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(*b*. Burbage, Leicestershire, England, 10 July 1682; *d*. Cambridge, England, 5 June 1716)

mathematics, astronomy.

Cotes was the second son of the Reverend Robert Cotes, rector of Burbage. His mother was the former Grace Farmer, of Barwell, Leicestershire. He was educated first at Leicester School, where he showed such a flair for mathematics that at the age of twelve his uncle, the Reverend John Smith, took him into his home to supervise his studies personally. Cotes later went to St. Paul's School, London, where he studied mainly classics while keeping up a scientific correspondence with his uncle. He was admitted as a pensioner to Trinity College, Cambridge, in 1699, graduating B.A. in 1702 and M.A. in 1706. He became fellow of his college in 1705 and a fellow of the Royal Society in 1711, and was ordained in 1713. In January 1706, Cotes was named the first Plumian professor of astronomy and natural philosophy at Cambridge on the very strong recommendation of Richard Bentley, master of Trinity. Cotes, who never married, died of a violent fever when only thirty-three. His early death caused Newton to lament; "Had Cotes lived we might have known something."

On his appointment as professor, Cotes opened a subscription list in order to provide an observatory for Trinity. This, with living quarters for the professor, was erected on the leads over King's Gate. Cotes spent the rest of his life here with his cousin Robert Smith, who was his assistant and successor. The observatory was not completed in Cotes's lifetime and was demolished in 1797.

Concerning his astronomical work Cotes supplied, in correspondence with Newton, a description of a heliostat telescope furnished with a mirror revolving by clockwork. He recomputed the solar and planetary tables of Flamsteed and J. D. Cassini and had intended to construct of the moon's motion, based on Newtonian principles. According to Halley (1714), he also observed the total solar eclipse of 22 April 1715, noticing the occultation of three spots.

Cotes formed a school of physical sciences at Trinity in collaboration with William Whiston. The two performed a series of experiments beginning in May 1707, the details of which can be found in a post-humous publication, *Hydrostatical and Pneumatical Lectures by Roger Cotes* (1738). These demonstration classes indicate a simple, straightforward style that is both stimulating and thorough. There was no thought of practical work by students at this time.

In 1709 Cotes became heavily involved in the preparation of the second edition of Newton's great work on universal gravitation, the *Philosophiae naturalis principia mathematica*. The first edition of 1687 had few copies printed. In 1694 Newton did further work on his lunar and planetary theories, but illness and a dispute with Flamsteed postponed any further publication. Newton subsequently became master of the mint and had virtually retired from scientific work when Bentley persuaded him to prepare a second edition, suggesting Cotes as supervisor of the work.

Newton at first had a rather casual approach to the revision, but Cotes took the work very seriously. Gradually, Newton was coaxed into a similar enthusiasm; and the two collaborated closely on the revision, which took three and a half years to complete. The edition was limited to only 750 copies, and a pirated version printed in Amsterdam met the total demand. Bentley, who had borne the expense of the printing, took the profits and rewarded Cotes with twelve free copies for his labors. Newton wrote a preface, remarking that in this edition the theory of the moon and the precession of the equinoxes had been more fully deduced from the principles, the theory of comets confirmed by several observations, the theory of comets confirmed by several observations, and the orbits of comets computed more accurately. His debt to Cotes for these improvements cannot be estimated.

Cotes's original contribution to this book was a short preface. He suggested to Newton that he write a description of the scientific methodology used and demonstrate, in particular, the superiority of these principles to the popular idea of vortices presented by Descartes. Cartesian ideas were still vigorous, not only on the Continent but also in England, and continued to be taught at Cambridge until 1730 at least. In particular, Cartesian critics alleged that Newton's idea of action at a distance required the conception of an unexplained, occult force. Newton and Bentley agreed that Cotes should write a preface defending the Newtonian hypothesis against the theory of vortices and the other objections.

Cotes began his preface by considering three possible methods of approaching celestial phenomena. The first, used mainly by the Greeks, was to describe motions without attempting a rational explanation; the second was to make hypotheses and, out of ignorance, to relate them to occult qualities; and the third was to use the method of experiment and observation. He vigorously asserted that category. Illustrating this by means of the inverse-square law of gravitation, he quoted Newton's discovery that the acceleration of the moon toward the earth confirms this theory and that Kepler's third law of motion, taken in conjunction with Huygen's rule for central forces, implies such a law. He asserted that the paths of comets could be observed as conics with the sun as focus and that in both planetary and cometary motion the theory of vortices conformed neither to reason nor to observation. Cotes concluded that the law of gravitation was confirmed by observation and did not depend on occult qualities.

But shall gravity be therefore called an occult cause, and thrown out of philosophy, because the cause of gravity is occult and not yet discovered? Those who affirm this, should be careful not to fall into an absurdity that may overturn the foundations of all philosophy. For causes usually proceed in a continued chain from those that are more compounded to those that are more simple; when we are arrived at the most simple cause we can go no father.… These most simple causes will you then call occult and reject them? Then you must reject those that immediately depend on them *[Mathematical Principles*, p. xxvii].

Cotes proceeded positively to imply the principle of action at a distance. "Those who would have the heavens filled with a fluid matter, but suppose it void of any inertia, do indeed in words deny a vacuum, but allow it in fact. For since a fluid matter of that kind can noways be distinguished from empty space, the dispute is now about the names and not the natures of things" (*ibid*., P. xxxi).

Leibniz later condemned Cotes's preface as "pleine d'aigreur," but it can be seen that Cotes argued powerfully and originally in favor of Newton's hypothesis.

Cote's major original work was in the field of mathematics, and the decline in British mathematics that followed his untimely death accentuated his being one of the very few British mathematicians capable of following on from Newton's great work.

His only publication during his life was an article entitled "Logometria" (1714). After his death his mathematical papers, then in great confusion, were edited by Robert Smith and published as a book, *Harmonia mensurarum* (1722). This work, which includes the "Logometria" as its first part, gives an indication of Cotes's great ability. His style is some what obscure, with geometrical arguments preferred to analytical ones, and many results are quoted without explanation. What cannot be obscured is the original, systematic genius of the writer. This is shown most powerfully in his work on integration, in which long sequences of complicated functions are systematically integrated, and the results are applied to the solution of a great variety of problems.

Cotes first demonstrates that the natural base to take of a system of logarithms is the number which he calculates as 2.7182818. He then shows two ingenious methods for computing Briggsian logarithms (with base 10) for any number and interpolating to obtain intermediate values. The rest of part 1 is devoted to the application of integration to the solution of problems involving quadratures, arc lengths, areas of surfaces of revolution, the attraction of bodies, and the density of the atmosphere. His most remarkable discovery in this section (pp. 27–28) occurs when he attempts to evaluate the surface area of an ellipsoid of revolution. He shows that the problem can be solved in two ways, one leading to a result involving logarithms and the other to arc sines, probably an illustration of the harmony of differnt types of measure. By equating these two results he arrives at the formula $i\phi = log(cos\ \phi + i\ sin\ \phi)$ where a discovery preceding similar equations obtained by Moivre (1730) and Euler (1748).

The second and longest part of the *Harmonia men surarmum* is devoted to systematic integration. In a preface to this section Smith explains that shortly before his death Cotes wrote a letter to D. Jones in which he claimed that any fluxion of the form

where *d, e, f* are constants, θ an integer (positive or negative), and δ/λ a fraction, had a fluent that could be expressed in terms of logarithms or trigonometric ratios. He claimed, further, that even fluxions of the forms

had fluents expressible in these terms.

Returning to the text, Cotes then proceeds to evaluate the fluents of no fewer than ninety-four types of such fluxions, working out each individual case as θ takes different values. His calculation was aided by a geometrical result now known as Cotes's theorem, which, expressed in analytical terms, is equivalent to finding all the factors of x^n - a^n where *n* positive integer. The theorem is that if the circumference of a circle is divided into *n* equal parts OO^1, O^1O^{11},… and any point *P* is taken on a radius *OC* then the circle *OC*, then $(PC)^n - (OC)^n = PO \times PO^1 \times PO^{11} \times$…. if *P* is outside the circle and $(OC)^n - (PC)^n = PO \times PO^1 \times PO^{11} \times$… if *P* is inside the circle. This result was proved by J. Brinkley(1797).

The third part consists of miscellaneous works, including papers on methods of estimating errors, Newton's differential method, the construction of tables by differences, the descent of heavy bodies, and cycloidal motion. There are two particularly interesting results here. The essay on Newton's differential method describes how, given *n* points at equidistant abscissae, the area under the curve of *n* th degree joining these points may be evaluated. Taking *A* as the sum of the first and last ordinates, *B* as the sum of the second and last but one, etc., he evaluates the formulas for the areas as

A modernized form of this result is known as the Newton-Cotes formula.

In describing a method for evaluating the most probable result of a set of observations, Cotes comes very near to the technique known as the method of least squares. He does not state this method as such; but his result, which depends on giving weights to the observations and then calculating their centroid, is equivalent. This anticipates similar discoveries by Gauss (1795) and Legendre (1806).

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