Jan De Witt | Encyclopedia.com

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(*b*. Dordrecht, Netherlands, 24 September 1625; *d*. The Hague, Netherlands, 20 August 1672)

mathematics.

De Witt was the son of Jacob de Witt, burgomeister of Dordrecht, and Anna van de Corput. Both families were prominent members of the regent class which governed the towns and provinces of the Netherlands. He entered Dordrecht Latin school in 1636, and went to the University of Leiden in 1641. There he studied law, leaving for France in 1645 to take his degree at Angers. At Leiden he studied mathematics privately with Frans van Schooten the Younger, and received from him an excellent training in Cartesian mathematics. De Witt was a talented mathematician who had little time to devote to mathematics. He became pensionary of Dordrecht in 1650, and grand pensionary of Holland in 1653, making him the leader of the States Party, and, in effect, the prime minister of the Netherlands. He was a statesman of unusual ability and strength of character who guided the affairs of the United Provinces during the twenty-year interregnum in the Stadtholdership during the minority of William of Orange. This was one of the most critical periods in Dutch history, with the three Anglo-Dutch wars; the hostility of the Orange faction culminated in the murder of de Witt and his brother Cornelis by a mob in 1672.

De Witt's most important mathematical work was his *Elementa curvarum linearum,* written before 1650 and printed in Van Schooten's second Latin edition of Descartes's *Géométrie* (1659–1661). It is in two books: the first, a synthetic treatment of the geometric theory found in the early books of Apollonius' *Conics* ; and the second, one of the first systematic developments of the analytic geometry of the straight line and conic. In the first book the *symptomae* (expressed a proportions) of the parabola, ellipse, and hyperbola are derived as plane loci, rather than as sections of the cone. His locus definitions of the ellipse are familiar to us today: the eccentric angle construction (a point fixed with respect to a rotating segment); the trammel construction (a fixed point on a given segment moving on two intersecting lines); and the "string" construction, based on the two-focus definition. For the hyperbola and parabola the locus is constructed as the intersection of corresponding members of two pencils of lines, one parallel and one concurrent. In modern terms these are interesting unintentional examples of the Steiner-Chasles projective definition of the conics, where the vertex of one pencil is at infinity.

De Witt is credited with introducing the term "directrix" for the parabola, but it is clear from his derivation that he does not use the term for the fixed line of our focus-directrix definition. Given fixed lines *DB* and *EF* intersecting at *D,* with *B* the pole and *EF* the directrix: for any point *H* on *EF,* if ∠HBL is constructed equal to ∠FDB, a line through *H* parallel to *BD* cuts *BL* in *G,* a point on the locus. *AC* is drawn through *B* with ∠DBC = ∠BDF, cutting *HG* in *I,* and *GK* is drawn parallel to *AC*. Since triangles *BDH* and *GKB* are similar, $(BI)^2 = (BD)(BK)$ or $y^2 = px$, a parabola with vertex at *B,* abscissa $BK = x$, and ordinate $KG = y$. If *EF* is perpendicular to *DB,* a rectangular coordinate system results, but *EF* is not our directrix.

In the first book of the *Elementa* de Witt not only freed the conics from the cone with his kinematic constructions, but satisfied the Cartesian criteria of constructibility. This book was written, as he reported to van Schooten, to give a background for the new analytic development of the second book. He began the analytic treatment by showing that equations of the first degree represent straight lines. As was usual at the time he did not use negative coordinates, graphing only segments or rays in the first quadrant. He carefully explained the actual construction of the lines for arbitrary coefficients

since they would be needed in his transformations reducing general quadratic equations to type conics. For each conic de Witt began with simplified equations equivalent to his standard forms in book I, and then used translations and rotations to reduce more complicatd equations to the canonical forms. For example, in the hyperbola

he lets

and then

$v = x + h$

where *h* is the coefficient of the linear term in *x* after the first substitution, giving

a standard hyperbola which cuts the new *v* or *z* axes according as *hh* is greater than or less than . Although de Witt seems to be aware of the characteristic of the general quadratic equation in choosing his examples, he does not explicitly mention its use to determine the type of conic except in the case of the parabola. There he states that, if the terms of the second degree are a perfect square, the equation represents a parabola.

The last chapter is a summing up of the various transfomations showing how to construct the graphs of all equations of second degree. Each case of positive and negative coefficients must be handled separately in a drawing, but the discussion for each curve is completely general, and both original and transformed axes are drawn.

In addition to the algebraic simplifications of the curves to normal form, book II contains the usual focus-directrix property of the parabola and the analytic derivations of the eilipse and hyperbola as the locus of points the sum or difference of whose distances from two fixed points is a constant. These are done in the modern manner, squaring twice, with the explicit use of the Pythagorean theorem in place of the more recent distance formula.

De Witt's *Elementa* and John Wallis' *Tractatus de sectionibus conicis* (1655) are considered the first textbooks in analytic geometry. Although Wallis raised the question of priority, their approaches were different and completely independent. Wallis first defined the conics as second-degree equations and deduced the properties of the curves from the equations, while de Witt dedfined them geometrically in the plane, and then showed that quadratic equations could be reduced to his normal forms.

Christiaan Huygens once wrote John Wallis of de Witt: "Could he have spared all his strength for mathematical works, he would have surpassed us all." His geometry was his only contribution to pure mathematics, but he truned his mathematical interests to the financial problems of the province of Holland throughout his long tenure as grand pensionary. The chief means of raising money for the Statres was by life or fixed annuities. In 1665 de Witt succeeded in redcucing the interest rate from 5 to 4 percent and established a sinking fund with the interest saved by the conversion accumulated at compound interest to be applied to the debt of Holland, which could thus be paid in forty-one years. The second Anglo-Dutch War(1665–1667), however, defeated this scheme. The English wars were a perpetual financial drain, and nore than half of the expenditure the costs of the war almost alone) was swallowed up in interest payments.

In April 1671 it was resolved to negotiate funds by life annuities, therby limiting the debt to one generation. De Witt prepared a treatise for the States of Holland demonstrating mathematically that life annuites were being offered at too high a rate of interest in comparison with fixed annuities. For many years the rule-of-thumb rates for life annuities had been twice the standard rate of interest.Holland had recently reduced the rate of interest to twenty-five years' purchase (4 percent) and was selling life annuities at fourteen years' purchase(7 percent). De Witt wanted to raise the price to sixteen years' purchase (6¼ percent). His *Waerdye van Lyf-renten naer proportie van Losrenten* (July, 1671) is certainly among the first attempts to apply the theory of probability to economic problems. It was written as a political paper, and remained buried in the archives for almost two hundred years. Since its discovery and publication by Frederick Hendriks in 1852 there have been many articles (some of which are listed in the bibliography)explaining or critizing it on the basis of modern actuarial science. It is actually a very simple and ingenious dissertation based only on the use of the principle of mathematical expectation to form equal contracts.

De Witt listed the present values at 4 percent of annuity payments of 10,000,000 stuyvers (to avoid decimals) per half year, and summed the mathematical expectations using hypothetical mortality rates for different ages. He first presupposed that a man is equally likely to die in the first or last half of any year, and then, since annuities were generally purchased on young lives, extended this to any half year of the "years of full vigor" from age three to fifty-three. For simplicity he considered the first hundred half years equally destructive or mortal, although he stated that the likelihood of decease is actually smaller in the first years. So too, he stopped at age eighty, although many live beyond that age. In the next ten years, fifty-three to sixtythree, the chance of dying does not exceed more than in the proportion of 3 to 2 the chance of dying in the first period; from sixty-three to seventy-three, the chance of dying is not more than 2 to 1; and from seventy-three to eighty, not more than 3 to 1.

De Witt gives many examples to explain the use of the concept of mathematical expectation. The following one is basic to his later calculations, and has been overlooked by many commentators. Consider a man of forty and a man of fifty-eight. According to his presuppositions the chances of the older man dying compared with the younger man are as 3 to 2. An equal contract could be devised:if the person of fifty-eight dies in six months, the younger man inherits 2,000 florins, but if the man of forty dies in six months, the elder inherits 3,000 florins. That is, chance of the man of fifty-eight gaining 3,000 florins. is as 2 to 3, or, in terms of de Witt's annuity calculations, the chance of receiving a particular annuity payment in the second period is two-thirds that in the first period.

From this reasoning de Witt' calculations are straightforward: he sums the present values for the first hundred half years; tow-thirds the present values for the next twenty half years; for the next twenty, one-half the present values; and one-third for the last fourteen. All these are summed and the average taken, giving a little more than sixteen florins as the present value of one florin of annuity on a young and healthy life. If the method had been applied to actual mortality tables, the labor would have been formidable. Later in 1671 de Witt and Jan Hudde corresponded on the problem of survivorship annuities on more than one life, and here both used actual mortality figures taken from the annuity records of Holland. Working with several groups of a least a hundred persons of a given age de Witt developed appropriate rates for annuities on two lives. These were extended a posteriori to any number of lives by a Pascal triangle, with a promise to Hudde to establish the results a priori. This was the culmination of de Witt's work with annuities, but for political reasons he suggested to Hudde that the public not be informed of the results of their study, since they were willing to buy annuities on more than one life at the current rate, which was favorable to the government.

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