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(*b*. Paris [?], France; *fl*. second half fifteenth century),

algebra.

Nicolas Chuquet is known only through his book, which as an entity has remained in manuscript; one part, the "Triparty," on the science of numbers, was published by Aristide Marre in 1880. The following year Marre published the statement of, and replies to, a set of 156 problems that follow the "Triparty" in the manuscript. The analysis of these problems remains unpublished, as do an application to practical geometry and a treatise on commercial arithmetic.

The conclusion of the "Triparty" indicates that it was composed by one Nicolas Chuquet of Paris, holder of the baccalaureate in medicine, at Lyons, in 1484. (Marre believes that he can date the work precisely: one problem in the treatise on business arithmetic permits fixing the work no later than May 1484.)

Only one copy of the book (the work of a firm of copyists) is known to exist; although it remained in manuscript until 1880, several passages from it were copied slavishly by "Master Étienne de la Roche, also called Villefranche, native of Lyons on the Rhone," in his arithmetic text of 1520, of which there was a second edition in 1538.

We owe confirmation of the existence and importance of Nicolas Chuquet to this unscrupulous Master Etienne, who appears to have been well established in Lyons, where his name was included on the tax rolls of 1493. He cites Chuquet and his "Triparty" at the beginning of his text—without, however, adding that he is plagiarizing outrageously.

Chuquet called himself a Parisian. He spent his youth in that city, where he was probably born and where the name is yet known. There he pursued his extensive studies, up to the baccalaureate in medicine (which implies a master of arts as well). It is difficult to say more about his life. He was living in Lyons in 1484, perhaps practicing medicine but more probably teaching arithmetic there as "master of algorithms." The significant place given to questions of simple and compound interest, the repayment of debts, and such in his work leads one to suppose this. However, he used these questions only as pretexts for exercises in algebra.

Chuquet's mathematical learning was solid. He cites by name Boethius—whom everyone knew at that time—Euclid, and Campanus of Novara. He knew the propositions of Archimedes, Ptolemy, and Eutocius, which he stated without indicating his sources (referring to Archimedes only as "a certain wise man"). In geometry his language seems to be that of a translator, transposing terms taken from Greek or Latin into French. By contrast, in the parts devoted solely to arithmetic or algebra there is no borrowing of learned terminology. Everything is written in simple, direct language, with certain French neologisms that have not been preserved elsewhere. The only exception is the ponderous nomenclature for the various proportions encumbering the first pages of the "Triparty," for Chuquet was respecting a style that goes back to Nicomachus and his translator Boethius and that still infested the teaching of mathematics in the seventeenth century.

On the whole Chuquet wrote a beautiful French that is still quite readable. His simple, very mathematical style does not lack elegance, although occasional affectation led to the use of three or four synonyms in order to avoid monotonous repetition. Marre purports to find many Italianisms in Chuquet's French. He attributes this peculiarity to the close relations between Lyons and the cities of northern Italy and to its large and prosperous Italian colony. Upon examination, many of these so-called Italianisms appear to be nothing more than Latinisms, however, and the French used by Chuquet seems as pure as that of his contemporaries.

As for his mathematical work, in order to judge it fairly, we must compare it with the work of such contemporaries as Regiomontanus, whose *De triangulis omnimodis* was written twenty years earlier, in 1464 (although it was not printed until 1533), and most notably with that of Luca Pacioli, whose *Summa de arithmetica, geometria proportioni et proportionalita* was published ten years later, in 1494.

Chuquet made few claims of priority. The only thing that he prided himself on as his personal discovery was his "règle des nombres moyens." On one further occasion he seemed also to be claiming for himself the "règle des premiers." ("What I call first numbers the ancients called 'things.'") But he says no more on this point—where, as far as we know, his originality is obvious.

Chuquet engaged in very little controversy, contenting himself with twice criticizing a certain Master Berthelemy de Romans, also cited by a contemporary French arithmetician, Jehan Adam (whose arithmetic manuscript is dated 1475). “Master Berthelemy de Romans, formerly of the Order of Preaching Friars [Dominicans] at Valence and Doctor of Theology” may well have been one of the mathematics professors of Chuquet and Adam. Nothing definite is known of this matter, however.

The “Triparty” is a treatise on algebra, although the word appears nowhere in the manuscript. This algebra deals only with numbers, but in a very broad sense of the term. The first part concerns rational numbers. Chuquet’s originality in his rules for decimal numeration, both spoken and written, is immediately obvious. He introduced the practice of division into groups of six figures and used, besides the already familiar million, the words billion (10^{12}), trillion (10^{18}), quadrillion (10^{24}), etc. Here is an example of his notation:

$745324^{3}804300^{2}700023^{1}654321$.

In this example Chuquet’s exponents are simply commas, but he points out that one can use ^{1} in place of the first comma, ^{2} in place of the second, ^{3} in place of the third, and so on.

Fractions, which Chuquet called “nombres routz” (*sic*), or broken numbers, were studied clearly and without complicated rules. Like all his contemporaries, however, he always used a numerator smaller than the denominator (and hence mixed numbers instead of improper fractions), a practice that led to unnecessary complications in his calculations.

Chuquet’s study of the rules of three and of simple and double false position, clear but commonplace, served as pretext for a collection of remarkable linear problems, expounded in a chapter entitled “Seconde partie d’une position.” Here he did not reveal his methods but reserved their exposition for a later part of the work, where he then said that after having solved a problem by the usual methods—double position or algebra (his “règle des premiers”)—one must vary the known numerical quantities and carefully analyze the sequence of computations in order to extract a canon (formula). This analysis generally led him to a correct formula, although at times he was mistaken and gave methods applicable only for particular values.

Another original concept occurred in this group of problems. In a problem with five unknowns, Chuquet concluded: “I find 30, 20, 10, 0, and minus 10, which are the five numbers I wished to have.” He then pointed out that zero added to or subtracted from a number does not change the number and reviewed the rules for addition and subtraction of negative numbers. In the thirteenth century Leonardo Fibonacci had made a similar statement but had not carried it as far as Chuquet in the remainder of his work.

The first part of the “Triparty” ends with the “règle des nombres moyens,” the only discovery to which Chuquet laid claim. According to him—and he was right—this rule allows the solution of many problems that are unapproachable by the classic rule of three or the rules of simple or double false position. It consisted of establishing that between any two given fractions a third can always be interpolated that has for numerator the sum of the numerators of the other two fractions, and for denominator the sum of their denominators. It has been demonstrated in modern times that by repeating this procedure it is possible to arrive at all the rational numbers included between the two given fractions. It is obvious, therefore, that this rule, together with a lot of patience, makes it possible to solve any problem allowing of a rational solution. Further on, Chuquet utilizes it in order to approach indeterminately the square roots, cube roots, and so on, of numbers that do not have exact roots. But here he used it to solve the equation

Successively interpolating five fractions between 5 and 6, he found the exact root, . Since he was, moreover, little concerned about rapid methods of approximation, Chuquet throughout his work used nothing but this one rule and only rarely at that.

The second part of the “Triparty” deals with roots and “compound numbers.” There is no trace of Euclidean nomenclature, of “binomials,” or of “apostomes.” The language has become simpler: there is no question of square roots or cube roots, but of second, third, fourth roots, “and so on, continuing endlessly.” The number itself is its own first root. Moreover, everything is called a “number”—whole numbers, rational numbers, roots, sums, and differences of roots—which, in the fifteenth century, was audacious indeed. The notation itself was original. Here are several examples:

Unfortunately, Chuquet, a man of his times after all, occasionally became involved in computations that to us might seem inextricable. For example, he wrote the product of as , whereas after Descartes it would be written .

The third part is by far the most original. It deals with the “règle des premiers,” a “truly excellent” rule that “does everything that other rules do and, in addition, solves a great many more difficult problems.” It is “the gateway and the threshold to the mysteries that are in the science of numbers.” Such were the enthusiastic terms in which Chuquet announced the algebraic method. First, he explained his notation and his computational rules. The unknown, called the “first number” *(nombre premier)*, is written as 1^{1}. Therefore, where Chuquet wrote 4^{0}, we should read 4; if he wrote 5^{1}, we should read $5x$; and if he wrote 7^{3}, we should read $7x^{3}$.

By a daring use of negative numbers, he wrote our $-12x$ as , our . x^{3} as . For $12x^{-1}$ or $12/x$, he wrote ; for $-12x^{-1}$, he wrote . In order to justify his rules of algebraic computation, and particularly those touching the product of the powers of a variable, he called upon analogy. He considered the sequence of the powers of 2 and showed, for example, that $2^{2} \times 2^{3} = 2^{5}$. He wished only to make clear, by an example that he considered commonplace and that goes back almost to Archimedes, the algebraic rule that if squares are multiplied by cubes the result is the fifth power.

For division he announced that the quotient of

36^3 by 6^1 is 6^2 $36x^2 \div 6x = 6x^2$
$72 \div 8x^3 = 9x^{-3}$
$84x^{-2} \div 7x^{-3} = 12x$.

Further on—but only once in the entire work—he wrote down a rational function:

In accordance with the custom of Chuquet's time, all these rules of computation were simply set forth, illustrated by a few examples, and at times justified by analogy with more elementary arithmetic, but never "demonstrated" in the modern sense of the term. Having set down these preliminaries, Chuquet dealt with the theory of equations, which he called the "method of equaling."

In setting up an equation, he specified that 1^1 should be taken as the unknown and that this "premier" should be operated on "as required in 'la raison,'" that is, the problem under study.

One can, moreover, take 2^1, 3^1, etc., as the unknowns in place of 1^1. One should end with two expressions equal or similar to each other. These are the two "parts" of the equation. This is followed by the classic rules for solving binomial equations of the type $ax^{m} = b$.

These rules include procedures for reducing equations to the binomial form. Chuquet emphasized the importance of recognizing "equipollent numbers," i.e., expressions of the same power as and x or and x^2. He notes that when the two parts of the equation are "similar" it may either have infinitely many solutions ($ax^{m} = ax^{m}$) or be impossible of solution ($ax^{m} = bx^{m}$; $a \neq b$). Several of the many numerical problems that follow—certain of them having several unknowns—come to one of these results. When the two parts are not similar, the equation has for Chuquet at most one solution. He is, however, of two minds regarding negative solutions; sometimes he accepts them, sometimes not.

Subsequently Chuquet solved problems that led to equations of the form

$ax^{2m} + bx^{m} + c = 0$.

Like all his contemporaries he distinguished three cases (a, b, c positive):

$c + bx_{m} = ax^{2m}$

$bx^{m} + ax^{2m} = c$

$c + ax^{2m} = bx^{m}$.

While he knew and stated that in the last case the equation might be insoluble, and admitted two solutions when it was soluble, he found only one solution in the other two cases. In this regard he did not show any progress over his predecessors.

On the subject of division into mean and extreme ratio, he showed, in fact, a profound ignorance of the theory of numbers.

Indeed, having to solve $144 + x^2 = 36x$, Chuquet obtained the answer , and, in effect, declared: "Campanus, in the ninth book of Euclid, at the end of Proposition 16, affirms that such a problem is impossible of solution in numbers, while in fact, as we have just seen, it is perfectly capable of solution."

Now in fact Campanus demonstrated, very elegantly, that the solution cannot be rational. Chuquet, who had a very broad concept of numbers, did not grasp the subtlety. In several other cases he also showed a lack of understanding, particularly regarding the problems that he included in his "rule of apposition and separation." These problems derive from Diophantine analysis of the first order, the exact theory of which had to await Bachet de Méziriac and the seventeenth century.

The "Triparty" ends with an admission of inadequacy: there remains the task of studying equations of a more general type, wrote the author, but he would leave them for those who might wish to go further with the subject. This attitude was common among the majority of the algebraists of the fifteenth century, who would have liked to go beyond the second degree but were not sure how to go about it.

Summarizing the "Triparty," we may say that it is a very abstract treatise on algebra, without any concrete applications, in which one can see a great extension of the concept of number—zero, negative numbers, and roots and combinations of roots all being included. There appear in it excellent notations that prefigure, in particular, those of Bombelli. But weak points remain. In linear algebra the cases of indetermination and insolubility were poorly set forth. In the theory of equations the importance of degree was not realized, and for equations of the second degree the old errors persisted. Moreover, Chuquet's taste for needlessly complicated computations—and beyond that the way in which the necessity for these computations arose in the shortcomings of numerical algebra—is painfully evident. At the end of a problem one may, for example, find

"which, if abbreviated by extraction of the second and third roots, comes to 3." Such an answer required a certain courage in the person making the calculation!

The "Triparty" has become well known since its publication in 1880. The same is not true of the sequel to the 1484 manuscript. It is true that Marre gave, in 1881, the statement of the 156 problems that follow the "Triparty," accompanied by answers and some remarks that Chuquet added to them. But this incomplete publication let several important points go by.

By contrast with the problems of the "Triparty," the greater part of those treated in this appendix appear to be concrete. But this is merely an appearance, and one can imagine the author coldly cutting an heir into several pieces if a problem concerning a will fails to come out even. Moreover, the questions dealt with were not all original; many were part of a long tradition going back at least as far as Metrodorus' anthology.

Marre indicated those problems that were taken over—almost word for word—by the plagiarist Étienne de la Roche and those that may be found in an almost contemporary treatise written in the Languedoc dialect of the region of Pamiers. One can find many other such duplications. Chuquet's originality lay not in his choice of topics but in his way of treating them. Indeed, he very often made use of his "règle des premiers," that is, of algebra.

A very important fact seems to have gone unnoticed by historians, no doubt because of the incomplete publication of this part of the work. It is known that, in his *Summa* of 1494, Luca Pacioli made use in certain cases of not one unknown but two: the *cosa* and the *quantita*. Now, what has passed unnoticed is that Chuquet employed the same device in 1484, and on at least five occasions. The first time was in the following problem:

Three men have some coins. If the first man took 12 coins from the two others, he would have twice the amount remaining to them, plus 6 coins. If the second man took 13 coins from the first and the third, he would have four times as many as remained to them, plus 2 coins. If the third man took 11 coins from the two others, he would have three times what remained to them, plus 3 coins. How many coins does each man have?

Chuquet solved the problem by a mixed method, a combination of the rule of two false positions and his "règle des premiers." Someone (it must be Étienne de la Roche) has written in the margin "by the rule of two positions and by the rule of the 'thing' together." This annotator, perhaps under the influence of Pacioli, used the language of the Italian algebraists.

Chuquet posited 6 as the holdings of the first man, and with the first datum of the problem deduced that the three men have 24 coins among them. Assigning to the third the value 1^1 and basing himself on the third condition, he found, using the "règle des premiers," that the third man has coins and therefore the second has .

By following the then classic method of false position, he established that the use of 6 for the first man's holdings gives, for the second condition, an excess of . He started again with 12, and by applying the classic rule, obtained the answer: the holdings are 8, 9, and 10.

But, a bit further on, he announced "another way of solving it, using only the 'règle des premiers.'" The annotator has written in the margin, "This rule is called the rule of quantity." Here it is plain that the annotator was familiar with Pacioli's *Summa*.

As we have already said, Chuquet liked to vary the numerical data of his problems. This was a necessity for him, and it mitigated to a degree the absence of literal computation. Thus he could better analyze the sequence of computations and draw from them the rules to follow for solving automatically problems of the same kind. Thus, he presented the following problem:

If the first of three men took 7 coins from the others, he would have five times what remained to them, plus 1. If the second took 9 coins from the others, he would have six times what remained to them, plus 2, and if the third took 11 coins from the first two, he would have seven times as many as remained to them, plus 3.

Chuquet then posited 1^1—i.e., x—for the holdings of the first, and for the first datum found that they have, in all,

"To find the portion belonging to the second man, I assign to him 1^2" This symbol no longer represents the square, x^2, of the unknown x but another unknown, y. Then, making use of the second condition of the problem, he found that

Finally, to find the third man's portion, he used the third condition and took 1^2 for the third unknown. This unknown is not the y of the preceding computation but a new unknown, z. He found he arrived at the final equation:

which remains only to be solved.

Several problems on geometric progressions and compound interest were not completely solved until the introduction of logarithms. Chuquet was aware of the difficulty without finding a way to solve it.

A vessel with an open spout loses one-tenth of its contents each day. In how many days will the vessel be half-empty?

After having answered that it will be days, he adds, “Many people are satisfied with this answer. However, it seems that between six days and seven, one should search for a certain proportional number that, for the present, is unknown to us.”

Among the numerous problems studied is a question of inheritance that is found in Bachet de Meziriac (1612) and again in Euler’s *Algebra* (1769) and that, by its nature, demands a whole number as the answer.

The oldest son takes 1 coin and one-tenth of the rest, the second takes 2 coins and one-tenth of the new remainder, and so on. Each of the children receives the same sum. How many heirs are there?

After having varied the numerical data, Chuquet gave a general rule for the solution of the problem, then treated eleven other numerical cases—in which the answers include fractions of an heir. One detects here not a number theorist, but a pure algebraist.

Many of Bachet de Méziriac’s *Problèmes plaisans et délectables* can be found in Chuquet, especially in the chapter entitled “Jeux et esbatemens qui par la science des nombres se font.” But they formed part of a thousand-year-old tradition, and a manuscript by Luca Pacioli, *De viribus quantitatis*, also dealt with them.

The geometric part of Chuquet’s work is entitled “Comment la science des nombres se peut appliquer aux mesures de geometrie.” It formed part of a tradition that goes back to the Babylonians and that developed over the centuries, in which figured notably Hero of Alexandria and, in the thirteenth century, Leonardo Fibonacci. It comprised, first of all, the measurement of length, area, and volume. Then it became more scientific and, in certain respects, a veritable application of algebra to geometry.

Rectilinear measure was either direct or effected by means of the quadrant, which was represented in practice by a figure on the back of the astrolabe that utilized a direct and inverse projection or shadow. No recourse was made to trigonometry, nor did any measurement of angles appear. In this we are removed from the astronomic tradition so brilliantly represented by Regiomontanus. The quadrant was used for measuring horizontal distances, depths, and heights. Other elementary techniques were also indicated, such as those of the vertical rod and the horizontal mirror.

As for curves, “the circular line is measured in a way that will be described in connection with the measuring of circular surfaces. Other curved lines are reduced as much as possible to a straight line or a circular line.” For the circle the approximations for π for $\pi/4$ were the only ones indicated.

Chuquet separated triangles into equilateral and nonequilateral triangles. Each triangle consisted of one base and two hypotenuses. The “cathete” descends perpendicular to the base. This terminology was quite unusual and certainly different from classical terminology.

Chuquet knew Euclid’s *Elements* very well, however. For him the following proposition was the fundamental one: “If the square of the perpendicular is subtracted from the square of the hypotenuse, the square root of the remainder will be the length of the part of the base that corresponds to the hypotenuse.” He gave no demonstration. The first part of this practical geometry was completed by a few notes on the “measurement of hilly surfaces” and the volumes of spheres, pyramids, and cubes. These are scarcely developed. This work does not have the amplitude of Pacioli’s treatment of the same topic in the *Summa* and is scrcely more than a brief summary of what can be found in the pages of Leonardo Fibonacci. But “the application of the aforesaid rules” brought forth a new spirit, and here the algebraist reappeared. For example, Chuquet applied Hero’s rule on the expression of the area of a triangle as a function of its three sides to the triangle 11, 13, 24. He found the area to be zero, and from this concluded the nonexistence of the triangle.

Again, two vertical lines of lengths 4 and 5 are horizontally distant by 12. Chuquet sought a point on the horizontal that was equidistant from the two apexes. This led him to the equation

There are still other, similar algebraic exercises. Two of them are important.

In the first it is proposed to find the diameter of the circle circumscribed about a triangle whose base is 14 and whose sides are 13 and 15. To do this it was first necessary to compute the projections x and y of the sides on the base, using the relation $x^2 - y^2 = 15^2 - 13^2$. The perpendicular then follows directly. Now, taking as the unknown the distance from the center to the base and expressing the center as equidistant from the vertices, one is led to the equation

In the second exercise Chuquet proposed to compute the diameter of a circle circumscribed about a regular pentagon with sides of length 4. He first took as the unknown the diagonal of the pentagon. Ptolemy's theorem on inscribed quadrilaterals then leads to the equation

which permitted him to make his computations. By this method he concluded that Euclid's proposition is correct: the square of the side of the hexagon added to that of the decagon gives that of the pentagon inscribed in the same circle.

Two analogous applications of algebra to geometry are in the *De triangulis* of Regiomontanus. Others are to be found in the geometrical part of Pacioli's *Summa*. Thus, it appears that this tradition was strongly implanted in the algebraists of the fifteenth century.

Without any vain display of erudition Chuquet showed his extensive learning in borrowings from Eutocius for the graphic extraction of cube roots. He also gave constructions with straight-edge and compasses and devoted a rather weak chapter to research on the squaring of the circle. Ramón Lull's quadrature was recalled here. Chuquet showed that it was equivalent to taking the value for π. As for the approximation 22/7, it is given by "a wise man.... But this is a thing that cannot be proved by any demonstration.

If Étienne de la Roche had been more insensitive—especially if he had plagiarized the "règle des premiers" and its applications—mathematics would perhaps be grateful to him for his larcenies. He was unfortunately too timid, and in his arithmetic text he returned to the classical errors of his time, and thus the most original part of Chuquet's work remained unknown. One cannot, however, assert that the innovations that Chuquet introduced were entirely lost. His notation can be found again, for example, in Bombelli. Was it rediscovered by the Italian algebraist? Was there a connection between the two, direct or indirect? Or did both derive from a more ancient source? These are the questions that remain open.

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