Fatou, Pierre Joseph Louis | Encyclopedia.com

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(*b*. Lorient, France, 28 February 1878; *d*. Pornichet, France, 10 August 1929)

mathematics.

Fatou attended the École Normale Supérieure from 1898 to 1901. The scarcity of mathematical posts in Paris led him to accept a post at the Paris observatory, where he worked until his death. He received his doctorate in 1907 and was appointed titular astronomer in 1928. Fatou worked in practical astronomy: on determining the absolute positions of stars and planets, on instrumental constants, and on measurements of twin stars.

In order to calculate the secular perturbations produced on a planet P' through the movement of another planet, P, Gauss had had the idea of spreading the mass of P' over its orbit, so that the mass of each arc is proportional to the time it takes for the planet to trace it. This proposition is valid only when the distinction between periodic and secular perturbations does not apply, i.e., when n' is very small (n and n' are the mean motions of the planets P and P', respectively). By means of general existence theorems of solutions of differential equations, Fatou studied these motions of material systems subjected to forces whose periods tend to zero. Gauss's intuitive result had often been used in practice but had never been rigorously justified.

Fatou also studied the movement of a planet in a resistant medium. This work was based on the probability that stellar atmospheres had previously been far more extensive than they are now and would thus have given rise to capture phenomena that can be used to explain the origins of twin stars and of certain satellites.

Along with this work Fatou did both related and general mathematical research. He contributed important results on the Taylor series, the theory of the Lebesgue integral, and the iteration of rational functions of a complex variable. When studying the circle of convergence of the Taylor series, several points of view are possible: (1) one can look for criteria of convergence or divergence of the series itself on the circumference; (2) one can consider the limit values of the circle of the analytic function represented by the series and try to determine where these limit values are finite or infinite, as well as the properties of the functions of the argument represented by the real and imaginary parts of the series when these functions are well defined; (3) one can consider what points on the circumference, singular in the Weierstrass sense, also determine the analytic extension of the series. The link between these problems led Fatou to formulate a fundamental theorem in the theory of the Lebesgue integral. He found that the theory of the Lebesgue integral allowed the first two of the above problems to be treated with more precision and more generality, with the following general result: If $f_n(x) \geq 0$ for all values of $n, x \in E$ and $f_n(x) \to f(x)$ as $n \to \infty$, then

The theorem implies that if the right-hand side is finite, then $f(x)$ is finite almost everywhere and integrable; if $f(x)$ is not integrable or is infinite in a set of positive measure, then

This work was advanced by Carathéodory Friedrich and Marcel Riesz, Griffith Evans, Leon Lichtenstein, Gabor Szegö and Nicolas Lusin.

Fatou also showed ways in which the algebraic signs of a_n affect the number and character of singularities in the Taylor series. Given the series $\sum a_n x^n$ $0 < R < \infty$, a sequence $\{\lambda_n\}$ exists such that the series obtained by changing the signs of $a_{\lambda n}$ has the circle of convergence as a cut. This theorem was proved in general by Hurwitz and George Pólya.

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The Société Mathématique de France expects to publish Fatou's papers.

II. Secondary Literature. On Fatou or his work, see Jean Chazy, "Pierre Fatou," in *Bulletin astronomique*, **8** , fasc. 7 (1934), 379–384; Griffith Evans, *The Logarithmic Potential* (New York, 1927); P. Fatou, *Notice sur les travaux scientifiques de M. P. Fatou* (Paris, 1929); A. Hurwitz and G. Pólya, "Zwei Beweise eines von Herrn Fatou vermuteten Satzes," in *Acta mathematica*, **40** (1916), 179–183; S. Mandelbrojt, *Modern Researches on the Singularities of Functions Defined by the Taylor Series* (Houston, Tex., 1929), chs. 9, 12; and M. Riesz, "Neuer Beweis des Fatouschen Satzes," in *Göttingensche Nachrichten* (1916); and "Ein Konvergenz satz für Dirichletsche Reihen," in *Acta mathematica*, **40** (1916), 349–361.

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