not absolutely conclusive, it seems highly probable that Bradwardine composed all of his philosophical and mathematical works between the onset of his regency in arts at Oxford and approximately 1335.

*Early Logical Works.* In spite of the lack of any direct testimony, it is nevertheless a reasonable assumption that the early logical works were the result of a youthful Bradwardine first trying his hand at a kind of activity common, even expected, among recent arts graduates in the earlier fourteenth century. A number of these logical treatises ascribed to Bradwardine are undoubtedly spurious, but at least two seem, to judge in terms of present evidence, to be most probably genuine: *De insolubilibus* and *De incipit et desinit*. Neither of these works has been edited or studied, yet the likelihood is not great that they will eventually reveal themselves to be much more than expositions of the *opinio communis* concerning their subjects. Both treatises are of course relevant to the history of medieval logic, but the *De incipit et desinit*, like the many other fourteenth-century tracts dealing with the same topic, had a direct bearing upon current problems in natural philosophy as well. For the medieval works grouped under this title (or under the alternative *De primo et ultimo instanti*) address themselves to the problem of ascribing what we would call intrinsic or extrinsic boundaries to physical changes or processes occurring within the continuum of time. Thus, to cite the fundamental assumption of Bradwardine's *De incipit* (an assumption shared by almost all his contemporaries), the duration of the existence of a permanent entity (*res permanens*) that lasts through some temporal interval is marked by the fact that it possesses a first instant of being (*primum instans in esse*) but no last instant of being (*ultimum instans in esse*); its termination is signified, rather, by an extrinsic boundary, a first instant of nonbeing (*primum instans in non esse*).

*Tractatus de proportionibus velocitatum in motibus.* It is this work, composed in 1328, that has firmly established Bradwardine's position within the history of science. As its title indicates, it treats of the "ratios of speeds in motions", a description of the contents of the *Tractatus* that becomes more properly revealing once one identifies the basic problem Bradwardine set out to resolve: How can one correctly relate a variation in the speeds of a mobile (expressed, as in the work's title, as a "ratio of speeds") to a variation in the causes, which is to say the forces and resistances, determining these speeds? The proper answer to this question is, without doubt, the fundamental concern of the *Tractatus de proportionibus*. In Bradwardine's own words, to find the correct solution is to come upon the *vera sententia de proportione Velocitatum in motibus, in comparatione ad moventium et motorum potentias* (Corsby ed., p. 64).

Answers to this question were, Bradwardine points out, already at hand. Yet they failed, he argues, to resolve the problem satisfactorily. Basically, their failure lay in that they would generate results which were inconsistent with the “postulate” of Scholastic-Aristotelian natural philosophy which stipulated that motion could ensue only when the motive power exceeded the power of resistance: when, to use modern symbols,  $F > R$ . Thus, for example, one unsatisfactory answer was that implied by Aristotle. For, although Aristotle was certainly not conscious of Bradwardine’s problem as such and, it can be argued, never had as a goal the firm establishment of any mathematical relation as obtaining for the variables involved, the medieval natural philosopher took much of what he had to say in the *De caelo* and the *Physics* (especially in bk. VII, Ch. 5) to entail what is now most frequently represented by  $V \propto F/R$ . But this will not do. For, if one begins with a given  $F_1 > R_1$ , and if one continually doubles the resistance (i.e.,  $R_2 = 2R_1$ ,  $R_3 = 2R_2$ , etc.), then  $F_1$  as a randomly given mover will be of infinite capacity (*quelibet potentia motiva localiter esset infinita*[*ibid.*, p. 98]). In our terms, what Bradwardine intends by his argument is that, under the continual doubling of the resistance, if we hold  $F_1$  constant, then at some point one will reach  $R_n > F_1$ , which, on grounds of the suggested resolution represented by  $V \propto F/R$ , implies that some “value” would still obtain for  $V$ ; this in turn violates the “motion only when  $F > R$ ” postulate. Therefore,  $V \propto F/R$  is an unacceptable answer to his problem. Bradwardine also sets forth related arguments against other possible answers, which are usually symbolized by  $V \propto F - R$  and  $V \propto (F - R)/R$ .

The correct solution, in Bradwardine’s estimation, is that “the ratio of the speeds of motions follows the ratio of the motive powers to the resistive powers and vice versa, or, to put the same thing in other words: the ratios of the moving powers to the resistive powers are respectively proportional to the speeds of the motions, and vice versa. And,” he concludes, “geometric proportionality is that meant here” (*Proportio velocitatum in motibus sequitur proportionem potentiarum moventium ad potentias resistivas, et etiam econtrario. Vel sic sub aliis verbis, eadem sententia remanente: Proportiones potentiarum moventium ad potentias resistivas, et velocitates in motibus, eodem ordine proportionales existunt, et similiter econtrario. Et hoc de geometrica proportionalitate intelligas* [*ibid.*, p. 112]).

Given just this much, it is not at all immediately clear what Bradwardine had in mind. His intentions reveal themselves only when one begins to examine his succeeding conclusions and, especially, the examples he uses to support them. If we generalize what we then discover, we can, in modern terms, say that his solution to his problem of the corresponding “ratios” of speeds, forces, and resistances is that speeds vary arithmetically while the ratios of forces to resistances determining these speeds vary geometrically. That is, to use symbols, for the series  $V/n$ , ...  $V/3$ ,  $V/2$ ,  $2V$ ,  $3V$ , ...  $nV$ , we have the corresponding series  $(F/R)^{1/n}$ , ...  $(F/R)^{1/3}$ ,  $(F/R)^{1/2}$ ,  $F/R$ ,  $(F/R)^2$ ,  $(F/R)^3$ , ...  $(F/R)^n$ . Or, straying an even greater distance from Bradwardine himself, we can arrive at the now fairly traditional formulations of his so-called “dynamical law”:

$$(F_1/R_1)^{V/R_1} = F_2/R_2 \text{ or } V = \log_a F/R, \text{ where } a = F_1/R_1.$$

Furthermore, if we continue our modern way of putting Bradwardine’s solution to his problem, we can more easily express the advantage it had over the medieval alternatives cited above. In essence, this advantage lay in the fact that Bradwardine’s “function” allowed one to continue deriving “values” for  $V$ , since such values—the repeated halving of  $V$ , for example—never correspond to a case of  $R > F$  (as was the case with  $V \propto F/R$ ); they correspond, rather, to the repeated taking of roots of  $F/R$ ; they correspond, rather, to the repeated taking of roots of  $F/R$ , and if the initial  $F_1 > R_1$  (as is always assumed), then for any such root  $F_n/R_n = (F_1/R_1)^{1/n}$ ,  $F_n$  is always greater than  $R_n$ . With this in view, it would seem that Bradwardine’s most notable accomplishment lay in discovering a mathematical relation governing speeds, forces, and resistances that fit more adequately than others the Aristotelian-Scholastic postulates of motion involved in the problem he set out to resolve.

It is of the utmost importance to note, however, that almost all we have said in expounding Bradwardine’s function goes well beyond what one finds in the text of the *Tractatus de proportionibus* itself. Notions of arithmetic versus geometric increase or of the exponential character of his “function” may well translate his intentions into our way of thinking, but they also simultaneously tend to mislead. Thus, to speak of exponents at once implies or suggests a mathematical sophistication that is not in Bradwardine, and also obscures the relative simplicity of his manner of expressing (by example, to be sure) his “function.” This simplicity derives from the symmetrical use of the relevant terminology: If one *doubles* a speed, he would say, then one *doubles* the ratio of force to resistance, and if one *halves* the speed, then one *halves* the ratio. Although we feel constrained to note that doubling or halving a *ratio* amounts, in our terms, to squaring or taking a square root, such an addendum was unnecessary for Bradwardine, since for him the effect of applying such operations as doubling or halving to ratios was unambiguous. To double  $A/B$  always gave—again in our terms— $(A/B)^2$ , and never  $2(A/B)$ . What is more, the examples Bradwardine utilizes to express his function deal merely with doubling and halving, a factor which makes it evident that he was still at a considerable remove from the general exponential function so often invoked in explaining the crux of the *Tractatus de proportionibus*.

This limitation not only derived from the fact that the relevant material in Aristotle so often spoke of doubles and halves, but may also be related to the possible origin of Bradwardine’s function itself. The locale of this origin is medieval pharmacology, where we find discussion of a problem similar to Bradwardine’s; in place of investigating the corresponding variations between the variables of motion, we have instead to do with an inquiry into the connection of variables within a compound medicine and its effects. Given any such medicine, how is a variation in the strength (*gradus*) of its effect related to the variation of the relative strengths (*virtutes*) of the opposing qualities (such as hot-cold or bittersweet) within the medicine which determine that effect? As early as the ninth century, the Arab philosopher al-Kindi replied to this question by stipulating that while the *gradus* of the effect increases arithmetically, the ratio of the opposing *virtutes* increases geometrically (where this geometric increase follows the progression of successive “doubling,” that is, squaring). Now, not only was the pertinent

text of al-Kindi translated into Latin, but the essence of his answer to this pharmacological puzzle was analyzed, developed, and, in a way, even popularized by the late thirteenth-century physician and alchemist [Arnald of Villanova](#). From Arnald's work al-Kindi's "function" found its way into early fourteenth-century pharmacological works, even into the *Trifolium* of Simon Bredon, fellow Mertonian of Bradwardine in the 1330's.

Now it is certainly possible, indeed even probable, that Bradwardine may have appropriated his function from the al-Kindian tradition (a borrowing that may also have occurred in the case of the use of "exponential" relations within certain fourteenth-century alchemical tracts as well). But even admitting this, Bradwardine did a good deal more than simply transfer the function from the realm of compound medicines to the context of his problem of motion. For, quite unlike his pharmacological forerunners, he developed the mathematics behind his function by axiomatically connecting it with the whole medieval mathematics of ratios as he knew it. Thus, the entire first chapter (there are but four) of his *Tractatus* is devoted to setting forth the mathematical framework required for his function. A beginning exposition of the standard Boethian division of particular numerical ratios (e.g., *sesquialtera*, *superpartiens*, etc.) furnishes him with the terminology with which he was to operate. Second, and of far greater insight and importance, he axiomatically tabulated the substance of the medieval notion of composed ratios. That is, to use our terms,  $A/C$  is composed of (*componiturex*)  $A/B$ ,  $B/C$ . Furthermore (and here lies the specific connection with Bradwardine's function), when  $A/B = B/C$ , then  $A/C = (A/B)^2$ . Or, as Bradwardine stated in general, if  $a_1/a_2 = a_2/a_3 = \dots a_{n-1}/a_n$ , then  $a_1/a_n = a_1/a_2 \cdot a_2/a_3 \cdot \dots a_{n-1}/a_n$  and  $a_1/a_n = (a_1/a_2)^{n-1}$ . In point of fact, the insertion of geometric means and the addition of continuous proportionals that are here manipulated were Bradwardine's, and the standard medieval, way of dealing with what are (for us), respectively, the roots and powers involved in his function.

The fact that Bradwardine was thus able to state in its general form the medieval mathematics behind his function suggests that, although his expression of the function itself in mathematical terms was never general, this was due to his inability to formulate such a general mathematical statement. The best he could do was, perhaps, to give his function in the rather opaque, and certainly mathematically ambiguous, form we have quoted in *extenso* above, and then merely to express the mathematics of it all by way of example.

Proper generalization of Bradwardine's function had to await, it seems, his successors. Hence, John Dumbleton, like Bradwardine a Mertonian, gave a more general interpretation of the function through a more systematic investigation of its connections with the composition of ratios (see his *Summa de logicis et naturalibus*, pt. III, chs. 6–7). He also extended Bradwardine by translating him, as it were, into the n current language of the latitude of forms (that is, equal latitudes of motion [V] always correspond to equal latitudes of ratio [F/R], where the corresponding "scales" of such latitudes are, respectively, what we would term arithmetic and geometric).

A further development of Bradwardine, in many ways the most brilliant, occurred in the *Liber calculationum* of [Richard Swineshead](#), yet another Mertonian successor. In *Tractatus XIV* (entitled *Demotu locali*) of this work, Swineshead elaborates his predecessor's function by setting forth some fifty-odd rules that, assuming Bradwardine to be correct, specify which different *kinds* of change (uniform, difform, uniformly difform, and so on) in  $F/R$  obtain relative to corresponding variations in  $V$ . Swineshead also extended Bradwardine in *Tractatus XI* (*De loco elementi*) of his *Liber calculationum*, where, in what is something of a fourteenth-century mathematical tour de force, he applies his function to the problem of the motion of a long, thin, heavy body (*corpus columare*) near the center of the universe.

Another significant medieval development of Bradwardine's function was effected by [Nicole Oresme](#) in his *De proportionibus proportionum*. Here one observes an extension of the mathematics implicit in the function into a whole new "calculus of ratios" in which rules are prescribed for dealing with what are, for us, rational and irrational exponents. Moreover, Oresme then applies this calculus to the problem of the possible incommensurability of heavenly motions and the consequences of such a possibility for astrological prediction.

Many other Scholastic legatees of the Bradwardinian tradition could be cited as well, but, unlike the three we have mentioned above, most appear to have concerned themselves chiefly with rather belabored expositions of what Bradwardine meant, although a few, such as [Blasius of Parma](#) and Giovanni Marliani, produced somewhat unimpressive dissents from his opinion.

One should not close an account of the *Tractatus de proportionibus* without some mention of its final chapter. Here, in effect, Bradwardine attacks the question of the appropriate measure of a body in uniform motion, a matter that becomes problematic when rotational movement is considered. Again investigating, and rejecting, proposed alternative solutions to his question, he argues that the proper measure must be determined by the fastest-moving point of the mobile at issue. Once more his decision bore fruit, especially in his English successors. The resulting "fastest moving point rule" gave birth to an extensive literature treating of the sophisms that arise when one attempts to apply the rule to bodies undergoing condensation and rarefaction or generation and corruption. (The work of the Mertonian [William Heytesbury](#) furnishes the best example of this literature.)

*Tractatus de continuo*. In book VI of the *Physics*, Aristotle had formulated a battery of arguments designed to refute, once and for all, the possible composition of any continuum out of indivisibles. Like all Aristotelian positions, this received ample confirmation and elaboration in the works of his Scholastic commentators. Yet two features of the medieval involvement with this particular segment of *Physics VI* are especially important as background to Bradwardine's entry, with his *Tractatus de continuo*, into what was soon to become a heated controversy among natural philosophers. To begin with, from the end of the thirteenth century on, Scholastic support and refortification of Aristotle's anti-indivisibles. Like all Aristotle's anti-indivisibilist position almost always included a series of mathematical arguments that did not appear in the *Physics* itself or in

the standard commentary on it afforded by Averroës Considerable impetus and authority were given to the inclusion of such arguments by the fact that Duns Scotus had seen fit to feature them, as it were, in his own pro-Aristotelian treatment of the “continuum composition” problem in book II of his *Commentary on the Sentences*. The second important medieval move in the history of this problem occurred in the early years of the fourteenth century, when we witness the eruption of anti-Aristotelian, proindivisibilist sentiments. These two factors alone do much to explain the nature and purpose of Bradwardine’s treatise, for he wrote it (sometime after 1328, since it refers to his *Tractatus de proportionibus*) to combat the rising tide of atomism, or indivisibilism, as personified by its two earliest adherents: Henry of Harclay (chancellor of Oxford in 1312) and Walter Chatton (an English Franciscan, *fl. ca.* 1323). Furthermore, in attacking the atomistic views of his two adversaries, Bradwardine used as his most lethal ammunition the appeal to mathematical arguments that, as we have noted, were by now standard Scholastic fare. But he developed this application of mathematics to the problem at issue far beyond that of his predecessors.

The *Tractatus de continuo* was, first of all, mathematical in form as well as content, for it was modeled on the axiomatic pattern of Euclid’s *Elements*, beginning with twenty-four “Definitions” and ten “Suppositions,” and continuing with 151 “Conclusions” or “Propositions,” each of them directly critical of the atomist position. These “Conclusions” purport to reveal the absurdity of atomism in all branches of knowledge (to wit: arithmetic, geometry, music, astronomy, optics, medicine, natural philosophy, metaphysics, logic, grammar, rhetoric, and ethics) but the nucleus of it all lies in the geometrical arguments Bradwardine brought to bear upon his opponents.

To understand, even in outline, the substance and success of what Bradwardine here accomplished, one should note at the outset that the atomism he was combating was, at bottom, mathematical. The position of the fourteenth-century atomistic thinker consisted in maintaining that extended continua were composed of nonextended indivisibles, of points. Given this, Bradwardine astutely saw fit to expand the mathematical arguments that were already popular weapons in opposing those of atomist persuasion. Such arguments can be characterized as attempts to reveal contradictions between geometry and atomism, in which the revelation takes place when assorted techniques of radial and parallel projection are applied to the most rudimentary of geometrical figures. For example, parallels drawn between all the “point-atoms” in opposite sides of a square will destroy the incommensurability of the diagonal, while the construction of all the radii of two concentric circles will, if both are composed of extensionless indivisibles, entail the absurdity that they have equal circumferences. In applying these and related arguments, Bradwardine effectively demolished the atomist contentions of his opponents, at least when they maintained that the atoms composing geometrical lines, surfaces, and solids were finite in number or, if not that, were in immediate contact with one another. His success (and that of others who employed similar arguments against atomism) was not as notable, however, in the case of an opponent who held that continua were composed of an infinity of indivisibles between any two of which there is always another. Here the argument by geometrical projection faltered due to a failure—which Bradwardine shared—to comprehend the one-to-one correspondence among infinite sets and their proper subsets (although this property was properly appreciated, it seems, by Gregory or Rimini in the 1340’s).

The major accomplishment of Bradwardine’s *Tractatus de continuo* lay, however, in yet another mathematical refutation of his atomist antagonist. To realize the substance of what he here intended, we should initially note that Aristotle’s own arguments in *Physics* VI against indivisibilism made it abundantly clear that the major problem for any prospective mathematical atomist was to account for the connection or contact of the indivisibles he maintained could compose continua. As if to grant his opponent all benefit of the doubt, Bradwardine suggests that this problematic contact of point-atoms might appropriately be interpreted in terms of the eminently respectable geometrical notion of superposition (*superpositio*), a respectability guaranteed for the medieval geometer in the application of this notion within Euclid’s proof of his fundamental theorems of congruence (*Elements*, I, 4 and 8; III, 24). However, immediately after this concession to the opposing view, Bradwardine strikes back and, in a sequence of propositions, conclusively reveals that the superposition of any two geometrical entities systematically excludes their forming a single continuum. Consequently, the urgently needed contact of atoms is geometrically inadmissible.

Finally, as if to reveal his awareness of the mathematical basis of his whole *Tractatus*, toward its conclusion Bradwardine puts himself the question of whether, in using geometry as the base of his refutation of atomism, he had perhaps not begged the very question at issue; does not geometry *assume* the denial of atomism from the outset? He replies by carefully pointing out that while some kinds of atomism are, by assumption, denied in geometry, others are not. And he explains why and how. In our terms, he has attempted to point out just which continuity assumptions are independent of the axioms and postulates, both expressed and tacit, of Euclid’s *Elements* and which are not. That he realized the pertinence of such an issue to the substance of the medieval continuum controversy is certainly much to his credit.

*Geometria speculativa and Arithmetica speculativa.* These two mathematical works, about which we lack information concerning the date of composition, are both elementary compendia of their subjects and were intended, it seems plausible to claim, for arts students who may have wished to learn something of the quadrivium, but with a minimal exposure to mathematical niceties. The *Arithmetica* is the two and appears to be little more than the extraction of the barest essentials of Boethian arithmetic. More interesting, both to us and to the medievals themselves, to judge from the far greater number of extant manuscripts, is the *Geometria speculativa*. From the mathematical point of view, it contains little of startling interest, although it does include elementary materials not developed in Euclid’s *Elements* (e.g., stellar polygons, isoperimetry, the filling of space by touching polyhedra [*impletio loci*], and so on). Of greater significance would seem to be Bradwardine’s concern with relating the mathematics being expounded to philosophy, even to selecting his mathematical material on the basis of its potential philosophical relevance. Such a guiding principle was surely in Bradwardine’s mind when he saw fit to have his compendium treat of such philosophically pregnant matters as the horn angle, the incommensurability of the diagonal of a

square, and the puzzle of the possible inequality of infinites. Indeed, it is precisely to passages of the *Geometria* dealing with such questions that we find reference in numerous later authors, such as Luis Coronel and John Major. Such authors were fundamentally philosophers—philosophers, moreover, with little mathematical expertise—and it would seem fair to conclude that Bradwardine had just this type of audience in view when he composed his *Geometria*.

*Theological Works.* Bradwardine's earliest venture into theology is perhaps represented by his treatment of the problem of predestination, extant in *questio* entitled *De futuris contingentibus*. His major theological work, indeed the *magnum opus* of his whole career, is the massive *De causa Dei contra Pelagium et de virtute causarum ad suos Mertonenses*, completed about 1344. Its primary burden was to overturn the contemporary emphasis upon [free will](#), found in the writings of those with marked nominalist tendencies (the "Pelagians" of the title), and to reestablish the primacy of the Divine Will. Although this reaffirmation of a determinist solution to the problem of [free will](#) is not of much direct concern to the history of science, brief excursions into sections of the *De causa Dei* have revealed that it is not without interest for the development of late medieval natural philosophy. Thus, to cite but two instances, Bradwardine discusses within the context of rejecting the possible eternity of the world, again struggles with the issue of unequal infinites. One is tempted to suggest that closer study of the *De causa Dei* will reveal that Bradwardine's theological efforts contain yet other matters of importance for the history of science.

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*Logical Works.* The unedited *De insolubilibus* is extant in at least twelve MSS, including Erfurt, Amplon. 8° 76, 6r–21v and Vat. lat. 2154, 13r–24r. For the equally unedited *De incipit et desinit*: Vat. lat. 3066, 49v–52r and Vat. lat. 2154, 24r–29v. Although Bradwardine's treatises are not considered, the kinds of problems they bear upon are dealt with (for the *De insolubilibus*) in I. M. Bochenski, *A History of Formal Logic* (Notre Dame, Ind., 1961), pp. 237–251; and (for the *De incipit*) in Curtis Wilson, *William Heytesbury. Medieval Logic and the Rise of Mathematical Physics* (Madison, Wis., 1956), pp. 29–56. A variety of other logical writings, although often ascribed to Bradwardine in MSS, are most likely spurious; they are too numerous to mention here.

*Tractatus de proportionibus velocitatum in motibus.* This has been edited and translated, together with an introduction, by H. Lamar Crosby as *Thomas of Bradwardine. His Tractatus de Proportionibus. Its Significance for the Development of Mathematical Physics* (Madison, Wis., 1955). Corrections to some of Crosby's views can be found in Edward Grant, ed., [Nicole Oresme. De proportionibus proportionum and Ad pauca respicientes](#) (Madison, Wis., 1966), pp. 14–24, a volume that also contains a text, translation, and analysis of Oresme's extension of the mathematics of Bradwardine's "function". For the problem and doctrine of Bradwardine's *Tractatus* one should also note Marshall Clagett, *The Science of Mechanics in the Middle Ages* (Madison, Wis., 1959), pp. 215–216, 220–222, 421–503; and Anneliese Maier, *Die Vorläufer Galileis im 14. Jahrhundert*, 2nd ed. (Rome, 1969), pp. 81–110. Discussion of some of the factors of the above-cited application of Bradwardine's function in *Liber Calculationum* of [Richard Swineshead](#) can be found in John E. Murdoch, "Mathesis in philosophiam scholasticam introducta: The Rise and Development of the Application of Mathematics in Fourteenth Century Philosophy and Theology", in *Acts of the IV th International Congress of Medieval Philosophy, Montreal, 1967* (in press); and M. A. Hoskin and A. G. Molland, "Swineshead on Falling Bodies: An Example of Fourteenth Century Physics", in *British Journal for the History of Science*, **3** (1966), 150–182, which contains an edition of the text of Swineshead's *De loco elementi* (*Tractatus XI* of his *Liber calculationum*). For a new interpretation of how Bradwardine's function should be understood, see A. G. Molland. "The Geometrical Background to the 'Merton School': An Exploration Into the Application of Mathematics to Natural Philosophy in the Fourteenth Century", in *British Journal for the History of Science*, **4** (1968), 108–125. A brief discussion and citation of the relevant texts in Dumbleton that treat of Bradwardine will appear in an article by John Murdoch in a forthcoming volume of the *Boston University Studies in the Philosophy of Science*. Finally, the issue of the probable pharmacological origin of Bradwardine's function is treated in Michael McVaugh, "[Arnald of Villanova](#) and Bradwardine's Law", in *Isis*, **58** (1967), 56–64.

*Tractatus de continuo.* The as yet unpublished text of this treatise was first indicated, and extracts given, in Maximilian Curtze, "Über die Handschrift R. 4° 2: Problematum Euclidis Explicatio, des Königl. Gymnasial Bibliothek zu Thorn", in *Zeitschrift für Mathematik und Physik*, **13** (1868), Hist.-lit. Abt., 85–91. A second article giving a partial analysis of the contents of the *Tractatus* is Edward Stamm, "Tractatus de Continuo von Thomas Bradwardina", in *Isis*, **26** (1936), 13–32, while V. P. Zoubov gives a transcription of the enunciations of the definitions, suppositions, and propositions of the *Tractatus*, with an accompanying analysis of the whole, in "Traktat Bradwardina 'O Kontinuume", in *Istoriko-mate-maticheskii Isslodovaniia*, **13** 1960, 385–440. A critical edition of the text has been made from the two extant MSS (Torun, Gymn. Bibl. R. 4° 2, pp. 153–192; Erfurt, Amplon. 4° 385, 17r–48r) by John Murdoch and will appear in a forthcoming volume on mathematics and the continuum problem in the later [Middle Ages](#). Some indication of the issues dealt with in the *De continuo* can be found in

Anneliese Maier, *Die Vorläufer Galileis im 14. Jahrhundert*, 2nd ed. (Rome, 1966), pp. 155–179; and John Murdoch, “*Rationes mathematicae*”. *Un aspect du rapport des mathématiques et de la philosophie au moyen âge*, Conférence, Palais de la Découverte (Paris, 1961), pp. 22–35; “Superposition, Congruence and Continuity in the Middle Ages”, in *Mélanges Koyré*, I (Paris, 1964), 416–441; and “Two Questions on the Continuum: Walter Chatton (?). O.F.M. and Adam Wodeham, O.F.M.,” in *Franciscan Studies*, **26** (1966), 212–288, written with E. A. Syanan.

*Mathematical Compendia. The Arithmetica speculativa* was first printed in Paris, 1495, and reprinted many times during the fifteenth and sixteenth centuries, *The Geometria speculativa* (Paris, 1495) was also republished, and has recently been edited by A. G. Molland in his unpublished doctoral thesis, “*Geometria speculativa* of [Thomas Bradwardine](#): Text with Critical Discussion” (Cambridge, 1967); cf. Molland’s “The Geometrical Background to the ‘Merton School’”, cited above. Brief consideration of the *Geometria* can also be found in Moritz Cantor, *Vorlesungen über Geschichte der Mathematik*, 2nd ed., II (Leipzig, 1913), 114–118. One might note that a good deal of Bradwardine’s *Geometria* was repeated in a fifteenth-century *Geometria* by one Wigandus Durnheimer (MS Vienna, Nat. Bib. 5257, 1r–89v). The *Rithmomachia* ascribed to Bradwardine (MSS Erfurt, Amplon. 4° 2, 38r–63r; Vat. Pal. lat. 1380, 189r–230v) is most probably spurious.

The *Questio de futuris contingentibus* was edited by B. Xiberta as “Fragments d’une question de Tomas Bradwardine”, in *Beiträge zur Geschichte der Philosophie des Mittelalters*, Supp. 3, 1169–1180. The *editio princeps* of the *De causa Dei* at the hand of Henry Savile (London, 1618) has recently been reprinted (Frankfurt, 1964). The basic works dealing with Bradwardine’s theology are Gordon Leff, *Bradwardine and the Pelagians* (Cambridge, 1957), and H. A. Obermann, *Archbishop Thomas Bradwardine. A Fourteenth Century Augustinian* (Utrecht, 1958), whose bibliographies give almost all other relevant literature. For the discussion of void space and infinity in the *De causa Dei*, see Alexandre Koyré, “Le vide et l’espace infini au XIV<sup>e</sup> siècle,” in *Archives d’histoire doctrinale et littéraire du moyen âge* **17** (1949), 45–91; and John Murdoch, “*Rationes mathematicae*” (see above), pp. 15–22. Also of value are A. Combes and F. Ruello, “Jean de Ripa I Sent. Dist XXXVII: De modo inexistendi divine essentie in omnibus creaturis,” in *Traditio*, **23** (1967), 191–267; and Edward Grant, “Medieval and Seventeenth-Century Conceptions of an Infinite Void Space Beyond the Cosmos,” in *Isis* (in press).

If one disregards the various epitomes of the *De causa Dei*, it would appear that the only remaining work which may well be genuine is a *Tractatus de meditatione* ascribed to Bradwardine (MSS Vienna, Nat. Bibl. 4487, 305r–315r; Vienna, Schottenkloster 321, 122r–131v). Of the numerous other works that are in all probability spurious, it will suffice to mention the *Sentence Commentary* in MS Troyes 505 and the *Questions physice* in MS Vat. Pal. lat. 1049, which is not by Bradwardine but, apparently, by one Thomas of Prague.

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