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(*b*. Huncoat, Lancashire, England, 30 October 1907; *d*. Cambridge, England, 9 June 1969)

mathematics.

Davenport was the only son of Percy Davenport, first office clerk and later company secretary of Perseverance Mill, and of Nancy Barnes, the mill owner's daughter. From Accrington Grammar School he won a scholarship in 1924 to Manchester University, from which he graduated with first-class honors in 1927, at the age of nineteen. Davenport proceeded to Cambridge University with a scholarship to Trinity College, where he graduated with a B.A. in mathematics in 1929, again with the highest honors in each part of the final examination. He wrote his Ph.D. dissertation under J. E. Littlewood. In 1931 he was Rayleigh prizeman, and in 1932 he became a research fellow at Trinity College.

In 1944, while he was professor of mathematics at the University College of North Wales, Bangor, Davenport married a colleague in modern languages, Anne Lofthouse; they had two sons. In 1938 Davenport received the Cambridge Sc.D., in 1940 he was elected a fellow of the Royal Society, and in 1941 he won the Adams Prize of Cambridge University. He won the senior Berwick Prize of the London Mathematical Society in 1954, and from 1957 to 1959 he was president of that society. He was elected an ordinary member of the Royal Society of Sciences in Uppsala in 1964, and in 1967 he was Sylvester medalist of the Royal Society. In 1968 he received an honorary degree from the University of Nottingham.

When Davenport embarked on research in mathematics under the supervision of Littlewood, he was not yet committed to any one branch of the subject. Among the problems from analysis and number theory that Littlewood proposed to him, however, was a question about the distribution of quadratic residues that attracted him; virtually all his subsequent work was devoted to the theory of numbers. The topic of Davenport's first piece of research led him directly to the study of character sums and exponential sums, which were then (and still are) central to many of the most profound inquiries in higher arithmetic, and influenced many of his own later researches.

This work brought Davenport to the attention of Louis J. Mordell and of Helmut Hasse; the latter invited him to Marburg for a long visit during 1931. Davenport learned fluent German during this time, and he and Hasse wrote an important paper that still is influential; also, it might be said that Hasse was led by their association to his proof of the Riemann hypothesis for elliptic curves. The natural culmination of this strand of ideas—Weil's proof in 1948 of the Riemann hypothesis for algebraic curves in general by deep methods from algebraic geometry—was at this time far in the future, and Davenport was probably ready for a change of direction.

During his travels in Germany, Davenport met H. Heilbronn, Edmund Landau's last assistant in Göttingen; the two soon became friends and embarked on several new lines of research, including one in the general area of Waring's problem via the celebrated circle method of Hardy, Littlewood, and Ramanujan. Their association deepened when Heilbronn joined Davenport at Cambridge in 1933; they wrote many joint papers over the years, the last appearing in 1971.

Working with Heilbronn or alone, Davenport made several novel adaptations of the circle method and also developed some important technical refinements that led to improved results for Waring's problem itself. While much of this work retains interest, its main importance derives from the fact that it prepared Davenport for the greatest mathematical achievements of his life: the adaptation, starting in 1956, of the circle method (which was invented to deal with additive problems) to nonadditive problems concerning values taken by quadratic and cubic forms in many variables. A critical feature of this adaptation was the use of ideas and results from the geometry of numbers, a branch of number theory that had originated with Hermann Minkowski.

In 1937, at the termination of his Cambridge fellowship, Davenport was appointed by Mordell to an assistant lectureship at the University of Manchester and, under his influence, took up the study of the geometry of numbers. This subject dominated his mathematical activities until 1956, by which time he had become a dominant figure in number theory, not only in the United Kingdom but also in most of the world. In 1941 Davenport, by then a fellow of the Royal Society, moved to a full professorship at the University College of North Wales at Bangor. In 1945 he was appointed Astor Professor of Mathematics at University College, London. In 1950 he became head of the department.

It was during the period 1945 to 1958 that Davenport achieved his full stature. His mathematical prose had been distinguished from the beginning by unusual grace and lucidity, and he now brought these gifts to the classroom and to the supervision of graduate students. Davenport was a superb teacher and an inspiring director of research, and his number theory seminar at University College became a mecca for aspiring number theoreticians from all over the world. It was here that Freeman Dyson

conceived his remarkable proof of Minkowski's conjecture for the product of four nonhomogeneous linear forms, that C. A. Rogers developed his deep researches in the theory of packing space, that K. F. Roth was led to his theorem on rational approximations to algebraic numbers, and that D. A. Burgess reported on his dramatic improvement of Ivan M. Vinogradov's estimate of the least quadratic nonresidue. Many other fine pieces of mathematics, not least many of Davenport's own results, first saw the light of day here.

Davenport had an unusually well-organized mind, and was ever ready to make available to his students and associates the wisdom he had gleaned from his mathematical experiences, Like Littlewood he was punctilious about having research problems available for his students, though he was also quick to encourage promising initiatives of their own. Davenport was always eager to discuss mathematics and more than willing to help fellow mathematicians with difficulties. No query ever went unanswered, and he conducted a voluminous correspondence.

Davenport's research activity never flagged, and therefore his expository writing was not on the scale that his literary powers warranted, although he did produce *The Higher Arithmetic* (1954), a small, elegant book that has gone through several editions.

In 1958 Davenport returned to Cambridge as Rouse Ball Professor. He was at the height of his powers, a worthy successor of Hardy, Littlewood, and Mordell, and the unquestioned leader of the British school of number theory. His research went from strength to strength. The original work on forms in many variables had overlapped in a number of ways with that of D. J. Lewis and B. J. Birch, and he embarked on a vigorous collaboration with both these mathematicians. His association with Lewis was especially close and endured for the rest of his life. He was a frequent visitor to Lewis at Ann Arbor; and his two other books, *Analytic Methods for Diophantine Equations and Diophantine Inequalities* (1962) and *Multiplicative Number Theory* (1967), grew out of graduate courses he presented there.

Davenport's activities as research director continued unabated. Among his students were A. Baker, J. H. Conway, P. D. T. A. Elliott, M. N. Huxley, and H. L. Montgomery. The talented young Enrico Bombieri came to his notice, and Davenport brought him to Cambridge for the first of several fruitful visits. Bombieri reawakened Davenport's interest in prime number theory, and they wrote several important joint papers. Baker and Bombieri both won Fields medals in later years. In this last period Davenport also collaborated with A. Schinzel of the Polish Academy of Sciences on properties of polynomials and with W. Schmidt of the University of Colorado on Diophantine approximation. Many achievements and honors seemed still ahead of him when lung cancer set in with awful suddenness and brought his life to an untimely end.

Davenport was shy and reserved, and his outlook on life was at all times conservative. Despite this his organizational gifts, and his willingness to help all who came to him for advice and to render service to the institutions and learned societies with which he was associated, made him a natural academic leader and one of the most influential mathematicians of his time.

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