Cesàro, Ernesto | Encyclopedia.com

(*b* Naples, Italy, 12 March 1859; *d* Torre Annunziata, Italy, 12 September 1906),

mathematics.

Cesàro was the son of Luigi Cesàro and Fortunata Nunziante, his second wife. The elder Cesàro owned a farm and shop in Torre Annunziata; he was one of the first farmers in Italy to introduce agricultural machinery, a supporter of Italian unification, and a backer of Garibaldi's revolution of 1860—all of which led him into financial difficulties.

Ernesto Cesàro completed the first class of the Gymnasium in Naples, studied in the seminary of Nola for two years, then returned to Naples to finish the fourth class of the Gymnasium in 1872. In 1873 his father sent him to Liège to join his older brother Giuseppe, who had gone there in 1867. Cesàro stayed for a year with his brother, who in the meantime had become lecturer in mineralogy and crystallography at the École des Mines, and then entered the É colehimself on a scholarship. He matriculated there, then applied unsuccessfully for admission to an Italian university, following which he was forced to enter the École des Mines of Liège. He studied mathematics with Eugène Catalan, who noticed Cesàxo's talent and helped him publish his first mathematical paper in *Nouvelle correspondence de mathématiques*, of which Catalan was editor.

In 1879 Cesàro's father died, the family's financial troubles increased, and Cesàro was forced to return for some time to Torre Annunziata. Nevertheless, he finished the fourth form at Liège in 1881, and at the same time prepared his first major mathematical work, "Sur diverses questions d'arithmétique," which was published in 1883 in *Mémoires de l'Académie de Liège*. This paper brought Cesàro to the attention of the mathematical public.

In 1882 Cesàro returned once more to Belgium, having won another scholarship to continue his studies at Liège. Shortly thereafter he returned to Italy, where he married his stepbrother's daughter Angelina. At this period he also accompanied a friend, the son of the Prince de Soissons, to Paris, where he spent several months. He attended the lectures of Hermite, Darboux, Serret, Briot, Bouquet, and Chasles at the Sorbonne; he attracted the attention of Hermite in particular. In 1883 the latter cited Cesàro's results. Darboux's lectures led Cesàro to formulate his "intrinsic geometry."

Cesàro did not finish his studies at Liège, perhaps because of a personal quarrel with a professor Deschamps. He returned to Torre Annunziata and once again sought to continue his work in Italy. His mathematical works and the recommendations of Cremona, Battaglini, and Dino secured him a scholarship to the University of Rome, which he entered in 1884 in the fourth form in pure mathematics. Here, in addition to attending a great many lectures, he wrote some eighty works—on infinite arithmetics, isobaric problems, holomorphic functions, theory of probability, and, particularly, intrinsic geometry— inthe two years 1884–1886. Despite his intensive activity, he did not earn an advanced degree at this time. (The University of Rome gave him a doctorate, with honors, in February 1887.)

In 1886 Cesàro won a competition for the position of professor of mathematics at the Lycée Terenzio Mamiani in Rome; in similar competitions at the universities of Messina and Naples he placed first and second, respectively. On Cremona's advice, Cesàro left the Lycée Mamiani after a month to fill the vacant chair of higher algebra at the University of Palermo. He stayed at Palermo until 1891, when he accepted the chair of mathematical analysis at Naples. He held this chair until his death, never realizing his intention of going over to the chair of theoretical mechanics.

Cesàro's bibliography is extensive; indeed, the author of the most complete bibliography available, A. Perna, mentions 259 works and expresses doubt whether his list is complete. Cesàro's topics are varied. In 1878, when he was nineteen, he attempted to master certain topological problems in a nontraditionalway in his *Forme poliedrichi regoiari e semiregolari in tutti gli spazii* (published in Lisbon in 1888). The most prominent of his early works, however, deal with the sums of divergent series, for Cesáro, Borel, Fejé r, and Voronoj were together creating the techniques for the elaboration of such problems. One of Cesàaro's first published works, *Sur diverses questions d'arithmétique* (Liège, 1882), and, more importantly, his series of nine artides published in *Annali di mathematica pura ed applicata* (**13** [1885], 235–351) are related to the theory of numbers. He was here concerned with such problems as the determination of the number of common divisors of two numerals, determination of the values of the sum totals of their squares, the probability of incommensurability of three arbitrary numbers, and so on; these he attempted to apply obtained results in the theory of Fourier series. Later he occupied himself with prime numbers of a certain type and tried to make Chebyshev's formulas more precise ("Sulla distributione dei numeri primi," in *Rendiconti dell'Accademia delle scienze fisiche e matemetiche***2** [1896], 297– 304).

Despite the generally sophisticated level of his mathematics, Cesàro reverted to such elementary problems as, for example, his work on constructions using limited geometrical means (1899) which repeats results already known. His textbooks, on the other hand, are rather exacting. They were successful and influential in their time; *Corso di analisi algebrica con introduzione al calculo infinitesimale* was published in Turin in 1894 and *Elementa di calcolo infinitesimale* appeared in Naples in 1899. Both texts were the outgrowth of Cesàro's lectures in Palermo and Naples and both were distinguished by the pertinent and novel exercises that they contained.

The two textbooks also reveal Cesàro's interest in the problems of mathematical physics. In addition, his textbook *Introduzione alla teoria matematica della elasticità* (Turin, 1894) had dealt with the theory of elasticity in an elementary way; there is no doubt that he planned to investigate mathematical physics in more detail, since he prepared two works on this subject, "Teoria matematica dels calore" and "Lezionesull'idrodinamica"; he died before he could publish these, however, and they remain unpublished to this day.

Cesàro's most important contribution remains his intrinsic geometry. It has been noted that he began to develop it while he was in Paris in 1883; it occupied him, with interruptions, from that time on. His earlier work on the subject is summed up in his monograph *Lezione di geometria intrinseca* (Naples, 1896), in which he proceeds from a utilization of Darboux's method of a mobile coordinate trihedral—formed by the tangent, the principal normal, and the binormal at a variable point of a curve—and used it to simplify the analytic expression and make it independent of extrinsic coordinate systems. By this means, Cesà rostressed the intrinsic qualities of the objects examined. This method proved fertile for him, and he systematically elaborated and propagated it, while at the same time pointing out further applications. The *Lezione* also describes the curves that bear Cesàro's name; Cesàro later expanded his method to the curves devised by H. von Koch, which are continuous but so constructed as to have no tangent at any point.

The last part of *Lezione* deals with the theory of surfaces and multidimensional spaces in general. Cesàro returned to this subject in the last years of his life and emphasized the independence of his geometry from the axiom of parallels. The special selection of the square of the linear element enabled him to extend the results to multidimensional spaces with constant curvature. He further established other bases on which to build non-Euclidean geometry, which he described in *Rendiconti della R. Accademia del Lincei* ("Sui fondamenti della interseca non-euelidea," **13** [1904], 438–446) and more especially in "Fondamento intrinseco della pangeometria," in *Memorie della R. Accademia del Lincei* (**5** [1904], 155–183).

Throughout his life, the variety of Cesàro's interests was always remarkable, ranging from elementary geometrical problems to the application of mathematical analysis; from the theory of numbers to symbolic algebra; from the theory of probability to differential geometry. Moreover, his admiration for Maxwell, whose faithful interpreter in theoretical physics he became, is worthy of note.

In recognition of his work he was named an honorary member of many learned and scientific societies.

Cesàro died of injuries sustained while coming to the aid of his seventeen-year-old son who was drowning in the rough sea near Torre Annunziata.

BIBLIOGRAPHY

I. Original Works. In addition to the individual works cited in the text, extensive lists of Cesàro's writings may be found in A. Perna and P. del Pezzo, below, and in Poggendorff.

II. Secondary Literature. Works on Cesàro include C. Alasia. "Ernesto Cesàro, 1859–1906," published simultaneously in French and Italian in *Rivista di fisica, matematica e scienze naturali*, **15** (1907), 23–46, and *Enseignement mathematique*, **9** (1907), 76–82; V. Cerruti, "Ernesto Cesàro, Commemorazione," in *Rendiconti della R. Accademia dei Lincei*, **16** (1907). 76–82; A. Perna, "Ernesto Cesàro." in *Giarnaledi matematiche di Battaglini*, **45** (1907), 299–319, which includes a bibliography of 259 items as well as a presentation of some of Cesàro's problems and are view of his solutions; and P. del Pezzo, "Ernesto Cesàro," in *Rendiconti dell'Accademia delle Scienze fisiche e matematiche*, 3rd ser., **12** (1906), 358–375, which includes a list of 254 of Cesàro's works.

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