Carathéodory, Constantin | Encyclopedia.com

(*b*. Berlin, Germany, 13 September 1873; *d*. Munich, Germany, 2 February 1950),

mathematics.

Carathéodory was descended from an old Greek family that had lived for several generations in Adrianople (now Edirne), Turkey. His grandfather had been a professor at the Academy of Medicine in Constantinople and attending physician to two Turkish sultans. His father, who had been a diplomat in the Turkish embassies in St. Petersburg and Berlin, was the Turkish ambassador in Brussels from 1875 on.

Carathéodory won prizes in mathematics while still in secondary school. From 1891 to 1895 he attended the École Militaire de Belgique. After completing his education, he went to Egypt in the employ of the British government as assistant engineer at the Asyut dam. In 1900, however, Carathéodory suddenly decided to go to Berlin and study mathematics. There he was stimulated by the students of Hermann Amandus Schwarz. In 1902 he followed his friend Erhard Schmidt to Göttingen, where, under Hermann Minkowski, he received the doctorate in 1904. Encouraged by Klein and Hilbert, who had recognized his genius, he qualified as a university lecturer a year later.

After having taught in Bonn, Hannover, Breslau, Göttingen, and Berlin, Carathéodory was called to Smyrna by the Greek government in 1920, to direct the completion of its university. In 1922, when Smyrna was burned by the Turks, Carathéodory was able to rescue the university library and take it to Athens. He then taught for two years at Athens University, and in 1924 accepted an invitation to the University of Munich as the successor of C. L. F. Lindemann. He remained there for the rest of his life.

For many years, Carathéodory was the editor of *Mathematische Annalen*. He was a member of scientific societies and academies in many countries; his membership in the Papal Academy was particularly noteworthy.

Carathéodory married a distant cousin, Euphrosyne Carathéodory. The marriage produced one son and one daughter.

Carathéodory was the most notable Greek mathematician of recent times, and the only one who does not suffer by comparison with the famous names of Greek antiquity. He made significant contributions to several branches of mathematics, and during the period of his activity he worked in all of them more or less simultaneously.

The first field was the calculus of variations, the theory of maxima and minima in curves. In his dissertation and in his habilitation thesis, Carathé odory drew up a comprehensive theory of discontinuous solutions (curves with corners). Previously, only the so-called Erdmann corner condition was known, but Carathéodory showed that all of the theory knownfor smooth curves can also be applied to curves with corners. He was also concerned with the fields of solution curves, which play a central part in the theory. Thoroughly familiar with the history of the subject, he drew upon many ideas of such mathematicians as Christiaan Huygens and Johann I Bernoulli. Inspired by these ideas, he restudied the relationship between the calculus of variations and first-order partial differential equations. The result of this was *Variationsrechnung and partielle Differentialgleichungen enter Ordnung*, which includes a quite surprising new “entry” to the theory.

By means of this method, Carathéodory was able to make significant progress in solving the so-called problem of Lagrange, i.e., variation problems with differential side conditions. He also wrote fundamental papers on the variation problems of m-dimensional surfaces in an n-dimensional space. Except for the case $m = 1$ of the curves, far-reaching results had until then been available only for the case $m = n - 1$. Carathéodory was the first to tackle the general case successfully. Here again, the consideration of fields played a decisive part.

Carathéodory was particularly interested in the application of the calculus of variations to geometrical optics. This interest is best shown in his “Elementare Theorie des Spiegelteleskops von B. Schmidt,” for which he carried out complete numerical calculations.

In the second main field, the theory of functions, Carathéodory’s achievements are in many areas: the problems concerning Picard’s theorem, coefficient problems in expansions in a power series, and problems arising from Schwarz’s lemma; he also significantly advanced the theory of the functions of several variables. His most important contributions, however, are in the field of conformal representation. The so-called main theorem of conformal representation of simply connected regions on the circle of unit radius had been proved rigorously for the first time shortly before World War I, and Carathéodory was able to simplify the proof greatly. His main achievement in this area was his theory of boundary correspondence, in which he investigated the geometrical-set theoretic properties of these boundaries in a completely new way.

The third main field consists of the so-called theory of real functions and the theory of the measure of point sets and of the integral. Carathéodory's book on this subject, *Vorlesungen über reelle Funktionen* (1918), represents both a completion of the development begun around 1900 by Borel and Lebesgue and the beginning of the modern axiomatization of this field. He returned to it in the last decade of his life, when he carried the axiomatization or, as he called it, the algebraization, of the concepts one step further.

In applied mathematics Carathéodory produced papers on thermodynamics and on Einstein's special theory of relativity.

BIBLIOGRAPHY

Carathéodory's articles were collected in *Gesammelte mathematische Schriften*. 5 vols. (Munich, 1954–1957); vol. V has autobiographical notes up to 1908 (pp. 387–408) and an obituary by Erhard Schmidt (pp. 409–419). Carathé odory's books are *Vorlesungen über reelle Funktionen* (Leipzig-Berlin, 1918; 2nd ed., 1928); *Conformal Representation*, Cambridge Tracts in Mathematics and Mathematical Physics, no. 28 (Cambridge, 1932); *Variationsrechnung and partielle Differentialgleichungen erster Ordnung* (Leipzig-Berlin, 1935; 2nd. ed 1956): *Geometrische Optik*, IV. Pt. 5 of the series Ergebnisse der Mathematik and ihrer Grenzgebiete (Berlin, 1937); *Reelle Funktionen*, I (Leipzig-Berlin, 1939); *Funktionentheorie*, 2 vols. (Basel, 1950); and *Mass and Integral and ihre Algebraisierung* (Basel, 1956).

An article on Carathéodory is Oskar Perron, "Constantin Carathéodory," in *Jahresbericht der Deutschen Mathematikervereinigung*, **55** (1952), 39–51.

H. Boerner