Dedekind, (Julius Wilhelm) Richard | Encyclopedia.com

(*b*. Brunswick, Germany, 6 October 1831; *d*. Brunswick, 12 February 1916)

mathematics.

Dedekind's ancestors (particularly on his mother's side) had distinguished themselves in services to Hannover and Brunswick. His father, Julius Levin Ulrich Dedekind, the son of a physician and chemist, was a graduate jurist, professor, and corporation lawyer at the Collegium Carolinum in Brunswick. His mother, Caroline Marie Henriette Emperius, was the daughter of a professor at the Carolinum and the granddaughter of an imperial postmaster. Richard Dedekind was the youngest of four children. His only brother, Adolf, became a district court president in Brunswick; one sister, Mathilde, died in 1860, and Dedekind lived with his second sister, Julie, until her death in 1914, neither of them having married. She was a respected writer who received a local literary prize in 1893.

Between the ages of seven and sixteen Dedekind attended the Gymnasium Martino-Catharineum in Brunswick. His interest turned first to chemistry and physics; he considered mathematics only an auxiliary science. He soon occupied himself primarily with it, however, feeling that physics lacked order and a strictly logical structure. In 1848 Dedekind became a student at the Collegium Carolinum, an institute between the academic high school and the university level, which Carl Friedrich Gauss had also attended. There Dedekind mastered the elements of analytic geometry, algebraic analysis, differential and integral calculus, and higher mechanics, and studied the natural sciences. In 1849–1850, he gave private lessons in mathematics to his later colleague at the Carolinum, Hans Zincke (known as Sommer). Thus, when he matriculated at the University of Göttingen at Easter 1850, Dedekind was far better prepared for his studies than were the majority of graduates from the academic high school. At Göttingen, a seminar in mathematics and physics had just been founded, at the initiative of Moritz Abraham Stern, for the education of instructors for teaching in the academic high school. The direction of the mathematics department was the duty of Stern and Georg Ulrich, while Wilhelm Weber and Johann Benedict Listing directed the physics department. Dedekind was a member of the seminar from its inception and was there first introduced to the elements of the theory of numbers. A year later Bernhard Riemann also began to participate in the seminar, and Dedekind soon developed a close friendship with him. In the first semester, Dedekind attended lectures on differential and integral calculus, which offered him very little new material. He attended Ulrich's seminar on hydraulics but rarely took part in the physics laboratories run by Weber and Listing; Weber's lectures on experimental physics, however, made a very strong impression on him throughout two semesters. Weber had an inspiring effect on Dedekind, who responded with respectful admiration. In the summer semester of 1850, Dedekind attended the course in popular astronomy given by Gauss's observer, Carl Wolfgang Benjamin Goldschmidt; in the winter semester of 1850–1851, he attended Gauss's own lecture on the method of least squares. Although he disliked teaching, Gauss carried out the assignment with his usual conscientiousness; fifty years later Dedekind remembered the lecture as one of the most beautiful he had ever heard, writing that he had followed Gauss with constantly increasing interest and that he could not forget the experience. In the following semester, Dedekind heard Gauss's lecture on advanced geodesy. In the winter semester of 1851–1852, he heard the two lectures given by Quintus Icilius on mathematical geography and on the theory of heat and took part in Icilius' meteorological observations. After only four semesters, he did his doctoral work under Gauss in 1852 with a thesis on the elements of the theory of Eulerian integrals. Gauss certified that he knew a great deal and was independent; in addition, he had prophetically "favorable expectations of his future performance."

Dedekind later determined that this knowledge would have been sufficient for teachers in secondary school service but that it did not satisfy the prerequisite for advanced studies at Göttingen. For instance, Dedekind had not heard lectures on more recent developments in geometry, advanced theory of numbers, division of the circle and advanced algebra, elliptic functions, or mathematical physics, which were then being taught at the University of Berlin by Steiner, Jacobi, and Dirichlet. Therefore, Dedekind spent the two years following his graduation assiduously filling the gaps in his education, attending—among others—Stern's lectures on the solution of numerical equations.

In the summer of 1854, he qualified, a few weeks after Riemann, as a university lecturer; in the winter semester of 1854–1855 he began his teaching activities as *Privatdozent*, with a lecture on the mathematics of probability and one on geometry with parallel treatment of analytic and projective methods.

After Dirichlet succeeded Gauss in Göttingen in 1855, Dedekind attended his lectures on the theory of numbers, potential theory, definite integrals, and partial differential equations. He soon entered into a closer personal relationship with Dirichlet and had many fruitful discussions with him; Dedekind later remembered that Dirichlet had made "a new man" of him and had expanded his scholarly and personal horizons. When the Dirichlets were visited by friends from Berlin (Rebecca Dirichlet was the sister of the composer Felix Mendelssohn-Bartholdy and had a large circle of friends), Dedekind was invited too and enjoyed the pleasant sociability of, for example, the well-known writer and former diplomat, Karl August Varnhagen von Ense, and his niece, the writer Ludmilla Assing.

In the winter semester of 1855–1856 and in the one following, Dedekind attended Riemann's lectures on Abelian and elliptic functions. Thus, although an instructor, he remained an intensive student as well. His own lectures at that time are noteworthy in that he probably was the first university teacher to lecture on Galois theory, in the course of which the concept of field was introduced. To be sure, few students attended his lectures: only two were present when Dedekind went beyond Galois and replaced the concept of the permutation group by the abstract group concept.

In 1858, Dedekind was called to the Polytechnikum in Zurich (now the Eidgenössische Technische Hochschule) as the successor to Joseph Ludwig Raabe. Thus Dedekind was the first of a long line of German mathematicians for whom Zurich was the first step on the way to a German professorial chair; to mention only a few, there were E. B. Christoffel, H. A. Schwarz, G. Frobenius, A. Hurwitz, F. E. Prym, H. Weber, F. Schottky, and H. Minkowski. The Swiss school counsellor responsible for appointments came to Göttingen at Easter 1858 and decided immediately upon Dedekind—which speaks for his power of judgment. In September 1859, Dedekind traveled to Berlin with Riemann, after Riemann's election as a corresponding member of the academy there. On this occasion, Dedekind met the initiator of that selection, Karl Weierstrass, as well as other leaders of the Berlin school, including Ernst Eduard Kummer, Karl Wilhelm Borchardt, and Leopold Kronecker.

In 1862, he was appointed successor to August Wilhelm Julius Uhde at the Polytechnikum in Brunswick, which had been created from the Collegium Carolinum. He remained in Brunswick until his death, in close association with his brother and sister, ignoring all possibilities of change or attainment of a larger sphere of activity. The small, familiar world in which he lived completely satisfied his demands: in it his relatives completely replaced a wife and children of his own and there he found sufficient leisure and freedom for scientific work in basic mathematical research. He did not feel pressed to have a more marked effect in the outside world; such confirmation of himself was unnecessary.

Although completely averse to administrative responsibility, Dedekind nevertheless considered it his duty to assume the directorship of the Polytechnikum from 1872 to 1875 (to a certain extent he was the successor of his father, who had been a member of the administration of the Collegium Carolinum for many years) and to assume the chairmanship of the school's building commission in the course of the transformation to a technical university. Along with his recreational trips to Austria (the Tyrol), to Switzer land, and through the Black Forest, his visit to the Paris exposition of 1878 should also be mentioned. On 1 April 1894 he was made professor emeritus but continued to give lectures occasionally. Seriously ill in 1872, following the death of his father, he subsequently enjoyed physical and intellectual health until his peaceful death at the age of eighty-four.

A corresponding member of the Göttingen Academy from 1862, Dedekind also became a corresponding member of the Berlin Academy in 1880 upon the initiative of Kronecker. In 1900, he became a correspondent of the Académie des Sciences in Paris and in 1910 was elected as *associé étranger*. He was also a member of the Leopoldino-Carolina Naturae Curiosorum Academia and of the Academy in Rome. He received honorary doctorates in Kristiania (now Oslo), in Zurich, and in Brunswick. In 1902 he received numerous scientific honors on the occasion of the fiftieth anniversary of his doctorate.

Dedekind belonged to those mathematicians with great musical talent. An accomplished pianist and cellist, he composed a chamber opera to his brother's libretto.

In character and principles, in style of living and views, Dedekind had much in common with Gauss, who also came from Brunswick and attended the Gymnasium Martino-Catharineum, the Collegium Carolinum, and the University of Göttingen. Both men had a conservative sense, a rigid will, an unshakable strength of principles, and a refusal to compromise. Each led a strictly regulated, simple life without luxury. Cool and reserved in judgment, both were warm-hearted, helpful people who formed strong bonds of trust with their friends. Both had a distinct sense of humor but also a strictness toward themselves and a conscientious sense of duty. Averse to any excess, neither was quick to express astonishment or admiration. Both were averse to innovations and turned down brilliant offers for other professorial chairs. In their literary tastes, both numbered Walter Scott among their favorite authors. Both impressed by that quality called modest greatness. Thus, it is not astonishing to find their similarity persisting in mathematics in the same preference for the theory of numbers, the same reservations about the algorithm, and the same partiality for "notions" above "notations." Although considerable, significant differences existed between Gauss and Dedekind, what they had in common predominates by far. Their kinship also received a marked visible expression: Dedekind was one of the select few permitted to carry Gauss's casket to the funeral service on the terrace of the Sternwarte.

Aside from Gauss the most enduring influences on Dedekind's scientific work were Dirichlet and Riemann, with both of whom he shared many inclinations and attitudes. Dedekind, Dirichlet, and Riemann were all fully conscious of their worth, but with a modesty bordering on shyness, they never let their associates feel this. Ambition being foreign to them, they were embarrassed when confronted by the brilliance and elegance of their intellect. They loved thinking more than writing and were hardly ever able to satisfy their own demands. Being of absolute integrity, they had in common the same love for plain, certain truth. Dedekind's own statement to Zincke is more revelatory of his character than any description could be: "For what I have accomplished and what I have become, I have to thank my industry much more, my indefatigable working rather than any outstanding talent."

When Dedekind is mentioned today, one of the first associations is the "Dedekind cut," which he introduced in 1872 to use in treating the problem of irrational numbers in a completely new and exact manner.

In 1858, Dedekind had noted the lack of a truly scientific foundation of arithmetic in the course of his Zurich lectures on the elements of differential calculus. (Weierstrass also was stimulated to far-reaching investigations from such observations in the course of preparing lectures.) On 24 October, Dedekind succeeded in producing a purely arithmetic definition of the essence of continuity and, in connection with it, an exact formulation of the concept of the irrational number. Fourteen years later, he published the result of his considerations, *Stetigkeit und irrationale Zahlen* (Brunswick, 1872, and later editions), and explained the real numbers as "cuts" in the realm of rational numbers. He arrived at concepts of outstanding significance for the analysis of number through the theory of order. The property of the real numbers, conceived by him as an ordered continuum, with the conceptual aid of the cut that goes along with this, permitted tracing back the real numbers to the rational numbers: Any rational number a produces a resolution of the system R of all rational numbers into two classes A_1, A_2, in such a way that each number a_1 of the class A_2 is smaller than each number a_2 of the second class A_2. (Today, the term "set" is used instead of "system.") The number a is either the largest number of the class A_1 or the smallest number of the class A_2. A division of the system R into the two classes A_1, A_2, whereby each number a_1 in A_1 is smaller than each number a_2 in A_2 is called a "cut"(A_1, A_2) by Dedekind. In addition, an infinite number of cuts exist that are not produced by rational numbers. The discontinuity or incompleteness of the region R consists in this property. Dedekind wrote, "Now, in each case when there is a cut (A_1A_2) which is not produced by any rational number, then we *create* a new, *irrational* number α, which we regard as completely defined by this cut; we will say that this numberα corresponds to this cut, or that it produces this cut" (*Stetigkeit*, § 4).

Occasionally Dedekind has been called a "modern Eudoxus" because an impressive similarity has been pointed out between Dedekind's theory of the irrational number and the definition of proportionality in Eudoxus' theory of proportions (Euclid, *Elements*, bk. V, def. 5). Nevertheless, Oskar Becker correctly showed that the Dedekind cut theory and Eudoxus' theory of proportions do not coincide: Dedekind's postulate of existence for all cuts and the real numbers that produce them cannot be found in Eudoxus or in Euclid. With respect to this, Dedekind said that the Euclidean principles alone—without inclusion of the principle of continuity, which they do not contain—are incapable of establishing a complete theory of real numbers as the proportions of the quantities. On the other hand, however, by means of his theory of irrational numbers, the perfect model of a continuous region would be created, which for just that reason would be capable of characterizing any proportion by a certain individual number contained in it (letter to Rudolph Lipschitz, 6 October 1876).

With his publication of 1872, Dedekind had become one of the leading representatives of a new epoch in basic research, along with Weierstrass and Georg Cantor. This was the continuation of work by Cauchy, Gauss, and Bolzano in systematically eliminating the lack of clarity in basic concepts by methods of demonstration on a higher level of rigor. Dedekind's and Weierstrass' definition of the basic arithmetic concepts, as well as Georg Cantor's theory of sets, introduced the modern development, which stands "completely under the sign of number," as David Hilbert expressed it.

Dedekind's book *Was sind und was sollen die Zahlen?* (Brunswick, 1888, and later editions) is along the same lines; in it he presented a logical theory of number and of complete induction, presented his principal conception of the essence of arithmetic, and dealt with the role of the complete system of real numbers in geometry in the problem of the continuity of space. Among other things, he provides a definition independent of the concept of number for the infiniteness or finiteness of a set by using the concept of mapping and treating the recursive definition, which is so important for the theory of ordinal numbers. (Incidentally, Dedekind regarded the ordinal number and not the cardinal number [*Anzahl*] as the original concept of number; in the cardinal number he saw only an application of the ordinal number [letter to Heinrich Weber, 24 January 1888].) The demonstration of the existence of infinite systems given by Dedekind—similar to a consideration in Bolzano's *Paradoxien des Unendlichen* (Prague, 1851, §13)—is no longer considered valid. Kronecker was critical because Dedekind, agreeing with Gauss, regarded numbers as free creations of the human intellect and defended this viewpoint militantly and stubbornly. Weierstrass complained that his own definition of a complex quantity had not been understood by Dedekind. Hilbert criticized his effort to establish mathematics solely by means of logic. Gottlob Frege and Bertrand Russell criticized Dedekind's opinion that the cuts were not the irrational numbers but that the latter produced the former. However, even his critics and those who preferred Cantor's less abstract procedure for the construction of real numbers agreed that he had exercised a powerful influence on basic research in mathematics.

Just as Kronecker and Weierstrass had edited the mathematical works of those to whom they felt obligated, so Dedekind worked on the erection of literary monuments to those who had stood close to him. Making accessible the posthumous works of Gauss, Dirichlet, and Riemann occupied an important place in his work. In doing this, he gave proof of his congeniality and the rare combination of his productive and receptive intellectual talents.

Publishing the manuscripts of Gauss on the theory of numbers (*Werke*, vol. II [Göttingen, 1863]) gave him the opportunity of not only making available to wider circles the papers of a man he so greatly respected, but also of commenting on them with deep understanding. Dirichlet's *Vorlesungen über Zahlentheorie*(Brunswick, 1863, and later editions) was edited by him. If, as has been said, Dirichlet was the first not only to have completely understood Gauss's *Disquisitiones arithmeticae* but also to have made them accessible to others, then the same is true, to a great extent, of Dedekind's relationship to Dirichlet's lectures on the theory of numbers. Finally, he collaborated in editing the *Werke Bernhard Riemanns* (Leipzig, 1876; 2nd ed., 1892) with his friend Heinrich Weber and with his accustomed modesty placed his name after Weber's.

The editing of Dirichlet's lectures led Dedekind into a profound examination of the theory of generalized complex numbers or of forms that can be resolved into linear factors. In 1871 he provided these lectures with a supplement, in which he established the theory of algebraic number fields, or domains, by giving a general definition of the concept of the ideal—going far beyond Kummer's theory of "ideal numbers"— that has become fruitful in various arithmetic and algebraic areas. In several papers Dedekind then, independently of Kronecker and with his approval, established the ideal theory that is held to be his masterpiece. Its principal theorem was that each ideal different from the unit ideal *R* can be represented unambiguously—with the exception of the order of factors—as the product of prime numbers. In his treatises concerning number fields, Dedekind arrived at a determination of the number of ideal classes of a field, penetrated the analysis of the base of a field, provided special studies on the theory of modules, and stimulated further development of ideal theory in which Emmy Noether, Hilbert, and Philipp Furtwängler participated. Paul Bachmann, Adolf Hurwitz, and Heinrich Weber in their publications also disseminated Dedekind's thoughts and expanded them.

That Dedekind did not stand completely apart from the applications of mathematics is shown by a treatise, written with W. Henneberg, which appeared as early as 1851 and concerns the time relationships in the course of plowing fields of various shapes, and also by the completion and publication of a treatise by Dirichlet concerning a hydrodynamic problem (1861).

Finally, we are indebted to Dedekind for such fundamental concepts as *ring* and *unit*.

It was indicative of the great esteem in which Dedekind was held even in foreign countries that, shortly after his death, in the middle of World War I, Camille Jordan, the president of the Académie des Sciences in Paris, warmly praised his theory of algebraic integers as his main work and expressed his sadness concerning the loss.

Although the association of mathematicians' names with concepts and theorems is not always historically justified or generally accepted, the number of such named concepts can provide an indication, albeit a relative one, of a mathematician's lasting accomplishments in extending the science. By this standard, Dedekind belongs among the greatest mathematicians; approximately a dozen designations bear his name.

BIBLIOGRAPHY

I. Original Works. For a bibliography of Dedekind's writings, see Poggendorff, I (1863), 534, 1555; III (1898), 340; IV (1904), 305; V (1926), 269; and VI (1936), 538.

His works include *Gesammelte mathematische Werke*, R. Fricke, E. Noether, and O. Ore, eds., 3 vols. (Brunswick, 1930–1932), with a bibliography in III, 505–507; and *Briefwechsel Cantor–Dedekind*. E. Noether and J. Cavaillès, eds.(Paris, 1937). Extracts from his works appear in Oskar Becker, *Die Grundlagen der Mathematik in Geschichtlicher Entwicklung* (Freiburg–Munich, 2nd ed., 1964), pp. 224–245, 316.

II. Secondary Literature. On Dedekind and his work, see Kurt-R. Biermann, "Richard Dedekind im Urteil der Berliner Akademie," in *Forschungen und Fortschritte*, **40** (1966), 301–302; Edmund Landau, "Richard Dedekind", in *Nachrichten von der Königlichen Gesellschaft der Wissenschaften zu Göttingen, Geschäftliche Mitteilungen* (1917), pp. 50-70, with a bibliography on pp. 66–70; Wilhelm Lorey, *Das Studium der Mathematik an den deutschen Universiäten seit Anfang des 19. Jahrhunderts* (Leipzig-Berlin, 1916), with personal recollections of Dedekind on pp. 81–83; Karl Mollenhauer, "Julie Dedekind," in *Braunschweigisches Magazin*, **21** (1916), 127–130; Richard Müller,"Aus den Ahnentafeln deutscher Mathematiker: Richard Dedekind," in *Familie und Volk*, **4** (1955), 143–145; Nikolaus Stuloff, "Richard Dedekind," in *Neue Deutsch Biographie*, **3** (1957), 552–553; Hans Zincke ("Sommer") "Erinnerungen an Richard Dedekind," in *Braunschweigisches Magazin*, **22** (1916), 73–81; "Die Akademien zu Paris und Berlin über Richard Dedekind," *ibid*, 82-84; and "Julius Wilhelm Richard Dedekind," in *Festschrift zur Feier des 25-jährigen Bestehens des Gesellschaft ehemaliger Studierender des Eidgenössischen polytechnischen Schule in Zürich* (1894), pp. 29–30.

Kurt-R. Biermann