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(*b*. St. Petersburg, Russia, 3 March 1845; *d*. Halle, Germany, 6 January 1918),

mathematics, set theory.

Cantor's father, Georg Waldemar Cantor, was a successful and cosmopolitan merchant. His extant letters to his son attest to a cheerfulness of spirit and deep appreciation of art and religion. His mother, Marie Böhm, was from a family of musicians. Her forebears included renowned violin virtuosi; and Cantor described himself also as "rather artistically inclined," occasionally voicing regrets that his father had not let him become a violinist.

Like his father, Cantor was a Protestant; his mother was Catholic. The link with Catholicism may have made it easier for him to seek, later on, support for his philosophical ideas among Catholic thinkers.

Cantor attended the Gymnasium in Wiesbaden, and later the Grossherzoglich-Hessische Realschule in Darmstadt. It was there that he first became interested in mathematics. In 1862 he began his university studies in Zurich, resuming them in Berlin in 1863, after the sudden death of his father. At that time Karl Weierstrass, famed as a teacher and as a researcher, was attracting many talented students to the University of Berlin. His lectures gave analysis a firm and precise foundation, and later many of his pupils proudly proclaimed themselves members of the "Berlin school" and built on the ideas of their teacher.

Cantor's own early research on series and real numbers attests to Weierstrass' influence, although in Berlin he also learned much from Kummer and Kronecker. His dissertation, *De aequationibus secundi gradus indeterminatis*, dealt with a problem in number theory and was presented to the department by Kummer. In those days it was still the custom for a doctoral candidate to have to defend his scholarly theses against some of his fellow students. Worthy of note is Cantor's third thesis, presented on receiving his doctor's degree in 1867: *In re mathematica ars proponendi pluris facienda est quam solvendi*. And indeed his later achievements did not always consist in *solving* problems. His unique contribution to mathematics was that his special way of asking questions opened up vast new areas of inquiry, in which the problems were solved partly by him and partly by successors.

In Berlin, Cantor was a member (and from 1864 to 1865 president) of the Mathematical Society, which sought to bring mathematicians together and to further their scientific work. In his later years he actively worked for an international union of mathematicians, and there can have been few other scholars who did as much as he to generate and promote the exchange of ideas among scientists. He conceived a plan to establish an Association of German Mathematicians and succeeded in overcoming the resistance to it. In 1890 the association was founded, and Cantor served as its first president until 1893. He also pressed for international congresses of mathematicians and was responsible for bringing about the first ever held, in Zurich in 1897.

Thus Cantor was no hermit living within his own narrow science. When, later, he did sever ties with many of his early friends—as with H. A. Schwarz in the 1880'—the reasons lay in the nature of his work rather than in his character. During the Berlin years a special friendship had grown up between him and Schwarz, who was two years his senior. Both revered their teacher, Weierstrass; and both were concerned to gain the good opinion of Kronecker, who frequently criticized the deductions of Weierstrass and his pupils as unsound. These first years of Cantor's early research were probably the happiest of his life. His letters from that period radiate a contentment seldom granted him in later times, when he was struggling to gain acceptance for his theory of sets.

In 1869 Cantor qualified for a teaching position at the University of Halle, soon becoming associate professor and, in 1879, full professor. He carried on his work there until his death. His marriage in 1874 to Vally Guttmann was born of deep affection, and the sunny personality of the artistically inclined "Frau Vally" was a happy counter to the serious, often melancholy, temperament of the great scholar. They had five children, and an inheritance from his father enabled Cantor to build his family a house. In those days a professor at Halle was so poorly paid that without other income he would have been in financial straits. It was Cantor's hope to obtain a better-endowed, more prestigious professorship in Berlin, but in Berlin the almost omnipotent Kronecker blocked the way. Completely disagreeing with Cantor's views on "transfinite numbers," he thwarted Cantor's every attempt to improve his standing through an appointment to the capital.

Recognition from abroad came early, however. Cantor's friend Mittag-Leffler accepted his writings for publication in his then new *Acta mathematica*. He became an honorary member of the London Mathematical Society (1901) and of other scientific societies, receiving honorary doctor's degrees from Christiania (1902) and St. Andrews (1911).

The closing decades of Cantor's life were spent in the shadow of mental illness. Since 1884 he had suffered sporadically from deep depression and was often in a sanatorium. He died in 1918 in Halle University's psychiatric clinic. Schoenfliess was of the opinion that his health was adversely affected by his exhausting efforts to solve various problems, particularly the continuum problem, and by the rejection of his pioneering work by other eminent mathematicians.

Cantor has gone down in history as the founder of set theory, but the science of mathematics is equally indebted to him for important contributions in classical analysis. We mention here his work on real numbers and on representation through number systems. In his treatise on trigonometric series, which appeared in 1872,[1] he introduced real numbers with the aid of "fundamental series." (Today we call them fundamental sequences or Cauchy sequences.) They are sequences of rational numbers $\{a_n\}$ for which, given an arbitrarily assumed (positive, rational) $\in$, we have an integer n_1, such that

if $n \geqq n_1$ and m is an arbitrary positive integer. To series having this property Cantor assigned a "limit" b. if is a second sequence of the same kind and if for sufficiently large n, then the same limit b is assigned to this second sequence. We say today that the real number b is defined as an equivalence class of fundamental sequences.

Cantor further showed[2] that any positive real number r can be represented through a series of the type

(1) with coefficients c_γ that satisfy the inequality

Series such as (1) are known today as Cantor series. The work also contains a generalization of representation (1) and representations of real numbers in terms of infinite products. With these writings (and with several remarkable studies on the theory of Fourier series) Cantor established himself as a gifted pupil of the Weierstrass school. His results extended the work of Weierstrass and others by "conventional" means.

In November 1873, in an exchange of letters with his colleague Dedekind in Brunswick, a question arose that would channel all of Cantor's subsequent scientific labor in a new direction. He knew that it was possible to "count" the set of rational numbers, i.e., to put them into a one-to-one correspondence with the set of natural numbers, but he wondered whether such one-to-one mapping were not also possible for the set of real numbers. He believed that it was not, but he "could not come up with any reason." A short time later, on 2 December, he confessed that he "had never seriously concerned himself with the problem, since it seemed to have no practical value," adding, "I fully agree with you when you say that it is therefore not worth very much trouble."[3] Never the less, Cantor did further busy himself with the mapping of sets, and by 7 December 1873 he was able to write Dedekind that he had succeeded in proving that the "aggregate" of real numbers was uncountable. That date can probably be regarded as the day on which set theory was born.[4] Dedekind congratulated Cantor on his success. The significance of the proof had meanwhile become clear, for in the interim Cantor (and probably Dedekind, independently) had succeeded in proving that the set of real algebraic numbers is countable. Here, then, was a new proof of Liouville's theorem that transcendental numbers do exist. The first published writing on set theory is found in *Crelle's Journal* (1874).[5] That work, "Über eine Eigenschaft des Inbegrifhes aller reellen algebraischen Zahlen," contained more than the title indicated, including not only the theorem on algebraic numbers but also the one on real numbers, in Dedekind's simplified version, which differs from the present version in that today we use the "diagonal process," then unknown.[6]

Following his initial successes, Cantor tackled new and bolder problems. In a letter to Dedekind dated 5 January 1874, he posed the following question:

Can a surface (say, a square that includes the boundaries) be uniquely referred to a line (say, a straight-line segment that includes the end points) so that for every point of the surface there is a corresponding point of the line and, conversely, for every point of the line there is a corresponding point of the surface? Methinks that answering this question would be no easy job, despite the fact that the answer seems so clearly to be "no" that proof appears almost unnecessary.[7]

The proof that Cantor had in mind was obviously a precise justification for answering "no" to the question, yet he considered that proof "almost unnecessary." Not until three years later, on 20 June 1877, do we find in his correspondence with Dedekind another allusion to his question of January 1874. This time, though, he gives his friend reasons for answering "yes." He confesses that although for years he had believed the opposite, he now presents Dedekind a line of argument proving that the answer to the following (more general) question was indeed "yes":

We let $x_1, x_2, \ldots, x^p$, be p independent variable real quantities, each capable of assuming all values $\geqq 0$ and $\leqq 1$. We let y be a st variable real quantity with the same free range . Does it then become possible to map the ϱ quantities $x_1, x_2, \ldots, x_\varrho$) onto the one y so that for every defined value system $(x_1, x_2, \ldots, x_\varrho,)$ there is a corresponding defined value y and, conversely, for every defined value y one and only one defined value system $(x_1, x_2, \ldots, x_\varrho)$?[8]

Thus for $\varrho = 2$ we again have the old problem: Can the set of points of a square (having, say, the coordinates x_1, and x_2, $0 \leqq x_\gamma \leqq 1$) be mapped in a one-to-one correspondence onto the points of a line segment (having, say, the coordinates y, $0 \leqq y \leqq 1$)?

Today we are in a position to answer this question affir matively with a very brief proof.[9] Cantor's original deduction[10] was still somewhat complicated, but it was correct; and with it he had arrived at a result bound to seem paradoxical to the mathematicians of his day. Indeed, it looked as if his mapping had rendered the concept of dimension meaningless. But

Dedekind recognized immediately that Cantor's map of a square onto a line segment was discontinuous, suspecting that a continuous one-to-one correspondence between sets of points of different dimensions was not possible. Cantor attempted to prove this, but his deduction[11] did not stand up. It was Brouwer, in 1910, who finally furnished a complete proof of Dedekind's supposition.

Cantor's next works, dealing with the theory of point sets, contain numerous definitions, the orems, and examples that are cited again and again in modern textbooks on topology. The basic work on the subject by Kuratowski[12] contains in its footnotes many historical references, and it is interesting to note how many of the now generally standard basic concepts in topology can be traced to Cantor. We mention only the "derivation of a point set," the idea of "closure," and the concepts "dense" and "dense in itself." A set that was closed and dense in itself Cantor called "perfect" and he gave a remarkable example of a perfect discontinuous set. This "Cantor set" can be defined as the set of all points of the interval J 0; 1["which are contained in the formula

where the coefficients c_γ must arbitrarily assume the values 0 or 2, and the series may consist of both a finite and an infinite number of terms."[13] It was Cantor, too, who provided the first satisfactory definition of the term "continuum," which had appeared as early as the writings of the Scholastics. A continuous perfect set he called a continuum, thereby turning that concept, until then very vague, into a useful mathematical tool. It should be noted that today a continuum is usually introduced as a compact continuous set, a definition no longer matching Cantor's. The point is, though, that it was he who provided the first definition that was at all usable.

With his first fundamental work, in 1874,[14] Cantor showed that, with the aid of the one-to-one correspondence, it becomes possible to distinguish differences in the infinite: There are *countable* sets and there are sets having the power of a *continuum*. Of root importance for the development of general set theory was the realization that for every set there is a set of higher power. Cantor substantiated this initially through his theory of ordinal numbers. It can be seen much more simply, though, through his subset theorem, which appears in his published writings in only one place,[15] and there for only one special case. But in a letter to Dedekind dated 31 August 1899,[16] we find a remark to the effect that the so-called "diagonal process," which Cantor had been using, could be applied to prove the general subset theorem. The essence of his proof was the observation that there can be no one-to-one correspondence between a set L and the set M of its subsets. To substantiate this, Cantor introduced functions $f(x)$ that assign to the elements x of a set L the image values 0 or 1. Each such function defines a subset of L—the set, say, for which $f(x) = 0$—and to each subset $L' \subset L$ there belongs a function $f(x)$, which becomes 0 precisely when x belongs to L'.

Now, if there were a one-to-one correspondence between L and M, the set of functions in question could be written in the form $\phi(x, z)$

…so that through each specialization of z an element $f(x) = \phi(x, z)$ is obtained and, conversely, each element $f(x)$ of M is obtained from $\phi(x, z)$ through a particular specialization of z. But this leads to a contradiction, because, if we take $g(x)$ as the single-valued function of x which assumes only the values 0 or 1 and is different from $\phi(x, x)$ for each value of x, then $g(x)$ is an element of M on the one hand, while, on the other, $g(x)$ cannot be obtained through any specialization $z = z_0$, because $\phi(z_0, z_0)$ is different from $g(z_0)$.[17]

According to the subset theorem, for each set there is a set of higher power: the set of subsets or the power set $P(M)$. The question of a general "set theory" thus became acute. Cantor regarded his theory as an expansion of classical number theory. He introduced "transfinite" numbers (cardinal numbers, ordinal numbers) and developed an arithmetic for them. With these numbers, explained in terms of transfinite sets, he had, as Gutzmer remarked on the occasion of Cantor's seventieth birthday in 1915, opened up "a new province" for mathematics.

Understandably, the first probing steps taken in this new territory were shaky. Hence, the definition of the basic concept has undergone some noteworthy modifications. In Cantor's great synoptic work in the *Mathematische Annalen of* 1895 we read:

We call a power or cardinal number that general concept which with the aid of our active intelligence is obtained from the set M by abstracting from the nature of its different elements m and from the order in which they are given. Since every individual element m, if we disregard its nature, becomes a 1, the cardinal number M itself is a definite set made up merely of ones.[18]

Modern mathematics long ago dropped this definition, and with good reason. Today two sets are called "equal" if they contain the same elements, however often they are named in the description of the set. So if between the braces customarily used in defining these sets we place several l's, we then have the set with the single element 1: e. g., {1,1,1,1,1} = {1}.

Cantor himself may have sensed the inadequacy of his first definition. In discussing a book in 1884, and later in 1899 in a letter to Dedekind, he called a power "that general concept that befits it and all its equivalent sets." Today we would say, more simply, that "a cardinal number is a set of equivalent sets." But this second definition turns out to be inadequate, too. We know, of course, that the concept of a "set of all sets" leads to contradictions. From this, though, it follows that the concept of a "set of all sets equivalent to a given set M" is also inconsistent. We let M, for example, be a given infinite set and where encompasses the set of all sets. The set {} then naturally has the power 1, and the sets M^* (which contain only one element more than M) are all equivalent to M. Accordingly, the system of the sets M is a genuine subset of the "set of all sets equivalent to M." But since we can map this system into the elements of the "set of all sets," we thus have a concept that must lead to antinomies.

In short, in all of Cantor's works we find no usable definition of the concept of the cardinal number. The same is true of the ordinal number.

But the story does not end there. A third Cantor definition of a cardinal number appears in a report by Gerhard Kowalewski, included in his biography, *Bestand and Wandel*, of meetings that he had had with Georg Cantor. The eighty-year-old Kowalewski wrote the book shortly before his death, around 1950. With graphic vividness he tells of events and meetings that had occurred a half century before. Around 1900, Kowalewski was a *Privatdozent* in Leipzig. In those days the mathematicians from Halle and Leipzig used to meet about twice a month, first in one city, then in the other. On these occasions they would discuss their work. Although by then Cantor was no longer publishing, he often, according to the report of a young colleague, impressively held forth at the meetings on his theory of manifolds. This included his studies on the "number classes," the set of ordinal numbers that belong to equivalent sets. The numbers of the second number class were, for example, the ordinal numbers of the countable sets. Kowalewski then discusses powers (which were called "alephs"):

This "power" can also be represented, as was Cantor's wont, by the smallest or initial number of that number class, and the alephs can be identified with these initial numbers so that such and such would be the case if we wished to use those designations for the initial terms of the second and third number class from the Schoenfliess report on set theory.[19]

In a modern book on set theory we find the following definition of a cardinal number: "A cardinal number is an ordinal number which is not similar to any smaller ordinal number."[20] So modern mathematics has adopted Cantor's belated definition, which is not to be found anywhere in his published writings. It is unlikely that Stoll, author of the above book, ever read Kowalewski's biography. The modern view of the cardinal-number concept lay dormant for a time, to be embraced by younger scholars. One should mention, though, that even Cantor himself finally did arrive at the definition of a cardinal number considered "valid" today. True, this modern version of the concept does presuppose the availability of an ordinal-number concept. Here again we cannot accept Cantor's classical definition, for roughly the same reasons that prevent our accepting the early versions of the cardinal-number concept. Today, according to John von Neumann, an ordinal number is described as a well-ordered set w in which every element $\nu \in w$ is equal to the segment generated by ν:

$\nu = A_\nu$.

To sum up, we are not indebted to Cantor only for his initiative in developing a theory of transfinite sets. He proved the most important theorems of the new theory himself and laid the groundwork for the present-day definitions of the concept. It would be silly to hold against him the fact that his initial formulations did not fully meet modern precision requirements. One who breaks new ground in mathematics needs a creative imagination, and his initial primitive definitions cannot be expected to stand up indefinitely. When Leibniz and Newton founded infinite simal calculus, their definitions were crude indeed compared with the elegant versions of centuries later, refined by Weierstrass and his pupils. The same is true of the beginnings of set theory, and it is worthy of note that Cantor himself was inching ever closer to the definitions that the present generation accepts as "valid."

By his bold advance into the realm of the infinite, Cantor ignited twentieth-century research on the fundamentals. Hilbert refused to be driven out of the "paradise" that Cantor had created. But Cantor himself was not an axiomaticist. His way of thinking belonged more to the classical epoch. In the annotations to his *Grundlagen einer allgemeinen Mannigfaltigkeitslehre* (1883) he expressly acknowledged his adherence to the "principles of the Platonic system," although he also drew upon Spinoza, Leibniz, and Thomas Aquinas.

For Cantor the theory of sets was not only a mathematical discipline. He also integrated it into metaphysics, which he respected as a science. He sought, too, to tie it in with theology, which used metaphysics as its "scientific tool."[21] Cantor was convinced that the actually infinite really existed "both concretely and abstractly." Concerning it he wrote: "This view, which I consider to be the sole correct one, is held by only a few. While possibly I am the very first in history to take this position so explicitly, with all of its logical consequences, I know for sure that I shall not be the last!"[22]

Mathematicians and philosophers oriented toward Platonic thought accepted actual infinity abstractly, but not concretely. In a remarkable letter to Mittag-Leffler, Cantor wrote that he believed the atoms of the universe to be countable, and that the atoms of the "universal ether" could serve as an example of a set having the power of a continuum. Present-day physicists are not likely to be much interested in these quaint opinions. Today his philosophical views also appear antiquated. When we now ask what is left of Cantor's work, we can answer very simply: Everything formalizable is left. His statements in the realm of pure mathematics have been confirmed and extended by subsequent generations, but his ideas and conceptions in that of physics would not be acceptable to most of the present generation.

To the end Cantor believed that the basis of mathematics was metaphysical, even in those years when Hilbert's formalism was beginning to take hold. Found among his papers after his death was a shakily written penciled note (probably from 1913) in which he reaffirms his view that “without some grain of metaphysics” mathematics is unexplainable. By metaphysics he meant “the theory of being.” There are several important theorems in set theory that were first stated by Cantor but were proved by others. Among these is the Cantor-Bernstein equivalence theorem: “If a set A is equivalent to a subset $B' \subset B$ and B to a subset $A' \subset A$, then A and B are equivalent.” A simple proof of this, first demonstrated by Cantor’s pupil Bernstein, is found in a letter from Dedekind to Cantor.[23] That every set can be well ordered was first proved by Zermelo with the aid of the axiom of choice. This deduction provoked many disagreements because a number of constructivists objected to pure “existence theorems” and were critical of the paradoxical consequences of the axiom of choice.

More fundamental, though, were the discussions about the antinomies in set theory. According to a theorem proved by Cantor, for every set of ordinal numbers there is one ordinal number larger than all the ordinal numbers of the set. One encounters a contradiction when one considers the set of all ordinal numbers. Cantor mentioned this antinomy in a letter to Hilbert as early as 1895. A much greater stir was caused later by the Russell antinomy, involved in the “set of all sets which do not contain themselves as an element.” It was chiefly Hilbert who was looking for a way out of the impasse, and he proposed a strict “formalization” of set theory and of all mathematics. He hoped thereby to save the “paradise” that Cantor had created. We do not have time or space here to dwell upon the arguments that raged around Hilbert’s formalism. Suffice it to say that today’s generally recognized structural edifice of mathematics is form alistic in the Hilbertian sense. The concept of the set is preeminent throughout.

When we pick up a modern book on probability theory or on algebra or geometry, we always read something about “sets.” The author may start with a chapter on formal logic, usually followed by a section on set theory. And this specialized discipline is described as the theory of certain classes of sets. An algebraic structure, say, is a set in which certain relations and connections are defined. Other sets, defined by axioms concerning “neighborhoods,” are called spaces. In probability theory we are concerned with sets of events and such.

In Klaua’s *Allgemeine Mengenlehre* there is a simple definition of mathematics: “Mathematics is set theory.” Actually, we can regard all mathematical disciplines as the theory of special classes of sets. True, a high price (in Cantor’s eyes) was paid for this development. Modern mathematics deals with formal systems; and Cantor, probably the last great Platonist among mathematicians, never cottoned to the then nascent formalism. For him the problem of the continuum was a question in metaphysics. He spent many years attempting to show that there can be no power between that of the countable sets and the continuum. In recent time it has been shown (by Gödel and Cohen) that the continuum hypothesis is independent of the fundamental axioms of the Zermelo-Fraenkel system. That solution of the problem would not have been at all to Cantor’s taste. Yet had he not defended, against his antagonist Kronecker, the thesis that the essence of mathematics consists in its freedom?[24] Does this not include the freedom for a theory created by Cantor to be interpreted in a way not in conformity with his original ideas? The fact that his set theory has influenced the thinking of the twentieth century in a manner not in harmony with his own outlook is but another proof of the objective significance of his work.

NOTES

1.*Gesammelte Abhandlungen*. pp. 92 ff.

2.*Ibid*., pp. 35 ff.

3. E. Noether and J. Cavailles, *Briefwechsel Cantor-Dedekind*, pp. 115–118.

4. The first version of Cantor’s proof (in his letter to Dedekind) was published in E. Noether and J. Cavaillès, *Briefwechsel Cantor Dedekind*, pp. 29–34, and by Meschkowski, in *Probleme des Unendlichen*, pp. 30 ff.

5.*Gesammelte Abhandlungen*, pp. 115 ff.

6. See, for instance. Meschkowski, *Wandlungen des mathematischen Denkens*, pp. 31 ff.

7. E. Noether and J. Cavaillès, *Briefwechsel Cantor℃Dedekind*, pp. 20–21.

8.*Ibid*, pp. 25–26.

9. Mcschkowski, *Wandlungen des mathematischen Denkens*, pp.32 ff.

10.*Crelle’s Journal*, **84** (1878), 242–258; *Gesammelte Abhandlungen*, pp. 119 ff.

11.*Gesammelte Abhandlungen*, pp. 134 ff.

12. K. Kuratowski, *Topologie I, II* (Warsaw. 1952).

13.*Gesammelte Abhandlungen*, p. 193.

14. "Über eine Eigenschaft des Inbegriffes aller reellen algebrais-chen Zahlen.'in *Crelle's Journal*, **77** (1874), 258–262," also in *Gesammelte Abhandlungen*, pp. 115 ff.

15.*Gesammelte Abhandlungen*, pp. 278 ff.; *Jahresbericht der Deautschen Mathematikervereinigung*, **1** (1890–1891). 75–78.

16.*Gesammelte Abhandlungen*, p. 448.

17.*Ibid*., p 280.

18.*Mathematische Annalen*, **46** (1895), 481

19. G. Kowalewski, *Bestand and Wandel*, p. 202.

20. R. Stoll. *Introduction to Set Theory and Logic* (San Francisco, 1961), p. 317.

21. See Meschkowski, *Probleme des Unendlichen*, ch. 8.

22.*Gesammelte Abhandlungen*, p. 371.

23.*Ibid*. p. 449.

24.*Ibid*. p. 182.

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