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(*b*. Jia Xin, Chekiang Province, China, 26 October 1911;

d. Tianjin, China, 3 December 2004), *mathematics, differential geometry*.

Chern was a highly influential figure in pure mathematics. From the 1940s onward he redefined the subject of differential geometry by drawing on, and contributing to, the rapid development of topology during the period. Despite spending most of his working life in the United States, he was also a source of inspiration for all Chinese mathematicians, and contributed in many ways to the development of the subject in China.

Early Life . Shiing-Shen Chern was born in the final year of the Qing dynasty, and educated at a time when China was only beginning to set up Western-style universities. His father was a lawyer who worked for the government. Chern first showed his mathematical ability when he was a student at the Fu Luen Middle School in Tsientsin, where he did all the exercises in classical English textbooks on algebra and trigonometry. He then went to Nankai University at the age of fifteen. There, mathematics was a one-man department run by Li-Fu Chiang, who had been a student of Julian Coolidge, and this ensured that Chern studied a great deal of geometry, particularly the works of Coolidge, George Salmon, Guido Castelnuovo, and Otto Staude. He became a postgraduate in 1930 at Tsinghua University in Beijing, where he met his wife Shih-Ning, the daughter of a professor. At Tsinghua Chern came under the influence of Dan Sun, one of the few mathematicians in China publishing research in mathematics. Sun's subject was projective differential geometry, which caught Chern's interest, and he studied in detail the works of the German mathematician Wilhelm Blaschke. After Blaschke visited Tsinghua in 1932 and lectured on differential-geometric invariants, Chern won a fellowship to study with him in Hamburg, Germany, for two years, and he received his DSc there in 1936 for work on the theory of webs, a subject central to Blaschke's work at the time. These were turbulent times in Germany: in Hamburg Chern met the mathematician Wei-Liang Chow, who had left Göttingen because of the flight of the best mathematicians from that university, and during the same period Blaschke was forced to resign from the German Mathematical Society for opposing the imposition of Nazi racial policies.

While in Hamburg, Chern studied the works of Elie Cartan and in 1936 spent a year in Paris with him. Cartan, who turned sixty-seven that year, was the dominant figure in geometry at the time, and had introduced new techniques that few people understood. The language in which to properly express Cartan's work was not then available, and it was ten years before the notation and terminology of fiber bundles allowed Chern to explain these concepts in a satisfactory way. The regular "Séminaire Julia" that year was devoted to expounding Cartan's work and Chern there met André Weil and other young French mathematicians who were the founders of the Bourbaki group that came to dominate French mathematics after World War II.

Move to the United States . In the summer of 1937 he took up the position of professor at Tsinghua, crossing the Atlantic, the United States, and the Pacific to do so, only to find that the Sino-Japanese war had begun. His university had moved, with the universities of Peking and Nankai, to Kunming. There, despite all the deprivations of war and virtually cut off as he was from the outside world, he found the time to work deeply through Cartan's work and came to his own vision of where geometry should be going. He was also able to teach many students who were to go on to make substantial contributions in mathematics and other fields—among them Chen Ning Yang, whose work in theoretical physics won him a Nobel Prize in 1957. Eventually, Chern was able to make his way to the Institute for Advanced Study in Princeton, New Jersey, on a series of military flights through India, Africa, Brazil, and Central America.

In Princeton, Hermann Weyl and Oswald Veblen already had a high opinion of Chern because of his papers. Chern soon got in touch with Claude Chevalley and Solomon Lefschetz and also with Weil in nearby Lehigh University. In Weil's words, "We seemed to share a common attitude towards such subjects, or towards mathematics in general; we were both striving to strike at the root of each question while freeing our minds from preconceived notions about what others might have regarded as the right or the wrong way of dealing with it" (Weil, 1992, p. 74). Chern and Weil worked and talked together to reveal the topological character of some of the new ideas in algebraic geometry. These included the Todd-Eger classes, whose definition was at the time derived in the old-fashioned spirit of Italian geometry, but which nevertheless caught Chern's imagination. These discussions provided the foundation of his most famous work on what became known as Chern classes (though he would always insist that the letter *c* by which they were denoted stood for "characteristic classes"). The ideas he developed at that time emerged in a concrete form in his new intrinsic proof (1944) of the general Gauss-Bonnet theorem— by his own account, one of his favorite theorems.

When World War II ended in 1945, Chern began another lengthy, complicated return to China, reaching Shanghai in March 1946. There, he was asked to set up an institute of mathematics as part of the Academia Sinica. He did this very successfully—several outstanding mathematicians were nurtured there—but the institute was located in Nanjing, and the turmoil of the civil war was making southern China ever more dangerous. As a result, Weil, by then in Chicago, and Veblen and Weyl in Princeton became concerned about his fate, and both Chicago and Princeton's Institute for Advanced Study offered Chern visiting positions, culminating in a full professorship at Chicago. So in 1949 he returned to the United States, this time with his family, to spend most of his working life there.

Chern's topological interests in Nanjing and Chicago deepened as he absorbed the rapid postwar development in algebraic topology, and his talk at the 1950 International Congress of Mathematicians (1952) shows how dramatic the interaction of differential geometry and topology had become by then. It is a thoroughly modern statement, totally different in outlook from the work of fifteen years earlier.

Work in California and China . In 1960, Chern moved again, to become a professor at the University of California at Berkeley—attracted by an expanding department and a milder climate. There he immediately started a differential geometry seminar that continues in the early twenty-first century, and he attracted visitors both young and old. His own PhD students included Shing-Tung Yau, who won a Fields Medal in 1982.

In 1978, the year he turned sixty-seven, Chern, Isadore Singer, and Calvin Moore prepared a response to the National Science Foundation's request for proposals for a mathematical institute that would reflect the "need for continued stimulation of mathematical research" in an environment that considered American mathematics to be in a "golden age." Their ideas were approved in 1981 and Chern became the first director of the Mathematical Sciences Research Institute, a post he held from 1982 until 1985. It was a huge success, and Chern supported it thereafter in many ways, not least from the proceeds of his 2004 Shaw Prize. A new building, Chern Hall, was dedicated in his memory on 3 March 2006.

Throughout his years in the United States Chern's interest in Chinese mathematicians continued. He aimed to put Chinese mathematics on the same level as its Western counterpart, "though not necessarily bending its efforts in the same direction" (Citation, Honorary Doctorate, Hong Kong University of Science and Technology, 7 November 2003, available from http://genesis.ust.hk/jan_2004/en/camera/congregate/citations_txt05.html). During the 1980s, he initiated three developments in China: an International Conference on Differential Geometry and Differential Equations, the Summer Education Centre for Postgraduates in Mathematics, and the Chern Programme, aimed at helping Chinese postgraduates in mathematics to go for further study in the United States. In 1984 China's Ministry of Education invited him

to return to his alma mater, Nankai University, and create the Nankai Research Institute of Mathematics. The university built a residence for him, "The Serene Garden," and he and his wife lived there every time they returned to China. While director he invited many overseas mathematicians to visit; he also donated more than 10,000 books to the institute, and his $50,000 Wolf Prize to Nankai University.

In 1999 Chern returned to China for good, where he continued to do mathematics, grappling until just before his death with an old problem about the existence or otherwise of a complex structure on the six-dimensional sphere. The finest testament to his achievement in his final years was to be seated next to President Jiang Zemin in the Great Hall of the People in Beijing at the opening of the 2002 International Congress of Mathematicians. During the course of his lifetime, mathematics in China had changed immeasurably.

Chern received many awards for his work including the U.S. National Medal of Science in 1975, the Wolf Prize in Mathematics in 1983, and the Shaw Prize in 2004. He died on 3 December 2004 at age ninety-three; his wife of sixty-one years had died four years earlier. He was survived by a son, Paul, and a daughter May Chu.

Proof of the Gauss-Bonnet Theorem . Chern's mathematical work encompasses a period of rapid change in geometry, and he was exceptionally able to capitalize on his extensive knowledge of the mathematics of both the first and the second half of the twentieth century. His subject of differential geometry had its origins in the study of surfaces inside the three-dimensional Euclidean space with which everyone is familiar. It involves the notions of the length of curves on the surface, the area of domains within it, the study of geodesics on the surface, and various concepts of curvature. By the late nineteenth century other types of geometry were being studied this way, such as projective geometry and web geometry, the subject on which Chern cut his mathematical teeth. An *n-web* in the plane consists of *n* families of nonintersecting curves that fill out a portion of the plane. For example, a curvilinear coordinate system such as planar Cartesian coordinates or polar coordinates defines a 2-web. By a change of coordinates any planar 2-web can be taken to the standard Cartesian system, but this is not so for webs of degree three and higher and invariants which have the nature of curvature obstruct this.

Most proofs related to the subject that appeared during this period involve intricate calculations, and Chern indeed was a master at such proofs. However, in the 1920s new inputs in differential geometry arrived from its importance in Einstein's theory of general relativity. One of these was the shift in emphasis from two-dimensional geometry to the four-dimensional geometry of space-time. Coupled with the nineteenth-century formulation of mechanics, which involved high-dimensional configuration spaces where kinetic energy defined a similar structure to a surface in Euclidean space, the ruling perspective in differential geometry was to work in *n* dimensions. A second change brought on by relativity was the requirement that the equations of physics should be written in a coordinate-independent way. This required the introduction of mathematical objects that had a life of their own, but which could still be manipulated by indexed quantities so long as one knew the rules for changing from one coordinate system to another. The most fundamental change, however, was the movement from extrinsic geometry to intrinsic geometry: four-dimensional space-time was not sitting like a surface in a higher-dimensional Euclidean space; its geometry could be observed and described only by the beings that lived within it. The intrinsic viewpoint also paved the way for the global viewpoint—the spaces one needed to study, not least space-time itself, could have quite complicated topology and one wanted to understand the interaction between the differential geometry and the topology: to see what constraints topology imposes on curvature, or vice-versa.

This was the context of Chern's proof of the general Gauss-Bonnet theorem (1944), which was a pivotal event in the history of differential geometry, not just for the theorem itself but also for what it led to. The classical theorem of the same name concerns a closed surface in Euclidean three-space. It states that the integral of the Gaussian curvature is 2π times the Euler number. The Euler number for a surface divided into F faces, E edges and V vertices is V-E+F. For a sphere this is 2, and the Gauss-Bonnet theorem gives this because the Gaussian curvature of a sphere is 1, and its area is 4π.

This link between curvature and topology has several features: one is Gauss's *theorema egregium*, which says that a certain expression of curvature of the surface, the Gauss curvature, is intrinsic—it can be determined by making measurements entirely within the surface. That being so, clearly whatever its integral evaluates to depends only on the intrinsic geometry. In contrast, there is a very natural and useful extrinsic interpretation of this integral as the degree of the Gauss map: the unit normal to the surface at each point defines a map to the sphere, and its topological degree (the number of points with the same normal direction) is the invariant. The problem was to extend this result to (even-dimensional) manifolds in higher dimension. In 1926 Heinz Hopf had generalized the Gauss map approach to hypersurfaces in Euclidean *n*-space, but the task was to prove the theorem for any even-dimensional Riemannian manifold. The concept of *manifold*, commonplace in mathematics today and signifying a higher-dimensional analogue of a surface, was by no means clear when Chern was working on this theorem. Indeed, the definition was formulated correctly by Hassler Whitney only in 1936, and Cartan even in 1946 considered that "the general notion of manifold is quite difficult to define with precision" (Cartan, 1949, p. 56).

The novel content of the proof came from studying the intrinsic tangent sphere bundle, and using the exterior differential calculus that Chern had learned at the hands of Cartan. The language of fiber bundles was necessary to describe in an intrinsic way the totality of tangent vectors to a manifold—it was what Cartan lacked and was only developed amongst topologists in the period 1935–1940. Chern's theorem, proved with the use of this concept, provided a link between topology and differential geometry at a time when the very basics of the topology of manifolds were being laid down.

Discovery of the Chern Classes . The successful attack on the Gauss-Bonnet theorem led him to study the other invariants of bundles, to see whether curvature could detect them. He started with Stiefel-Whitney classes but their more algebraic properties "seemed to be a mystery" (Weil, 1992, p. 74), and what are now called Pontryagin classes, where curvature could make an impact, were not known then, so Chern moved into Hermitian geometry and discovered the famous Chern classes whose importance in algebraic geometry, topology, and index theory cannot be underestimated. As he pursued his work on characteristic classes and curvature, Chern always recognized that there was more than just the topological characteristic class to be obtained, and this emerged later in a strong form in his work on Chern-Simons invariants with James Simons (1971). Nowadays the Chern-Simons functional is an everyday tool for theoretical physicists.

The Chern classes, coupled with the Hodge theory that in the postwar period was given a more rigorous foundation by Kunihiko Kodaira and Weyl, provided a completely new insight into the interaction of algebraic geometry and topology. But Chern was always happy to work in algebraic geometry. His studies in Hamburg involved webs obtained from algebraic curves—a plane curve of degree *d* meets a general line in *d* points. There is a duality between points and lines in the plane: the one-parameter family of lines passing through a point describes a line in another plane. So the curve describes *d* families of lines, which is a web. Chern in fact later returned to this theme in far more generality in collaboration with the algebraic geometer Phillip Griffiths (1978). Nevertheless, it was the new differential and topological viewpoint on the traditional geometry in the complex domain that motivated most of his contributions. One of these was his work in several complex variables on value distribution theory. In joint work with Raoul Bott (1965) he introduced the use of connections and curvature on vector bundles into this area. In fact, their formulation of the notion of a connection in that paper is so simple and manageable that it has become the standard approach in the literature. In this context a vector bundle is a smooth family of abstract vector spaces parameterized by the points of a manifold (like the tangent spaces of a surface) and a connection is an invariant way of taking the derivative of a family of vectors.

Another link between the algebraic geometric and differential geometric world that Chern contributed to is in the area of minimal surfaces, the simplest examples of which are the surfaces formed by the soap films spanning a wire loop. Chern was the first to attempt a rigorous proof of what is classically known as the existence of isothermal coordinates on a surface. On any surface, such as a surface sitting in Euclidean space, one can find two real coordinates which are described by a single complex number. This immediately links the differential geometry of surfaces with complex analysis, and the most direct case is that of a minimal surface. The physicist Yang learned about this taking a course from Chern in China in 1940: "When Chern told me to use complex variables … it was like a bolt of lightning which I never later forgot" (Yang, 1992, p. 64). In later work, Chern discussed minimal surfaces in higher dimensional Euclidean spaces and in spheres and showed how in quite intricate ways the algebraic and differential geometry intertwine.

Other Mathematical Work . Chern's work on characteristic classes earned him a large audience of mathematicians in a variety of disciplines, but he did not neglect the other aspects of differential geometry, especially where unconventional notions of curvature were involved. Some of this arose from early attempts to extend general relativity—for example, Weyl geometry and path geometry. The latter considers a space which has a distinguished family of curves on it that behave qualitatively like geodesics— given a point and a direction there is a unique curve of the family passing through the point and tangent to the direction. Veblen and his school in Princeton had worked on this and it was through this work that they probably first heard of Chern. Curvature invariants in complex geometry also came up in his work with Jürgen Moser (1974) on the geometry of real hypersurfaces in a complex vector space, picking up on a problem once considered by Cartan. When, in the mid-1970s, soliton equations such as the KdV equation, together with its so-called Bäcklund transformations, began to be studied, he was well prepared to apply both his expertise in exterior differential systems and his knowledge of classical differential geometry to provide important results.

Sometimes his choice of topics was unorthodox, but reflected both his curiosity and respect for the mathematicians of the past. Bernhard Riemann in his famous inaugural lecture of 1854, *On the Hypotheses which Lie at the Basis of Geometry*, discussed various competing notions of infinitesimal length but concluded that it would "take considerable time and throw little new light on the theory of space, especially as the results cannot be geometrically expressed; I restrict myself therefore to those manifoldnesses in which the line element is expressed as the square root of a quadric differential expression" (Riemann, 1873, p. 17). His "restricted" theory is what is known as Riemannian geometry in the early twenty-first century. The alternatives have come to be called Finsler metrics; Chern took Riemann at face value and set out with collaborators to investigate the geometry of these (2000).

In a life as long and full as Chern's, there are many more highly significant contributions. He also returned to some favorite themes over the decades. One was Blaschke's use of integral geometry and generalizations of the attractive Crofton's formula, which measures the length of a curve by the average number of intersections with a line. Despite his geometrical outlook, Chern's proofs were usually achieved by the use of his favorite mathematical objects—differential forms. He had learned this skill with Cartan and was an acknowledged master at it.

One of the enduring features in Chern's life was his accessibility and offers of encouragement to young mathematicians: As Bott remarked, "Chern treats people equally; the high and mighty can expect no courtesy from him that he would not also naturally extend to the lowliest among us" (Bott, 1992, p. 106). His relaxed style and willingness to help young researchers earned him loyalty from generations of mathematicians. One such appreciative student bought his weekly California State Lottery tickets with the single thought "If I win, I will endow a professorship to honor Professor Chern." In 1995 he won $22 million and the Chern Visiting Professors became a regular feature on the Berkeley campus.

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