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(*fl*. Caunus [?], Asia Minor, third-second centuries b.c.)

mathematics.

The Dionysodorus who is the subject of this article is recorded by Eutocius as having solved, by means of the intersection of a parabola and a hyperbola, the cubic equation to which (in effect) Archimedes had reduced the problem of so cutting a sphere by a plane that the volumes of the segments are in a given ratio. Of the many bearers of this name in Greek literature, he has usually been identified with the Dionysodorus who is described by Strabo (XII, 3,16) as a mathematician and is included among the men noteworthy for their learning who were born in the region of Amisene in Pontus, on the shore of the Black Sea. But since Wilhelm Cronert published in 1900 hitherto unknown fragments from the Herculaneum roll no. 1044, and especially since Wilhelm Schmidt commented on them in 1901, it has seemed more probable that he should be identified with Dionysodorus of Caunus, son of a father of the same name, who was probably an Epicurean. One fragment (no. 25) indicates that this Dionysodorus succeeded Eudemus as the teacher of Philonides, and another (no. 7) that Philonides published some lectures by Dionysodorus. Eudemus is obviously the Eudemus of Pergamum to whom Apollonius dedicated the first two books of his *Conics*, and Philonides is the mathematician to whom Apollonius asked Eudemus to show the second book. When we recollect that Caunus in Caria is near Apollonius' birthplace, Perga in Pamphylia, it is clear that this Dionysodorus moved in distinguished mathematical company and would have been capable of the elegant construction that Eutocius has recorded. If this identification is correct, he would have lived in the second half of the third century b.c. If he is to be identified with Dionysodorus of Amisene, all that can be said about his date is that he wrote before Diodes, say before 100 b.c. It is clear that he is not the same person as the geometer Dionysodorus of Melos, who is mentioned by Pliny *(Natural History*, II, 112.248) as having arranged for a message to be put in his tomb saying that he had been to the center of the earth and had found the earth's radius to measure 42,000 stades. Strabo, indeed, specifically distinguishes them.

In the passage quoted by Eutocius, *Commentarii in libros II De sphaera et cylindro* (Archimedes, Heiberg ed., III, 152.28–160.2), Dionysodorus says: Let AB be a diameter of a given sphere which it is required to cut in the given ratio CD:DE. Let BA be produced to F so that $AF = AB/2$, let AG be drawn perpendicular to AB so that FA:AG = CE: ED, and let H be taken on AG produced so that $AH^2 = FA \cdot AG$. With axis FB let a parabola be drawn having AG as its parameter; it will pass through H. Let it be FHK where BK is perpendicular to AB. Through G let there be drawn a hyperbola having FB and BK as asymptotes. Let it cut the parabola at L—it will, of course, cut at a second point also—and let LM be drawn perpendicular to AB. Then, proves Dionysodorus, a plane drawn through M perpendicular to AB will cut the sphere into segments whose volumes have the ratio CD:DE.

It will be more instructive to turn the procedure into modern notation rather than reproduce the prolix geometrical proofs. In his treatise *On the Sphere and Cylinder*, II, 2 and 4, Archimedes proves geometrically that if r be the radius of a sphere and h the height of one of the segments into which it is divided by a plane, the volume of the segment is equal to a cone with the same base as the segment and height

If h' is the height of the other segment, and the volumes of the segments stand in the ratio m: n, then

Eliminating h' by the relationship $h + h' = 2r$, we obtain the cubic equation in the usual modern form

If we substitute $x = 2r–h$ (= h') we may put the equation in the form solved by Dionysodorus:

Dionysodorus solves it as the intersection of the parabola

and the hyperbola.

It seems probable (despite Schmidt) that this mathematician is the same Dionysodorus who is mentioned by Hero as the author of the book Πϵρίτη̂ςσαπϵνίϱας, "On the Tore" (*Heronis opera omnia*, H. Schöne, ed., III, 128.1–130.11), in which he gave a formula for the volume of a torus. If BC is a diameter of the circle $BDCE$ and if BA is perpendicular to the straight line HAG in the same plane, when AB makes a complete revolution around HAG, the circle generates a spire or torus whose volume, says Dionysodorus, bears to the cylinder having HG for its axis and EH for the radius of its base the same ratio as the circle $BDCE$ bears to half the parallelogram $DEHG$.

That is to say, if r is the radius of the circle and $EH = a$

whence

Volume of torus = $2\pi a.\pi r^2$

In an example, apparently taken from Dionysodorus, $r = 6$ and $a = 14$, and Hero notes that if the torus be straightened out and treated as a cylinder, it will have 12 as the diameter of its base and 88 as its length, so that its volume is 9956 4/7. This is equivalent to saying that the volume of the torus is equal to the area of the generating circle multiplied by the length of the path traveled by its center of gravity, and it is the earliest example of what we know as Guldin's theorem (although originally enunciated by Pappus).

Among the inventors of different forms of sundials in antiquity Vitruvius (IX, 8; Krohn, ed., 218.8) mentions a Dionysodorus as having left a conical form of sundial—"Dionysodorus conum (reliquit)." It would no doubt, as Frank W. Cousins asserts, stem from the hemispherical sundial of Berossus, and the cup would be a portion of a right cone, with the nodal point of the style on the axis pointing to the celestial pole. Although there can be no certainty, there seems equally no good reason for not attributing this invention to the same Dionysodorus; it would fit in with his known use of conic sections.

BIBLIOGRAPHY

On Dionysodorus or his work, see Eutocius, *Commentarii in libros II De sphaera et cylindro*, in Archimedes, Heiberg ed., III, 152.27–160.2; Hero of Alexandria, *Metrica*, II, 13—*Heronis opera omnia*, H. Schöne, ed.. III, 128.1–130.11; Wilhelm Cronert, "Der Epikur Philonides," in *Sitzungsberichte der K. Preussischen Akademie der Wissenschaften zu Berlin* (1900), 942–959, esp. frag. 7, p. 945, and frag. 25, p. 952. Wilhelm Schmidt, "Uber den griechischen Mathematiker Dionysodorus," in *Bibliotheca mathematica*, 3rd ser., **4** (1904), 321–325; Sir Thomas Heath, *A History of Greek Mathematics*, II (Oxford, 1921), 46, 218–219, 334–335; Ivor Thomas, *Selections Illustrating the History of Greek Mathematics*, II (London-Cambridge, Mass., 1941), pp. 135, 163, 364, 481; René R. J. Rohr, *Les cadrans solaires* (Paris, 1965), pp. 31–32, trans, by G. Godin as *Sundials: History, Theory and Practice* (Toronto-Buffalo, 1970), pp. 12, 13; and Frank W. Cousins, *Sundials* (London, 1969), pp. 13, 30 (correcting Cdynus to Caunus).

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