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(*b*. Munich, Germany, 17 February 1891; *d*. Jerusalem, Israel, 15 October 1965)

mathematics.

Fraenkel studied at the universities of Munich, Marburg, Berlin, and Breslau. From 1916 to 1921 he was a lecturer at the University of Marburg, where he become a professor in 1922. In 1928 he taught at the University of Kiel, and then from 1929 to 1959 he taught at the Herbrew University of Jerusalem. A fervent Zionist with a deep interest in Jewish culture, he engaged in many social activites. His interest in the history of mathematics appears in his papers “Zahlbegriff und Algebra bei Gauss” (1920), Georg Cantor” (1930), and “Jewish Mathematics and Astronomy” (1960). As a mathematician he was intereseted in the axiomatics of Hensel’s *p*-adic numbers and on the theory of rings. He soon turned to the theory of sets, and in 1919 his remarkable *Einletitung in die Mengenlehre* appeared, which was reprinted several times. Engaged in a proof of the independence of the axiom system of Ernst Zermelo (1908), Fraenkel noticed that the system did not suffice for a foundation of set theory and required stronger axioms of infinity. At the same time he found a way to avoid Zermelo’s imprecise notion of definite property.

Briefly stated, Zermelo’s set theory is about a system *B* of objects closed under certain principles of set production (axioms). One of these axioms, the axiom of subsets, states that if a property *E* is definite in a set *M*, then there is a subset consisting precisely of those elements *x* of M for which *E(X)* is true. property *E* is define for *x* if it can be decided systematically whether *E (x)* is true or false. Another one is the famous axiom of choice, stating that the union of a set *T* of nonvoid disjoint sets contains a subset that has precisely one element in common with the sets of *T*

Instead of Zermelo’s notion of definite property Fraenkel used a notion of function, introduced by definition; and he replaced Zermelo’s axiom of subsets by the following; if M is a set and ϕ and ψ are functions, then there are subsets M_E and M_E. consisting of those elements *x* of *M* for which $\phi(x)$ is an elements of $\psi(x)$, and $\phi(x)$ is not an element of $\psi(x)$ respectively. Using this axiom Fraenkel proved the independence of the axiom of choice, having recourse to an infinite set of objects that are not sets themselves. A proof avoiding such an extraneous assumption proved to be far more difficult and was given in 1963 by P. J. Cohen for a slightly revised system, ZFS, named after Zermelo, Fraenkel, and Thoralf Skolem. This system devies from a modification proposed by Skolem in 1922, consisting in the interpretation of definite property as property expressible in first-order logic.

In a series of papers Fraenkel developed ZF set theory to include theories of order and well-order. His encyclopedic knowledge of set theory is preserved in his works *Abstract Set Theory* (1953) and *Founda- tions of Set Theory* (1958). As early as 1923 he emphasized the importance of a through investigation of predicativism, based on ideas of H. Poincare and undertaken much later by G. Kreisel, S. Feferman, and K. Schütte, among others

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