Du Bois-Reymond, Paul David Gustav | Encyclopedia.com

(*b*, Berlin, Germany, 2 December 1831; *d*. Freiburg, Germany, 7 April 1889)

mathematics.

Paul du Bois-Reymond was the younger brother of the famous physiologist Emil du Bois-Reymond. He studied first at the French Gymnasium in Berlin, then at the *collège* in Neuchâtel and the Gymnasium in Naumburg. Following the example of his brother, he began to study medicine at the University of Zurich in 1853 and by the next year had published four articles that dealt basically with physiological problems. But soon du Bois-Reymond began to apply his talents to the mathematical and physical sciences. He continued his studies at the University of Königsberg, where, mainly through the influence of Franz Neumann, he turned to the study of mathematical physics, joining his talent for observation with that for making theoretical analyses. He specialized in the study of liquids, especially the areas of liquidity and capillarity. In 1859 he received his doctorate at the University of Berlin on the basis of his dissertation, "De aequilibrio fluidorum." Du Bois-Reymond then became a professor of mathematics and physics at a secondary school in Berlin and continued to devote himself systematically to mathematics until his appointment at the University of Heidelberg in 1865. In 1870 he went from Heidelberg to Freiburg as a professor, and thence to the University of Tübingen, in 1874, as the successor to H. Hankel. From 1884 until the end of his scientific career he occupied a chair of mathematics at a technical college in Berlin.

Du Bois-Reymond worked almost exclusively in the field of infinitesimal calculus, concentrating on two aspects: the theory of differential equations and the theory of the functions of real variables.

Studying the problems of mathematical physics led du Bois-Reymond to the theory of differential equations. He was concerned with these problems at the start of his scientific career and returned to them in the last years of his life. His basic study, *Beiträge zur Interpretation der partiellen Differentialgleichungen mit drei Variablen* (part 1, *Die Theorie der Charakteristiken* [Leipzig 1864]), was one of the first to follow up Monge's idea of the "characteristic" of a partial differential equation. This idea, expressed by Monge for equations of the second order as early as 1784, depended on the geometric expression of the integral of a differential equation as the surface defined by a system of curves. Du Bois-Reymond generalized this for partial differential equations of the nth order. These ideas, such as the simple case of a study of contact transformations, led in generalized form to the studies by Lie and Scheffers.

The chief means of solving partial differential equations at that time was by Fourier series. As early as the 1820's Cauchy, Abel, and Dirichlet had pointed out some of the difficulties of the expansion of "arbitrary" function in a Fourier series and of the convergence of this series. These problems contributed substantially to the rebuilding of the foundations of mathematical analysis. One of the first to deal with them systematically was du Bois-Reymond. He published his main results toward the end of the 1860's and in the 1870's—at first under the influence of Riemann's ideas and at a time when the results of Weierstrass' work had not been published and were little known.

He achieved a number of outstanding results. As early as 1868, when he was studying some properties of integrals, du Bois-Reymond expressed both precisely and generally and demonstrated the meanvalue theorem for definite integrals, which was then an important aid in the study of Fourier series. This theorem was later expressed independently by Dini (1878), who ascribed it to Weierstrass. The latter, however, developed a similar but more specialized proposition and made no claim to du Bois-Reymond's discovery.

Like other mathematicians who relied on Dirichlet, du Bois-Reymond also originally tried to show that each continuous function in a given interval is necessarily representable by its Fourier series (or another series analogous to a trigonometric one). A decisive turn came in 1873, when du Bois-Reymond published "Über die Fourier'schen Reihen." This article contains an exposition of the chief idea of a construction of a continuous function with a divergent Fourier series at any point. Later he also attempted to show the properties of this continuous function (which has a very complicated construction) using considerations, difficult to comprehend, that concern the infinitely small and the infinitely large (the so-called *Infinitärcalcul*).

Two of the other results of du Bois-Reymond's work should be mentioned. First and foremost is the solution of the problem of the integrability of Fourier series, which he proposed (but did not publish until 1883) and for which he demonstrated certain conditions that made it possible to distinguish Fourier's from other trigonometric series. The second is the solution to a

question that concerned mathematicians of that time: the publication (1873) of an example and the precise demonstration of the properties of the function that is continuous in a given interval but without derivatives. This achievement was inspired by Weierstrass.

Du Bois-Reymond was then led to attempt a general exposition of the fundamental concepts of the theory of functions in his book *Die allgemeine Functionentheorie*, the first part of which was published in 1882. Among other things, this work shows that its author was aware that a precise theory of real numbers was needed for the further progress of the theory of functions, but he did not make any real contributions to that progress. Instead, he wrote of the problems of the philosophy of mathematics, recognizing the advantages of different approaches and expressing grave doubts about the usefulness of formalism.

Du Bois-Reymond's work was directed at the basic questions of the mathematical analysis of the time and is marked by both the personality of the author and the state of the mathematics of the period. It appeared before completion of the revision of the foundations of mathematical analysis for which he was striving. Led by sheer mathematical intuition, he did not hesitate to publish even vague considerations and assertions that were later shown to be false. Further developments, some while du Bois-Reymond was still alive, disclosed these weaknesses (e.g., Pringsheim's criticism) and also rapidly outdated his results, even on the main questions. Among them is his attempt to give a general theory of convergence tests. This meant that his work, which had been greatly appreciated by his contemporaries, soon sank into oblivion, although it had included very important questions and notions that were later reflected in the work of such mathematicians as W. H. Young, A. Denjoy, and H. Lebesgue.

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