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(*b*. Mendoza, Argentina, 14 September 1920;

d. Chicago, Illinois, 16 April 1998), *mathematics, analysis, partial differential equations, singular integrals*.

Calderón was one of the most original mathematicians of the twentieth century. He had a profound influence in the development of a wide range of topics, from harmonic analysis to partial differential equations and their multiple applications. His first fundamental contribution was the theory of singular integrals, which he developed jointly with the Polish mathematician Antoni Zygmund, at the University of Chicago. The Calderón-Zygmund school projected its intellectual influence all over the mathematical world.

From Mendoza to Chicago . Calderón was born in Mendoza, Argentina. Since he was gifted in mathematics and was keenly interested in mechanics, his father sent him to a preparatory Swiss school, with the idea of having him study engineering in Zürich. But after two years he was called home, and he finished high school in Mendoza. He attended the University of Buenos Aires, from where he graduated in 1947 with a degree in civil engineering. Calderón took a research position at YPF, the Argentine state oil corporation, where he studied mathematical problems related to oil prospecting. Fortunately for mathematics, he quit YPF in 1948, just at the time when Professor Zygmund was visiting the University of Buenos Aires.

Zygmund was offering a seminar for students, and gave each participant a topic taken from his fundamental treatise *Trigonometric Series* (1935). Calderón joined the seminar, together with two other future distinguished mathematicians, Mischa Cotlar and Luis Santaló. According to Cotlar, the way Zygmund “discovered” Calderón was a memorable event. At the seminar, Calderón had to make a presentation of the Marcel Riesz theorem on the continuity of the Hilbert transform in L^p for 1 <*p* <?. His presentation, short and elegant, was liked by his fellow students, but as it went on, Zygmund grew increasingly restless and began to grimace. Finally, Zygmund interrupted Calderón to ask where he had learned that proof. A subdued Calderón answered that he had read it in *Trigonometric Series*, but Zygmund vehemently informed the audience that such proof was not in his book. After the seminar, Calderón explained to Zygmund that he had tried first to prove the theorem by himself, but thinking it was too difficult, had read a couple of lines in *Trigonometric Series* without turning the page, assuming he had figured the rest of the proof. In fact what Calderón had figured was a shorter and elegant new proof of the Marcel Riesz theorem.

The episode of the unread proof illustrates a constant in Calderón’s research attitude: he always approached a problem in his own way, found his own proofs, and developed his own methods–finding new insights in the process. Zygmund realized immediately Calderón’s potential, and they wrote together two joint papers while still in Buenos Aires. Not surprisingly, Zygmund invited Calderón to visit the Department of Mathematics of the University of Chicago on a Rockefeller Fellowship.

From an Outstanding Thesis to a New Theory . Calderón arrived at the University of Chicago in 1949, when its department of mathematics was considered the best in the world. The chairman of the department was Marshall Stone, and the senior faculty included A. Adrian Albert, Shiing-Shen Chern, Saunders McLane, Lawrence Graves, and André Weil. Paul Halmos, Irving Kaplansky, Irving Segal, and Edward Spanier were the junior faculty. Calderón was so awed by such star-studded firmament that he considered returning to Argentina, but Zygmund did his best to retain him—fortunately with success.

Calderón was not seeking a doctoral degree, and did not feel inclined to write a dissertation. One day Stone called him to his office and asked for his three recent papers in harmonic analysis. Stone then stapled the three papers together and said "This is your thesis!" It was a revolutionary PhD thesis indeed, since it dealt with harmonic analysis in the circle bypassing complex methods, thus opening the door to the extensions to n-dimensional Euclidean space, which were at the core of Zygmund’s new program.

The Calderón-Zygmund collaboration started in Buenos Aires in 1948 and continued until Zygmund’s death in 1992. It resulted in the famous theory of singular integrals in several variables. An integral is singular if it converges while its integrand tends to infinity. The classical example in one variable is the Hilbert transform of a function $f(x)$ given by , which is known to be convergent by the theory of analytic functions in one complex variable. Calderón and Zygmund devised a real-variable method for the convergence of the Hilbert transform. The generalization to several real variables are the multidimensional singular integrals.

One of the first and most useful results of this fertile collaboration was the C-Z decomposition lemma, to deal specifically with singular integrals of integrable functions. The C-Z decomposition lemma became an indispensable tool in itself, which permeated analysis and probability theory.

Later came the memorable Calderón program, starting with the boundedness of the “first commutator,” to follow with his study of the Cauchy integral on Lipschitz curves. From the successes of the Calderón program stemmed the more precise theory of Ronald R. Coifman, Yves Meyer, and Alan McIntosh, the remarkable “T-1 theorem” of Guy David and Jean-Lin Journé, and the fundamental work of Michael Lacey and Christopher Thiele on the bilinear Hilbert transform.

The methods of Calderón and Zygmund, seen at first as marginal by most analysts, became mainstream in the later 1960s. This was due to the epoch-making contributions of Calderón to the theory of partial differential equations, obtained through singular integral operators.

International Fame . Calderón achieved international fame first with the proof of the uniqueness of the Cauchy problem and then with his demonstration of the existence and uniqueness theory for hyperbolic differential equations. Both results were obtained by using singular integral operators (SIOs) and the method of the Calderón projector, which reduced elliptic boundary value problems to singular equations on the boundary. Furthermore, Calderón introduced the algebras of SIOs, which play a significant role in nonsmooth problems. Such algebras, developed by Robert Seeley, Calderón’s first student, proved crucial to the proof of the Atiyah-Singer index theorem. The applications of algebras of SIOs to the theory of partial differential equations led directly to the theory of pseudodifferential operators, developed first by Joseph J. Kohn and Louis Nirenberg, and then by Lars Hormander. Calderón and Rémi Vaillancourt made important contributions to that theory.

The SIOs with kernels infinitely differentiable off the diagonal are a special case of the pseudodifferential operators. Yet Calderón was more interested in SIOs with non-smooth kernels for potential applications to quasi-linear and nonlinear problems. In his final remarks at the 1977 international conference in his honor, held at the University of Chicago, he insisted on his viewpoint that algebras of SIOs with nonsmooth kernels (and thus not pseudo-differentials), are the ones suitable for solving concrete problems in physics and engineering, where lack of smoothness prevails.

Interest in Applied Problems . Calderón was keenly interested in the application of mathematics to physics and engineering problems. He was proud of the applied results he obtained during his brief stint at YPF in Argentina during 1947–1948, and even prouder when questions unsolved at that time were still unanswered. Calderón wrote on the applications of the Radon transform and on the phase problem 3-D Fourier transforms, crucial to 3-D crystallography. He felt good when his friend Paul Malliavin attributed his program of stochastic analysis to the influence of Calderón’s paper on the uniqueness of the Cauchy problem. He was intrigued that the Calderón decomposition formula—a solution of the identity developed for complex interpolation in Banach spaces in the early 1960s—was rewritten as a decomposition of a wavelet and its inverse. In sum, he enjoyed very much seeing how his pure and applied work had influenced applied areas as diverse as signal processing, impedance tomography, geophysics, and wavelet theory.

Academic Career . The first academic position held by Calderón was as assistant professor at Ohio State University (1950–1953). After two years at the Institute for Advanced Study in Princeton, New Jersey, he joined the Massachusetts Institute of Technology (MIT) as associate professor (1955–1959). Calderón returned to the University of Chicago as professor (1959–1972; department chairman 1970–1972). He went back to MIT for three years and returned to Chicago in 1975 as university professor of mathematics until his retirement in 1985. During the late 1960s and early 1970s, he spent time in Argentina as a professor at the University of Buenos Aires and as director of the Argentine Institute of Mathematics (Instituto Argentino de Matemáticas).

Calderón had twenty-seven PhD students, among them some well-known mathematicians. In chronological order by school, they are Robert Seeley, Jerome Neuwirth, Irwin Bernstein, Israel Katz, and Robert Reitano, at MIT; Earl Berkson, Evelio Oklander, Cora Sadosky, Stephen Vagi, Umberto Neri, John Polking, Néstor Rivière, Carlos Segovia Fernández, Miguel de Guzmán, Daniel Fife, Alberto Torchinsky, Keith Powls, Carlos Kenig, Kent Merryfield, Michael Christ, and Gerald Cohen at the University of Chicago; and Josefina Dolores Álvarez Alonso, Telma Caputti, Angel Gatto, Cristián Gutiérrez, María Amelia Muschietti, and Marta Urciolo at the University of

Buenos Aires. Now the Calderón-Zygmund school comprises mathematicians from many countries.

Honors . Calderón received the National Medal of Science (United States, 1991; Zygmund had received the same honor in 1986), the Wolf Prize (Israel, 1989, together with John Milnor), and the Böcher (1989) and Steele (1979) prizes from the American Mathematical Society, and the Consagración Nacional prize (Argentina, 1989). He was elected a Fellow of the American Academy of Arts and Sciences in 1957; a member of the National Academy of Sciences of the United States in 1968; and corresponding member of the national academies of Argentina, France, and

Spain. He also received doctor honoris causa diplomas from the University of Buenos Aires (1969), the Technion Institute of Haifa (1989), Ohio State University (1995), and the Universidad Autónoma de Madrid (1997).

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