Cramer Gabriel | Encyclopedia.com

Complete Dictionary of Scientific Biography COPYRIGHT 2008 Charles Scribner's Sons
15-19 minutes

(*b*. Geneva, Switzerland, 31 July 1704; *d*. Bagnols-sur-Céze, France, 4 January 1752)

geometry, probability theory.

Gabriel was one of three sons born to Jean Isaac Cramer, whose family had moved from Holstein to Strasbourg to Geneva in the seventeenth century, and his wife, Anne Mallet. The father and one son, Jean-Antoine, practiced medicine in Geneva. The other two sons, Jean and Gabriel, were professors Of law and of mathematics and philosophy, respectively. All three sons were also active in local governmental affairs.

Gabriel Cramer was educated in Geneva and at the age of eighteen defended a thesis dealing with sound. At twenty he competed for the chair of philosophy at the Académie de Calvin in Geneva. The chair was awarded to the oldest of the three contestants, Amédée de la Rive; but the magistrates making the award felt that it was important to attach to their academy two such able young men as Cramer and Giovanni Ludovico Calandrini, the other contestant, who was twenty-one. To do this they split off a chair of mathematics from philosophy and appointed both young contestants to it. This appointment provided that the men share both the position's duties and its salary. It was also provided that they might take turns traveling for two or three years "to perfect their knowledge," provided the one who remained in Geneva performed all the duties and received all the pay. Calandrini and Cramer, called Castor and Pollux by their friends, secured permission for the innovation of using French rather than Latin, not for courses *ex cathedra* but for recitations, "in order that persons who had a taste for these sciences but no Latin could profit." Calandrini taught algebra and astronomy; Cramer, geometry and mechanics. In 1734 Calandrini was made professor of philosophy and Cramer received the chair of mathematics. In 1750 he was made professor of philosophy when Calandrini entered the government.

Cramer's interests and activities were broad, both academically and in the daily life of his city. He wrote on such topics as the usefulness of philosophy in governing a state and the added reliance that a judge should place on the testimony of two or three witnesses as compared with one. He wrote against the popular idea that wheat sometimes changed to tares and also produced several notes on the history of mathematics. As a citizen Cramer was a member of the Conseil des Deux-Cents (1734) and Conseil des Soixante (1749) and was involved with artillery and fortification. He instructed workers repairing a cathedral and occupied himself with excavations and the search of archives. He was reported to be friendly, good-humored, pleasant in voice and appearance, and possessed of good memory, judgment, and health. He never married.

The encouragement to travel played an important role in Cramer's life. From 1727 to 1729 he traveled, going first to Basel, where he spent five months with Johann I Bernoulli and his students, including Daniel Bernoulli and Leonhard Euler. From Basel he went to England, Leiden, Paris, meeting Nicholas Saunder-son, Christopher Middleton, Halley, Sloane, Moivre, James Stirling, s'Gravesande, Fontenelle, Réaumur, Maupertuis, Buffon, Clairaut, and Mairan. In 1747 Cramer visited Paris again with the young prince of Saxe-Gotha, whom he had taught for two years. During the trip he was invited to salons frequented by Réaumur, d'Alembert, and Fontenelle. The friendship with the Bernoullis, formed during the first trip, led to much of Cramer's later editorial work, and the acquaintanceships formed during his travels produced an extended correspondence in which he served as an intermediary for the spread of problems and as a contributor of questions and ideas.

Cramer's major publication. *Introduction à l'analysedes lignes courbes algébriques*, was published in 1750. During the previous decade he had edited the collected works of Johann I and Jakob I Bernoulli, Christian Wolff's five-volume *Elementa*, and two volumes of correspondence between Johann I Bernoulli and Leibniz. Overwork and a fall from a carriage brought on a decline in health that resulted in his being bedridden for two months. The doctor then prescribed a rest in southern France. Cramer left Geneva on 21 December 1751 and died while traveling.

Cramer received many honors, including membership in the Royal Society of London; the academies of Berlin, Lyons, Montpellier; and the Institute of Bologna. In 1730 he was a contestant for the prize offered by the Paris Academy for a reply to the question "Quelle est la cause de la figure elliptique des planètes et de la mobilité de leur aphélies?" He was the runner-up *(premier accessit)* to Johann I Bernoulli.

This last fact is perhaps typical of Cramer's status in the history of science. He was overshadowed in both mathematics and philosophy by his contemporaries and correspondents. He is best-known for Cramer's rule and Cramer's paradox, which were neither original with him nor completely delineated by him, although he did make contributions to both. His most original contributions are less well-known: the general content and organization of his book on algebraic curves and his concept of mathematical utility.

In the preface to the *Introduction à l'analyse*, Cramer cites Newton's *Enumeration of Curves of the Third Order*, with the commentary by Stirling, as an "excellent model" for the study of curves. He comments particularly on Newton's use of infinite series and of a parallelogram arrangement of the terms of an algebraic equation of degree ν in two unknowns. He also refers to a paper by Nicole and one on lines of the fourth order by Christophe de Bragelogne. Cramer gives credit to Abbé Jean Paul de Gua de Malves for making Newton's parallelogram into a triangular arrangement in the book *L'usage de l'analyse de Descartes pour découvrir…les propriétés des lignes géométriques de tous les ordres* (1740).

Cramer also says that he would have found Euler's *Introductio in analysin infinitorum* (1748) very useful if he had known of it earlier. That he made little use of Euler's work is supported by the rather surprising fact that throughout his book Cramer makes essentially no use of the infinitesimal calculus in either Leibniz' or Newton's form, although he deals with such topics as tangents, maxima and minima, and curvature, and cites Maclaurin and Taylor in footnotes. One conjectures that he never accepted or mastered the calculus.

The first chapter of the *Introduction* defines regular, irregular, transcendental, mechanical, and irrational curves and discusses some techniques of graphing, including our present convention for the positive directions on coordinate axes. The second chapter deals with transformations of curves, especially those which simplify their equations, and the third chapter develops a classification of algebraic curves by order or degree, abandoning Descartes's classification by genera. Both Cramer's rule and Cramer's paradox develop out of this chapter. The remaining ten chapters include discussions of the graphical solution of equations, diameters, branch points and singular points, tangents, points of inflection, maxima, minima, and curvature. Cramer claims that he gives no example without a reason, and no rule without an example.

The third chapter of Cramer's *Introduction* uses a triangular arrangement of the terms of complete equations of successively higher degree (see Fig. 1) as the basis for deriving the formula $\nu^2/ + 3\nu/2$ for the number of arbitrary constants in the general equation of the νth degree. This is the sum of ν terms of the arithmetic progression 2 + 3 + 4 +… derived from the rows of the triangle by regarding one coefficient, say a, as reduced to unity by division. From this he concludes that a curve of order ν can be made to pass through $\nu^2/2 + 3\nu/2$ points, a statement that he says needs only an example for a demonstration. In his example Cramer writes five linear equations in five unknowns by substituting the coordinates of five points into the general second-degree equation. He then states that he has found a general and convenient rule for the solution of a set of ν linear equations in ν unknowns; but since this is algebra, he has put it into appendix 1. Figure 2 shows the first page of this appendix. The use of raised numerals as indices, not exponents, applied to co-Solent plufieurs inconnues *z, y, x, v,&c & autant d'equations*

oú les lettres *A', A', A', A', &c*. ne mar-quent pas, comme A I'ordinaire, les puifTances d'A, mais le premier membre, fuppofe connu, de Ia premiere, fe-conde, troifieme, quatrieme &c. equation. De meme Z', Z', &c.font les coefficients de z; $'Y_1$, $'Y_2$, &c. ceux de y; X_1, X_2, &c, ceux de x; V_1, V_2, &c. ceux de v; &c. dans Ia premiere, feconde, &c. equation.

Cette Notation fuppofee, s'il n'y a qu'une equation & qu'une inconnue z; on aura . S'iI y a deux equations & deux inconnues z & y; on trouvera z == . S'ily a trois equation & trois inconnues *z, y, & x;* on trois

efficients represented by capital letters enabled Cramer to state his rule in general terms and to define the signs of the products in terms of the number of inversions of these indices when the factors are arranged in alphabetical order.

Although Leibniz had suggested a method for solving systems of linear equations in a letter to L'Hospital in 1693, and centuries earlier the Chinese had used similar patterns in solving them, Cramer has been given priority in the publication of this rule. However, Boyer has shown recently that an equivalent rule was published in Maclaurin's *Treatise of Algebra* in 1748. He thinks that Cramer's superior notation explains why Maclaurin's statement of this rule was ignored even though his book was popular. Another reason may be that Euler's popular algebra text gave Cramer credit for this "très belle regle."

Cramer's paradox was the outgrowth of combining the formula $\nu^2/2 + 3\nu/2$ with the theorem, which Cramer attributes to Maclaurin, that *m* th- and *n* th- order curves intersect in *mn* points. The formula says, for example, that a cubic curve is *uniquely* determined by nine points; the theorem says that two *different* cubic curves would intersect in nine points. Cramer's explanation of the paradox was inadequate. Scott has shown that Maclaurin and Euler anticipated Cramer in formulating the paradox and has outlined later explanations and extensions by Euler, Plücker, Clebsch, and others. Plücker's explanation, using his abridged notation, appeared in Gergonne's *Annales des mathématiques pures et appliquées* in 1828.

Cramer's work as an editor was significant in the preservation and dissemination of knowledge and reflects both the esteem in which he was held and the results of his early travel. Cantor says he was the first scholar worthy of the name to undertake the thankless task of editing the work of others. Johann I Bernoulli authorized Cramer to collect and publish his works, specifying that there should be no other edition. At the request of Johann, Cramer also produced a posthumous edition of the work of Jakob I Bernoulli, including some unpublished manuscripts and additional material needed to understand them. He also edited a work by Christian Wolff and the correspondence between Johann I Bernoulli and Leibniz.

Throughout his life Cramer carried on an extensive correspondence on mathematical and philosophical topics. His correspondents included Johann I Bernoulli, Charles Bonnet, Georges L. Leclerc, Buffon, Clairaut, Condillac, Moivre, Maclaurin, Maupertuis, and Réaumur. This list shows his range of interests and the acquaintanceships formed in his travels and is further evidence of his function as a stimulator and intermediary in the spread of ideas. For example, a letter from Cramer to

Nikolaus I Bernoulli is cited by Savage as evidence of Cramer's priority in defining the concept of utility and proposing that it has upper and lower bounds. This concept is related to mathematical expectation and is a link between mathematical economics and probability theory. Cramer's interest in probability is further revealed in his correspondence with Moivre, in which he at times served as an intermediary between Moivre and various Bernoullis.

Cramer was a proposer of stimulating problems. The Castillon problem is sometimes called the Castillon-Cramer problem. Cramer proposed to Castillon (also called Castiglione, after his birthplace) that the problem of Pappus, to inscribe in a circle a triangle such that the sides pass through three given collinear points, be freed of the collinearity restriction. Castillon's solution was published in 1776. Since then analytic solutions have been presented and the problem has been generalized from a triangle to polygons and from a circle to a conic section.

Gabriel Cramer deserves to have his name preserved in the history of mathematics even though he was outshone by more able and single-minded contemporary mathematicians. He himself would probably have accepted the rule, if not the paradox, as meriting his name but would regret that his major work is less well-known than it merits.

BIBLIOGRAPHY

I. Original Works. Cramer's chief and only published book is *Introduction à l'analyse des lignes courbes algébriques* (Geneva, 1750). Speziali (see below), pp. 9, 10, refers to two unpublished manuscripts: "Éléments d'arithmétique," written for the young prince of Saxe-Gotha, and "Cours de logique," which was sold in London in 1945 under the mistaken idea that it was by Rousseau.

Two source books have excerpts from Cramer's *Introduction* that present Cramer's rule and Cramer's paradox, respectively: Henrietta O. Midonick, *The Treasury of Mathematics* (New York, 1965), pp. 269–279; and D. J. Struik, *A Source Book in Mathematics* (Cambridge, Mass., 1969), pp. 180–183.

Cramer's minor articles, published chiefly in the *Mémoires* of the academies of Paris (1732) and Berlin (1748, 1750, 1752), include two on geometric problems, four on the history of mathematics, and others on such scattered topics as the aurora borealis (in the *Philosophical Transactions of the Royal Society)*, law and philosophy, and the date of Easter. These are listed by Isely, Le Roy, and Speziali.

As editor, Cramer was responsible for the following: Johann I Bernoulli, *Opera omnia*, 4 vols. (Lausanne-Geneva, 1742,); Jakob I Bernoulli, *Opera*, 2 vols. (Geneva, 1744), which omits *Ars Conjectandi; Virorum celeberr Got. Gul. Leibnitii et Johan. Bernoullii commercium philosophicum et mathematicum* (Lausanne-Geneva, 1745), edited with Castillon; and Christian Wolff, *Elementa matheseos universae*, new ed., 5 vols. (1743–1752). Cantor and Le Roy also give Cramer credit for the 1732–1741 edition.

There is a portrait of Cramer by Gardelle at the Bibliothèque de Genève as well as a collection of 146 letters, many unpublished, according to Speziali. Many of Crame's letters may be found published in the works of his correspondents. His correspondence with Moivre is listed in Ino Schneider, "Der Mathematiker Abraham de Moivre (1667–1754)," in *Archive for History of Exact Sciences*, **5** (1968/1969), 177–317.

II. Secondary Literature. The best and most recent account of Cramer's life and works is M. Pierre Spezialli, *Gabriel Cramer* (1704–1752) *et ses correspondants*, Conférence du Palais de la Découverte, ser. D, no. 59 (Paris, 1959), A short account of his life is to be found in Georges Le Roy, *Condillac, lettres inédites à Gabriel Cramer* (Paris, 1953). L. Isely, *Histoire des sciences mathématiques dans la Suisse française* (Neuchâtel, 1901), gives an extended account of Cramer, beginning on p. 126.

Probably the best account of Cramer's *Introduction* is that in Carl B. Boyer, *History of Analytic Geometry* (New York, 1956), pp. 194–197. Extended discussions are also to be found in Moritz Cantor, *Vorlesungen über Geschichte der Mathematick*, 2nd ed. (Leipzig, 1901), pp. 605–609, 819–842; and Gino Loria, *Storia delle mathematiche* (Milan, 1950), pp. 739–741.

See also Carl B. Boyer, "Colin Maclaurin and Cramer's Rule," in *Scripta mathematica*, **27** (Jan. 1966), 377–379; Leonard J. Savage, *The Foundations of Statistics* (New York, 1954), pp. 81, 92–95; and Charlotte Angas Scott, "On the Intersections of Plans Curves," in *Bulletin of the American Mathematical Society*, **4** (1897–1898), 260–273.

Phillip S. Jones