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(*b*. Paris, France, 1 April 1776; *d*. Paris, 27 June 1831)

mathemtics.

Sophie Germain, France's greatest female mathematician prior to the present ear, was the the daugther of Ambroise-François Germain and Marie-Madeleine Gruguelu. Her father was for a time deputy to the State-General (later the Constituent Assembly). In his speeches he referred to himself as a merchant and ardently defended the rights of the Third Estate, which he represented, Somewhat later he became one of the directors of the Bank of France. His extensive library enabled his daughter to educate herself at home. Thus it was that, at age thirteen, Sophie read an account of the death of Archimedes at the hands of a Roman soldier. The great scientist of antiquity became her hero, and she conceived the idea that she too must become a mathematician. After teaching herself Latin and Greek, she read Newton and Euler despite her parent's opposition to a career in mathematics.

The Germain library sufficed until Sophie was eighteen. At that time she was able to obtain the lecture notes of courses at the recently organized École Polytechnique, in particular the *cahiers* of Lagrange's lectures on analysis. Students at the school were expected to prepare end-of-term reports. Pretending to be a student there and using the pseudonym Le Balanc, Sophie Germain wrote a paper on analysis and sent it to Lagrange. He was stounded at its originally, praised it publicly, sought out its author, and thus discovered that M. Le Blanc was Mlle. Germain, From then on, he became her sponsor and mathemtical counselor.

Correspondence with great scholars became the means by which she obtained ther higher education in mathematics, literature, biology, and philosophy, She wrote to Legendre about problems suggested by his 1798 *Théorie des nombres*. The subsequent Legendre-Germain correspondence was so voluminous that it was virtually a collaboration, and Legendre included some of her discoveries in a supplement to the second edition of the *Théorie*. In the interim she had read Gauss's *Disquisitiones arithmeticate* and, under the pseudonym of Le Blanc, engaged in corrrespondent with its author.

That Sophie Germain was no ivory-tower mathematician became evident in 1807, when French troops were occupying Hanover. Recalling Archimedes' fate and fearing for Gausss's safety, she addressed an inquiry to the French commander, General Pernety, who was a friend of the Germain family. AS a result accorded even more praise to her number-theoretic proofs.

One of Sophie Germain's theorems is related to the baffling and still unsolved problem of obtaining a general proof for "Fermat's last theorem," which is the conjecture that $X^n + Y^n = Z^n$ has no positive integral solutions if n is an integer greater than 2. To prove the theorem, one need only establish its truth for $n = 4$ (accomplished by Fermat himself) and for all values of n that are odd primes. Euler proved it for $n = 3$ and Legendre for $n= 5$. Sophie Germain's contribution was to show the impossibility of postive integral solutions if x, y, z are prime to one another and to n, where n is any prime less than generalized her theorem to all primes less than 1,700, and more recectly Barkley Rosser extended the upper limit to 41,000,000. In his history of the theory of numbers, Dickson describes her other discoveries in the higher arithmetic.

Parallel with and subsquent to her pure mathematical research, she also made contributions to the applied mathematics of acoustics and elasticity. This came about in the follwing manner. In 1808 the German physicist E. F. F. Chladniu visited Paris, where he conducted experiments on vibrating plates. He exhibited the so-called Chladniu figures, which can be produced when a metal or glass plate of any regular shape, the most or glass plate of any of the circle, is placed in a horizontal position and fastened at its center to a supporting stand. Sand is scattered lightly over the plate, which is then set in vibration by drawing a violin bow rapidly up and down along the edge of the plate. The sand is thrown from the moving points to those which remain at rest (the nodes), forming the nodal lines or curves constituting the Chladnui figures.

Chladni's results were picturesque, but their chief effect on French mathematicians was to emphasize that there was no pure mathematical model for such phenomena. Hence, in 1811 the Académie des Sciences offered a prize for the best answere to the following challenge: Formulate a mathematical theory of elastic surfaces and indicated just how it agrees with empirical evidence.

Most mathematicians did not attempt to solve the problem because Lagrange assured them that the mathematical methods available were inadequate for the task. Neverthless, Sophie Germain submitted an anonymous memoir. No prize was awarded to any one; but Lagrange, using her fundamental hypotheses, was able to deduce the correct partial differential equation for the vibrations of elastic plates. In 1813 the Academy reopened the contest, and Sophie Germain offered a revised paper which included the question of experimental verification. That memoir received an honorable mention. When, in 1816, the third and final contest was held, a paper bearing her own name and treating vibrations of general curved as well as plane elastic surfaces was awarded the grand prize—the high point in her scientific career.

After further enlargement and improvement of the prize memoir, it was published in 1821 under the title *Remarques sul la nature, les bornes et l'étendue de la question des surfaces élastiques et éequation générale de ces surfaces*. In that work Sophie Germain stated that the law for the general vibrating elastic surface is given by the fourth-order partial differential equation.

Here *N* is a physical constant if the "surface" is an elastic membrane of uniform thickness, The generality us achieved because *S*, the radius of mean curvature, varies from point to point of a general curved surface. The very concept of mean curvature (*l/S*) was created by Sophie Germain.

The notion of the curvature of a surface generalizes the corresponding concept for a plane curve by considering the curvatures of all plane sections of surface through the normal at a given point of the surface and then using only the largest and smallest of those curvatures. The extremes, called the principal curvatures, are multiplied to give the Gaussian total curvature. Sophie Germain, however, defined the mean curvature as half the sum, that is, the arithmetic mean, of the principal curvature. Her definition seems more in accordance with the term "mean," Moreover, she indicated that her measure is a representative one, an average in the statistical sense, by demonstrating that if one passes such that through the normal at a pint of surface such that the angel between successive planes in $2\pi/n$ where *n* very large (thus yielding sample sections in many different directions), the arithmetic mean of the curvatures of all the sections is the same as the mean of the two principal curvatures, a fact that remains true in the limits *n* best larger and larger. Also, while the Gaussian curvature completely characterizes the local metric geometry of a surface, the mean cruvature is more suitabe for applications in elasticity theory. A plane has zero mean curvature at all points. Hence $4/S^2 = 0$ in Germain's differential equation, and it reduces to the equation which she and Lagrange had derived for the vibration of flat plates. The same simplification holds for all surfaces of zero mean curvature, the so-called minimal surfaces (such as those formed by a soap film stretched from wire contours).

In later papers Sophie Germain enlarged on the physics of vibrating curved elastic surfacves and considered the effect of variable, thickness (which emphasizes that one is, in fact, dealing with elastic solids).

She also wrote two philosophic works entitled *Pensées diverses and Consideé'rations générales sur l'état des sciencs et des lettres*, which were published post humously in the *Owuvres philosophiques*. The first of these, probably written in her youth, contains, capsule summaries of scientific subjects, brief comments on physicsts throughout the ages, and personal opinions. The *État des sciences et des lettres*, which was praised by Auguste Comte, is an extremely shcolarly development of the theme of the unity of thought, that is, the idea that there always has been and always will be not basic difference between the sciences and the humanities with respect to their motivation, their methodology, and their cultural importance.

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