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(*b*. Brünn, Moravia lsqbnow Brno, Czechoslovakiarsqb, 28 April 1906; d, Princeton, New Jersey, 14 January 1978)

mathe matical logic, set theory, general relativity.

The character and achievement of Gödel, the most important logician since Aristotle, bear comparison with those of the most eminent mathematicians. His aversion to controversy is reminiscent of Newton's, while his relatively small number of publications — each quite precise and almost all making a major contribution—echo Gauss's motto of "few but ripe." Like Newton, he revolutionized a branch of math ematics—in this case, mathematical logic—giving it a structure and a Kuhnian paradigm for research. His notebooks, like those of Gauss, show him to be well in advance of his contemporaries. While Newton made substantial efforts in theology as well as in mathematical physics, Gödel contributed to philosophy as well as to mathematical logic. But during the formative period of his career, unlike either Gauss or Newton, he was torn between two cultures, the Austrian and the American, and he spent most of his life on foreign soil.

Life. Gödel's family belonged to the German speaking minority in Brünn, a textile-producing city in the Austro-Hungarian province of Moravia, where they had lived for several generations. His grand father, Josef Gödel, after marrying a Viennese wife, Aloisia Keimel, moved from Brünn to Vienna, where he worked in the leather industry. He allegedly committed suicide in the 1890's. Subsequently his wife left their son, Rudolf, with a sister-i–law, "Aunt Anna," who raised him in Bruumlnn and later lived with Kurt's family until her death. Rudolf was first sent to a gymnasium, where the classical education bored him, and then to a weavers'school, from which he graduated with honors. An energetic and practical man intrigued by weaving machinery, he worked until his death for the Friedrich Redlich textile factory in Br&ünn, eventually becoming man ager and part owner of it—a man of property who belonged to the upper middle class. He married Marianne Handschuh, the daughter of a weaver originally from the Rhineland. She had been educated at a French institute in Br&ünn and had broad cultural interests. They had two children: the elder, Rudolf, became a radiologist in Vienna; the younger was Kurt Göel.

According to his brother, Kurt had a happy child hood, though he was shy and easily upset. His family dubbed him Herr Warum (Mr. Why) because of his continual questions. When two weeks old, he was baptized a Lutheran, his mother's religion, with Friedrich Redlich as his godfather and as the source of his middle name, Friedrich; his father remained nominally Old Catholic. At the age of five, according to his brother, he experienced a mild anxiety neurosis, from which he recovered com pletely. Three or four years later he contracted rheumatic fever, which, despite his recovery, left him convinced that his heart had been permanently damaged. World War I did not directly affect him, since Brünn was far from the fighting, but the es tablishment of Czechoslovakia as a nation in 1918 tended to isolate the Germa–speaking minority there. In 1929 he was to renounce his Czechoslovakian citizenship and officially become an Austrian.

Göumldel's education began in september 1912, when he enrolled in the Evangelische Privat-Volks-und Bürgerschule, a Lutheran school in Brünn, from which he graduated in 1916. That fall he entered the Staats-Realgymnasium, a Germa–language high school in Bruumlnn, where he excelled in mathematics, languages, and religion. (He took religious questions more seriously than the rest of his family, later describing his belief as "theistic,") As an adolescent he became interested first in foreign languages, then in history, and finally, around 1920, in mathematics; about a year later he was attracted to philosophy as well, and in 1922 he read Kant. According to his brother, he had mastered a good deal of university mathematics before graduating from the gymnasium in 1924.

That year G&ödel matriculated at the University of Vienna, intending to study mathematics, physics, and philosophy, and to take a degree in physics. He had become interested in science by reading Goethe's theory of colors. At the university he at tended lectures on theoretical physics by Hans Thirring, who had just published a book on the theory of relativity, and his interests focused on the foundations of physics. But about 1926, influ enced by the number theorist Philipp Furtwängler, he changed to mathematics. During 1927 he attended Karl Menger's course in dimension theory. About this time his philosophical interests were greatly stimulated through a course in the history of phi losophy given by Heinrich Gomperz.

Soon Hans Hahn, an analyst intrigued by general topology and logic, became his principal teacher and introduced him to the group of philosophers around Moritz Schlick, the logical positivists later known abroad as the Vienna Circle. He wrote in the curriculum vitae submitted with his *Habilita tionsschrift:* "At that time (1926–1928), stimulated by Prof. Schlick, whose philosophical circle I fre quently attended, I was also occupied with modern works in epistemology." In 1928 he grew less in volved with the Vienna Circle and attended its meetings sporadically until 1933, Göumldel's Platonism conflicted with their positivism. "I only agreed with some of their tenets," he later noted; "I never believed that mathematics is syntax of language."

All the same, he remained in contact with one mem ber of the Vienna Circle, Rudolf Carnap, whose 1928 lectures on mathematical logic and the philo sophical foundations of arithmetic strongly influenced the future direction of his research.

In February 1929, Göumldel's father died from an abscess of the prostate, a medical condition that Kurt later experienced. Soon his mother moved with her two sons to Vienna. The three of them often went to the theater together, since Kurt was greatly interested in it at that time; his taste in music ran to light opera and Viennese operettas. In 1937 his mother returned to the family villa in Brünn: she died three decades later, in Vienna.

After Gödel completed his dissertation in the summer of 1929, it was approved by his supervisor Hahn and by Furtwängler. He received a doctorate in mathematics from the University of Vienna in Feb ruary 1930. During the academic year 1931ndsh1932 he was assistant in Hahn's seminar in mathemati cal logic, selecting much of the material discussed.

By invitation, in October 1929 Gödel began at tending Menger's mathematics colloquium, which was modeled on the Vienna Circle. There in May 1930 he presented his dissertation results, which he had discussed with Alfred Tarski three months ear lier, during the latter's visit to Vienna. From 1932 to 1936 he published numerous short articles in the proceedings of that colloquium (including his only collaborative work) and was coeditor of seven of its volumes. Gödel attended the colloquium quite regularly and participated actively in many discus sions, confining his comments to brief remarks that were always stated with the greatest precision.

At a conference in September 1930. Gödel an nounced his startling first incompleteness theorem: there are formally undecidable propositions in num ber theory. He sent a paper on his incompleteness results to *Monatshefte für mathematik und Physik* in November 1930; it was published two months later. The theorem constituted his *Habilitations schrift*, which he submitted to the University of Vienna in June 1932 and for which Hahn served as referee. In December 1932 Hahn wrote an evaluation of it for the university, praising G&ödel's submission as

a scientific achievement of the first rank which … will find its place in the history of mathe matics.… The work submitted by Dr. Gödel surpasses by far the standard usually required for *Habilitation*. Today Dr. G&ödel is already the principal authority in the field of symbolic logic and research on the foun dations of mathematics.

The following March, G&ödel was made *Privatdozent*, and during the summer he gave his first course, on the foundations of arithmetic. Extremely shy as a teacher, he always lectured to the blackboard; he had few students. During the next seven years he gave only two more courses at the University of Vienna.

The most obvious reason for this situation was Gödel's increasing association with the Institute for Advanced Study at Princeton, of which he was a member for three years during the 1930's; a less obvious reason was the state of his mental health. When the institute first began operation in the fall of 1933, he was a visiting member for the academic year, thanks to the efforts of Oswald Veblen, and from February to May 1934, he lectured on the incompleteness results. There he made Einstein's acquaintance, but came to know him well only a decade later. Lonely and depressed while at Princeton, he had a nervous breakdown after re turning to Europe in June 1934, and was treated for this condition by the eminent psychiatrist Julius von Wagner-jauregg. that fall he again stayed briefly in a sanatorium (the first stay had been in 1931, for suicidal depression), postponing an invitation to re turn to the Institute for Advanced Study for the spring term of 1935 and informing Veblen that the delay was due to an inflammation of the jawbone. At Vienna, during the summer semester of 1935, he gave a course on topics in mathematical logic, then traveled to Princeton in September, suffereing from depression and overwork, he resigned suddenly from the institute in mid-November, returned to Europe in early December, and spent the winter and spring of 1936 in a sanatorium. Veblen who had seen him to the boat, wrote to Paul Heegaard (who was on the organizing committee for the 1936 International congress of Mathematicians), urging that Gödel "be invited to give one of the principal addresses. There is no doubt that his work on the foundations of mathematics is the most important which has been done in this field in our time."

In 1935 Goumldel had made the first breakthrough in his new area of research: set theory. During May and June 1937 he lectured at Vienna on his striking result that the axiom of choice is relatively consistent. That summer he obtained the much stronger result that the generalized continuum hypothesis is rela tively consistent; and in September 1937 John von Neumann, an editor of the Princeton journal *Annals of Mathematics*, urged him to publish his new dis coveries there. Yet G&ödel did not announce them until November 1938, and then not in the *Annals* but in a brief summary communicated to the *Pro ceedings of the National Academy of Sciences*.

Two weeks after marrying Adele Porkert Nim bursky, a nightclub dancer, on 20 September 1938, G&ödel left Austria to work for a term at the Institute for Advanced Study. At Menger's invitation he spent the first half of 1939 as visiting professor at Notre Dame. Both at the institute and at Notre Dame, he gave a lecture course on his relative consistency results in set theory; he also presented them in December 1938 at the annual meeting of the Amer ican Mathematical Society. In February 1939, Gödel wrote to Veblen, asking him to submit the manuscript containing the proof to the *Proceedings of the National Academy of Sciences*. and promising a lengthy article with a detailed proof for the *Annals of Mathematics*. No such article ever appeared in the *Annals* and the standard source for his proof remained notes of his lectures taken by George W. Brown in the fall/winter of 1938–1939 and published in 1940.

When he returned to Vienna in June 1939, both Gödel's personal situation and the global one were darkening. After the *Anschluss* of March 1938. When Nazi Germany forcibly annexed Austria, the position of *Privatdozent* had been abolished and replaced by that of *Dozent neuer Ordnung*. Most lecturers at the University of Vienna were quickly transferred to the new position, but Gödel was not. He formally applied for it in September 1939. Within a week he received a letter responding to his application and nothing that he moved in Jewish-liberal circles, although he was not known to have spoken for or against the Nazis. Finally in June 1940, he was granted the position of *Dozent neuer Ordnung*, when it was no longer of any use to him. Ironically, from 1941 to 1945 he was listed at the University of Vienna as *Dozent Für Grundlagen der Mathematik und Logik*.

During the summer of 1939 Austria, as a part of Germany, was preparing for war. Gödel, ordered by the military authorities to report for a physical examination, was declared "fit for garrison duty" in September and feared that he would soon be called into service. In late November he wrote to Veblen, urgently seeking an American nonquota immigrant visa. During the summer, in Vienna, ultrarightist students had physically assaulted him. While attempting to obtain German exit permits for himself and his wife, he lectured at Göttingen in December on the continuum problem. By early January 1940, thanks to the vigorous efforts of Veblen and of Abraham Flexner, those permits had arrived, as had the U.S. visas and the Soviet transit visas. Since the war made it dangerous to cross the Atlantic, he took the Trans-Siberian Railway, then a ship from Yokohama to San Francisco, finally reaching Princeton, where he was appointed an ordinary member of the Institute. He never returned to Europe.

In Princeton, Gödel had a quiet social life, his closest friends being Albert Einstein and, later, Oskar Morgenstern, who was also from Vienna. Both of them served as witnesses when he became an American citizen in 1948. The gregarious Einstein and the reclusive Gödel often walked home together from the institute. During 1942 Gödel had begun to be close to Einstein, and their conversations generally concerned philosophy, physics, or politics. Einstein regularly informed him of advances in unified field theory, but Gödel did not collaborate with Einstein because he remained skeptical of this theory.

Gödel had no formal duties at the Institute for Advanced Studies, and thus was free to pursue his research. Several of his working notebooks from that time (in the now archaic Gabelsberger shorthand) record the development of his ideas. In particular, they chronicle his attempts around 1942 to prove the independence of the continuum hypothesis and of the axiom of choice. Slightly earlier, he found some essential errors in Jacques Herbrand's proof that there is a quantifier-free interpretation of firstorder logic; these errors were independently rediscovered two decades later by Peter Andrews and others.

In 1943, however, Gödel began to turn his research from mathematics to philosophy—at first, philosophy of mathematics, and later, philosophy in general. He expressed his Platonist views publicly in two well-known articles, one on Russell's mathematical logic in 1944 and the other on Cantor's continuum problem in 1947. Later he wrote that "the greatest philosophical influence on me came from Leibniz, whom I studied about 1943–1946." adding that Kant influenced him to some degree. According to Gödel, his Platonism had earlier led him to give a philosophical analysis that culminated in the discovery of both his completeness theorem and his incompleteness results.

Moreover, it was Kant who stimulated Gödel during the period 1947–1951, to consider new cosmological models, in some of which "time travel" into the past was theoretically possible. These models, so far removed from mathematical logic, echoed his work on differential geometry in Menger's colloquium in the early 1930's.

Late in his career, honors were showered upon Gödel in abundance. In 1946 he was finally made a permanent member of the Institute for Advanced Study. In 1950 he gave an invited address on relativity theory to the International Congress of Mathematicians, and the following year he was one of two recipients of the first Einstein Award. He was also asked to deliver the annual Gibbs Lecture to the American Mathematical Society, and did so in December 1951, arguing against mechanism in the philosophy of mind. That same year Yale University granted him an honorary D. Litt., and Harvard followed with an honorary Sc.D. in 1952 (Later there were two more honorary doctorates, one from Amherst College in 1967 and the other from Rockefeller University in 1972.) In 1955 Gödel was elected a member of the National Academy of Sciences. In 1957 he was elected a fellow of the American Academy of Arts and Sciences and, in 1967, an honorary member of the London Mathematical Society. He was made a foreign member of the Royal Society a year later, and a corresponding member of the Institut de France in 1972. The United States honored him in 1975 with a National Medal of Science.

Gödel repeatedly refused honors from Austrian academic institutions (for instance, in 1966, honorary membership in the Vienna Academy of Sciences), apparently because he was still perturbed by his treatment after the Anschluss. He could not, however, refuse the honorary doctorate that the University of Vienna awarded him posthumously.

After his promotion to professor of mathematics in 1953, Gödel spent a great deal of his time on institute business. particularly to ensure that aspiring young logicians were made visiting members. His promotion had been considerably delayed for fear of burdening him with administrative duties (and for fear that he would be overzealous in carrying them out).

Gödel's personality was idiosyncratic. Shy and solemn, short and slight of build, he was very courteous but lacked warmth and sensitivity. Since he was very much an introvert, he and his wife had guests infrequently; he found the excitement tiring, A hypochondriac whose health actually was relatively poor, he often complained of stomach trouble but remained distrustful of doctors, believing throughout his life that his medical judgment was better than theirs. (In the 1940's he delayed treatment of a bleeding duodenal ulcer for so long that blood transfusions were required to save his life.) Keenly sensitive to the cold, he was often seen in Princeton wearing a large overcoat, even on warm summer days.

In 1976 Gödel retired from the Institute for Advanced Study as professor emeritus. Soon illness and death visited those near to him. In 1977 his wife underwent major surgery, and Oskar Morgenstern, to whom he had become especially close since Einstein's death, died. His own health, uncertain throughout his life, had turned worse near the end of the 1960's. At that time he experienced a severe prostate condition, but he refused to have surgery performed, despite the urgings of his doctor and friends. During the last year of his life, he suffered from depression and paranoia. Late in December 1977, at his wife's insistence, he was hospitalized; two weeks later he died. According to the death certificate, this was due to "malnutrition and inanition" caused by a "personality disturbance," Fearing that his food would be poisoned, he refused to eat, and thus starved himself to death. His wife died three years later. They had no children.

Work. When, in 1928, Gödel began to do research in mathematical logic, it was not a well-defined field. It was still a part of the "foundations of mathematics"—a subject that belonged more to philosophy than to mathematics. As Ernst Zermelo described the situation in logic about that time, apropos of his axiomatization of set theory two decades earlier, "A generally accepted mathematical logic" …did not exist then, any more than it does today, when every foundational researcher has his own logistical system. "The state of foundations was epitomized by the 1930 conference at Königsberg at which Gödel announced his incompleteness result. The conference was primarily devoted to a discussion of the three competing foundational schools—formalism, intuitionism, and logicism—that had dominated the subject for more than a decade.

Formalism, developed by David Hilbert, treated mathematics as a purely formal and syntactical subject in which meaning was introduced only at the metamathematical level. "Hilbert's program," as it was called, was concerned primarily with proving the consistency of classical mathematics and the existence of a decision procedure for it. Intuitionism, developed by L.E.J. Brouwer, stressed the role of intuition as opposed to the formal aspect, and rejected much of both classical logic and classical mathematics. Logicism, developed by Bertrand Russell and embodied in his *Principia Mathematica*, asserted that mathematics was a part of logic and could be developed as a logical system without any further recourse to intuition. All three schools were to influence Gödel, who followed none of them but established a new paradigm: mathematical logic as a part of classical mathematics, answering questions of genuine mathematical interest and only indirectly of philosophical import.

Gödel's dissertation, completed in 1929 and published the following year, grew out of a problem that Hilbert and Wilhelm Ackermann had posed in their book *Grundzüge der theoretische Logik*. (He became interested in this problem in 1928, a year before he first read *Principia Mathematica*.) Hilbert and Ackermann remarked that it was not known whether every valid formula of first-order logic is provable and, moreover, whether each axiom of first-order logic is independent. In his dissertation Gödel solved both problems. considering only a countable set of symbols, since uncountable languages had not yet been introduced. His solution to the first problem is now known as the completeness theorem for first-order logic. In addition to showing that every valid first-order formula is provable, and then extending this result to countably infinite sets of first-order formulas, he gave a different form of the theorem: A first-order formula (or a countable set of such formulas) is consistent if and only if it has a model. In this form the theorem made rigorous along-standing fundamental belief of Hilbert's.

In 1930 a revised version of Gödel's dissertation was published, now supplemented by a theorem that became very important two decades later, the compactness theorem for first-order logic: A countably infinite set A of first-order formulas has a model if and only every finite subset of A has a model. The published version omitted the philosophical remarks found at the beginning of his dissertation, where he cast doubt on Hilbert's program (even hinting at possible incompleteness) at the same time that he furthered the program by his proof of the completeness theorem. Apparently his doubts about Hilbert's program had been nurtured by Brouwer's lectures, given at Vienna in March 1928, on the inadequacy of consistency as a criterion for mathematical existence.

Hilbert and Ackermann had made the theory of types their framework for logic, considering firstorder logic and second-order logic as subsystems. Hence it is not surprising that Gödel, after publishing his completeness results, soon turned to considering higher-order logic. He began by an attack on Hilbert's problem of finding a finitist consistency proof for analysis (that is, second-order number theory). His attack first divided the problem into two parts: (1) to establish the consistency of number theory by means of finitist number theory, and (2) to show the consistency of analysis by means of (the truth of) number theory. But he found that truth in number theory cannot be defined within number theory itself, and so his plan of attack failed. Thereby he was led to his first incompleteness theorem, which he announced privately to Carnap on 26 August 1930.

On 7 September, Gödel made his first public announcement at the königsberg conference. By 23 October. When he submitted an abstract of the result to the Vienna Academy of Sciences, he had obtained the second incompleteness theorem: The consistency of a formal system S cannot be proved in S if S contains elementary number theory, unless S is inconsistent. (Von Neumann, who had heard him discuss the first incompleteness theorem at Königsberg and was keenly interested in it, wrote to him on 20 November with an independent discovery of the second theorem.) Although Gödel proved his incompleteness results for higher-order logic— in particular, for the theory of types— he pointed out that they applied equally to axiomatic set theory. Adding finitely many new axioms would not change the situation, nor would adding infinitely many new axioms, as long as the resulting system was omegaconsistent.

Although Gödel's incompleteness theorems were eventually recognized as the most important theorems of mathematical logic, at first they had a mixed reception. At Princeton. Where von Neumann soon lectured on them, Stephen Kleene was enthusiastic about them but Alonzo Church believed, mistakenly, that his new formal system (which included the lambda calculus) could escape incompleteness. In Europe, Paul Bernays corresponded with Gödel about the results and, after some initial reservations, came to accept them. By contrast, Zermelo was skeptical both in correspondence and in person. As late as 1934, Hilbert denied in print that the incompleteness results refuted his program. During the correspondence with Bernays and Zermelo, Gödel showed that for a language truth cannot be defined in the same language, a theorem that was found independently by Tarski in 1933.

In 1932 Gödel published his formulation of the incompleteness results from the standpoint of firstorder logic. If number theory is regarded as a formal system in first-order logic, then the above results about incompleteness and unprovability of consistency apply to *S*. If, however, *S* is extended by variables for sets of numbers, for sets of sets of numbers, and so on (together with the corresponding comprehension axioms). then we obtain a sequence of systems S_n the consistency of each system is provable in all subsequent systems. But in each subsequent system there are undecidable propositions. Going up in type in this way, he noted, corresponds in a type-free system of set theory to adding axioms that postulate the existence of larger and larger infinite cardinalities. This was the beginning of Gödel's interest in large cardinal axioms, an interest that he elaborated in 1947 in regard to the continuum problem.

One consequence of Gödel's research was that in 1931 the editors of *Zentralblatt für Mathematik* invited him and Arend Heyting to prepare a joint report on the foundations of mathematics. Although Gödel labored for some time at this report, he eventually withdrew from the venture, and Heyting published his version alone in 1934. Gödel's fragmentary draft survives in his *Nachlass*.

From 1932 to 1936 Gödel published a variety of brief but substantial papers on logic as well as on differential and projective geometry, primarily for Menger's colloquium. The articles on geometry, all published in 1933, dealt with curvature in convex metric spaces, projective mappings, and coordinatefree differential geometry.

Articles on logic were more numerous and diverse, treating certain cases of Hilbert's decision problem (*Entscheidungsproblem*), the propositional calculus, intuitionistic logic and arithmetic, and speed–up theorems. In 1933 Gödel's results on the decision problem extended work by Ackermann and by Thoralf Skolem to obtain a sharp boundary between decidable and undecidable first–order formulas. In the propositional calculus he solved a problem of Hahn's by showing that some independence results in this calculus require infinite truth tables, and he answered a question of Menger's by formulating the propositional calculus so as to have uncountably many symbols. He did not, however, introduce a first–order language with uncountably many symbols, as Anatolii Ivanovich Maltsev was to do in 1936.

As for intuitionism, Gödel established that if only finitely many truth–values are permitted, then there is no completeness theorem for the intuitionistic propositional calculus; moreover, there are infinitely many systems of logic between the intuitionistic and the classical propositional calculi, each stronger than the previous system. In another paper he proved the philosophically important result that if intuitionistic number theory is consistent, then so is classical number theory; this was done by interpreting the latter theory within the former. Then Gödel reversed direction, showing that the intuitionistic propositional calculus can be given a "provability" interpretation within the classical propositional calculus— an interpretation that, by his second incompleteness theorem, cannot represent provability in a formal system.

What turned out to be an important contribution occurred in Gödel's 1934 lectures on incompleteness. There, refining a suggestion made by Herbrand three years earlier, he introduced the notion of a general time that this notion captured the general informal concept of computability, and was convinced that it did so ("Church's thesis') only by Alan Turing's work in 1936.

In 1936 Gödel published the first example of a speed–up theorem— pointing out that if one goes from a logic $S_n$of order *n* to a logic S_{n+1} of next higher order, there are infinitely many theorems of S_n, each of whose shortest proof in S_{n+1}. Such speedup theorems later were studied extensively in computer science.

Gödel's next major accomplishment occurred in set theory. Around 1930 he began to think about the continuum hypothesis and learned of Hilbert's attempt, during the period 1925— 1928, to prove it. In contrast with Hilbert, he felt that one should not build up the sets involved in a strictly constructive way. Then he reconsidered the question from the standpoint of relative consistency and of models of set theory, in which his discoveries in time became just as famous as his incompleteness theorems. The first breakthrough came in 1935. In October, while at the Institute for Advanced Study, he informed von Neumann of his new result, obtained by means of his "constructible" sets, that the axiom of choice is consistent relative to the other axioms of set theory. When he gave a course on axiomatic of set theory at Vienna in 1937, he used what was later called Bernays–Gödel set theory;this is a slight variant of Bernays'system (about which Bernays had informed him in a 1931 letter) used to identify a set with the corresponding class. One of those attending the course was Andrzej Mostowski, who later recalled that Gödel "constructed a model in which the axiom of choice was valid; at that time, I am sure that he did not have the consistency proof for the continuum hypothesis."

During this period Gödel was trying to prove that the generalized continuum hypothesis is true in the model of constructible sets. On 14 June 1937 he found the crucial step in establishing this fact. By September he had communicated his new result to von Neumann, but refrained from publishing it for over a year. When it appeared, late in 1938, it was clear that he had taken Zermelo's cumulative type hierarchy in set theory and had treated it from the standpoint of Russell's ramified theory of types, particular, Gödel stated that the generalized continuum hypothesis can be proved consistent relative to von Neumann's system of set theory, ZermeloFraenkel set theory, or the system of *Principia Mathematica*. At that time he did not mention firstorder logic as the basis of his notion of "constructible set" but merely regarded it as excluding impredicative definitions.

When he communicated his second brief paper on the subject, in February 1939, Gödel spelled out in detail how to prove that the generalized continuum hypothesis holds in the model. The hierarchy of constructible sets was introduced by transfinite recursion in such a way that its definition differed from Zermelo's cumulative hierarchy only at successor ordinals, where $M_{\alpha+1}$, the next level after M_α, was defined as the set of all subsets of $M_\alpha$that are first-order definable from parameters in M_α. The critical step, later called the condensation lemma, used a form of the Löwenheim-Skolem theorem to show that each constructible subset of $M\ \omega_\alpha$is $\aleph_\alpha$ Then the generalized continuum hypothesis holds in the model, since the cardinality of $M\ \omega_\alpha$ is $\aleph_\alpha$ Finally. Goöel gave two set models, $M\ \omega_\omega$ and M_Ω where Ω was the first inaccessible cardinal. The first of these was a model of Zermelo set theory. while the second was a model of Zermelo-Fraenkel set theory.

In the 1938 paper $Gö_a$ introduced the axiom of constructibility, which stated that every set is a constructible set. He asserted that this proposition. "added as a new axiom. Seems to give a natural completion of the axioms of set theory, in so far as it determines the vague notion of an arbitrary infinite set in a definite way." The axiom of constructibility, as he showed in the 1939 paper and in the 1940 monograph, implies both the axiom of constructibility holds in the model. To do so, he introduced and developed the critical notion of being "absolute." That is, a formula is absolute for a model if it holds in the model precisely when it is true.

Gödel's 1938 paper also asserted that the axiom of constructibility implies that there is a nonmeasurable set of real numbers (as well as an uncountable set of real numbers having no perfect subset) that occurs low in the projective hierarchy. These results followed from an observation of Stanislaw Ulam, who noticed that in the model there is a projective well-ordering of the real numbers.

Gödel's results on constructible sets became known largely through his 1940 monograph, based on lectures delivered at the Institute for Advanced Study during October-December 1938. There he did not use the hierarchy M_α but presented the constructible sets as built up presented the from eight fundamental operation on sets, formulated within Bernays-Gödel set theory. Later set theorists tended to find this second approach less intuitive than the first.

It is uncertain why Gödel refrained from submitting for publication his results on the relative consistency of the axiom of choice and the generalized continuum hypothesis, known by the summer of 1937, until November 1938. But a clue can be found in a letter he wrote to menger in December 1937:

I have continued my work on the continuum problem last summer and I finally succeeded in proving the consistency of the continuum hypothesis (even the generalized form) with respect to general set theory. But for the time being please do not tell anyone of this. So far I have communication this. beside to yourself, only to von Neumann.... Right now I am trying to prove also the independence of the continuum hypothesis, but do not yet know whether I will succeed with it.(Wang, *Reflections*.p.99)

Gödel persisted in his attempts to prove that the continuum hypothesis is independent. During the summerof 1942, while on vacation in Maine, he obtained a proof of a relative to the independence of the axiom of choice relative to the theory of types as well as the independence of the axiom of constructibility. But he did not succeed in showing the independence of the continuum hypothesis.

Gödel never published his result on the independence of the axiom of choice. According to comments he made later to John Addison and others he feared that such independence results would lead research in set theory "in the wrong direction." what he considered to be the right direction became apparent in his expository paper of 1947 on the continuum problem.

It was very likely, he observed in that paper, that the continuum hypothesis would eventually be proved independent of the axioms of set theory: to seek such a proof was the best way to attack the continuum problem. But.he insisted. even such a proof of independence would not solve the continuum problem because, he argued platonistically, the cumulative hierarchy of sets forms a well-determined reality, and thus it should be possible to establish the truth or falsity of the continuum hypothesis. The way to proceed was to search for new true axioms (especially large cardinal axioms) that would settle its truth or falsity. It was very likely, he believed that the continuum hypothesis was false.

In 1946 Gödel had delivered a paper on set theory at the princeton Bicentennial conference on Problems of Mathematics. There he introduced the notion of ordinal-definable set (which was related to, but distinct from, his notion of constructible set), and conjectured that the ordinal-definable sets would providea model of set theory in which the axiom of choice held, thereby providing a new proof for its relative consistency.Moreover. Gödel asserted, it would be impossible to prove that the continuum hypothesis help, in that model.Because this paper circulated only in manuscript at the time. the ordinaldefinable sets were rediscovered independently in 1962 by John Myhill and Dana Scott, who used them in the way proposed by Gödel.

During the 1940's Gödel turned increasingly to philosophy. The first fruits were found in his 1994 article on Russell's mathematical logic. Like almost all of his publications from this date on, it was requested by an editor—in this case by Paul Schilpp for the volume *The Philosophy of Bertrand Russell*. The article, solicited in November 1942, was sent to Schilpp six months later. In September 1943 Gödel wrote to Russell, urging him to reply in detail to the criticisms contained in the article. But Russell, who had not worked in logic for three decades, declined to answer the criticisms and merely granted that they had some merit.

This article was Gödel's first public defense of his Platonism, which was unfashionable in philosophy of mathematics at the time. He put forward suggestions for further research in foundations, especially in regard to the theory of types, and considered (though he was somewhat dubious) the possibility of infinitely long logical formulas. Such infinitary logics were fully developed only a decade later by Alfred Tarski and others.

In July 1946, Schilpp asked Gödel to write an article (completed three years later) for the volume *Albert Einstein: Philosopher-scientist*. This request prompted Gödel to return to his early interest in the foundations of physics, and therby led him to publish two technical papers on the general theory of relativity (1949).Highly original and eventually quite influential, these papers gave the first rotating solutions to Einstein's cosmological equations. The original solution put forward by Gödel permitted an observer, in principle, to travel into the past. while his later solutions(for an expanding universe) did not allow this. The article for Schilpp's volume dealt with the relationship between relativity theory and Kantian philosophy. Gödel elaborated on this subject in a still unpublished paper, in which he argued that relativity justified certain aspects of Kant's view of time. "Einstein told me," Oskar Morgenstern wrote in May 1972, "that Gödel's papers were the most important ones on relativity theory since his own [Einstein' s] original paper appeared. On the other hand, other cosmologists such as Robertson did not like Gödel's work at all."

In December 1951, G'del delivered to the American Mathematical Society the Gibbs Lecture, "Some Basic Theorems on the Foundations of Mathematics is and their Implications," which remains unpublished. In it he discussed the implications of his incompleteness theorems for mathematics and philosophy, His chief result was that "either mathematics is incompletable in the sense that its evident axioms can never be comprised in a finite rule, i.e. the human mind (even within the realm of pure mathematics) infinitely surpasses the powers of any finite machine, or else there exist absolutely unsolvable Diophantine problems." From this time on Gödel made it a policy to refuse any invitation to lecture, turning down many important ones.

In May 1953 Schilpp requested a third article from Gödel, this time for the volume *The Philosophy of Rudolf Carnap*. Gödel worked for an extended period on this article, "Is Mathematics Syntax of Language?" which was devoted to showing why the answer was no. He was however, not satisfied with any of his six versions of the article, and it remains unpublished. Indeed, because he published so little after the 1940's. Gödel appeared rather unproductive to many of the mathematicians interested in his work.

In 1958 there appeared the last of Gödel's published papers, solicited two years earlier for a volume honoring Bernays'seventieth birthday. Known as the "Dialectica Interpretation" (after the journal *Dialectica*, in which it was published), the paper supplied a new quantifier-free interpretation for intuitionistic logic by using primitive recursive functionals of any finite type. The paper extended Hilbert's program by not confining the notion of *"finitary,"* as Hilbert had, to concrete objects, but permitting abstract objects as well. This was an instance, however, of a result that was first published decades after Gödel found it. For he discovered it in 1941 and lectured on it that year at both Princeton and Yale. Late in the 1960's he wrote an expanded version of this paper, but never published it—despite Bernays' repeated pleas.

In 1963, when Paul Cohen discovered the method of forcing and used it to prove the independence of the axiom of choice as well as that of the continuum hypothesis, he went to Princeton to seek Gödel's *imprimatur*. At Cohen's request, Gödel submitted Cohen's article containing these results to the *Proceedings of the National Academy of Sciences*, in which Gödel's relative consistency results had appeared a quarter-century earlier. Their correspondence makes it clear that Gödel made many revisions in Cohen's paper.

That correspondence also reveals Gödel's concern with showing the existence of a "scale" of length $\aleph_1$ majorizing the real numbers (treated as the set of all sequences of natural numbers). He regarded this question as, "once the continuum hypothesis is dropped, the key problem concerning the structure of the continuum," In 1970 Gödel drafted a paper, intended for the *Proceedings*, on this problem and gave in it some axioms on scales (the square axioms) that, he claimed, imply that the continuum hypothesis is false and that the power of the real numbers is $\aleph_2$. Gödel submitted this paper to Tarski for his judgment and, when D. A. Martin found an error in it, withdrew it. Thus his final contribution to Cantor's continuum problem ended inconclusively.

During the 1970s, and perhaps earlier, in his philosophical research Gödel pursued the ideal of establishing metaphysics as an exact axiomatic theory, but he never achieved what he regarded as a satisfactory treatment. His work in this area was stimulated by Leibniz's *Monadology*. Leibniz also influenced his version, which began to circulate about 1970, of the ontological argument for the existence of God.

Since much on Gödel's philosophical work remains unpublished, his philosophical influence will likely increase as more of his work becomes available. By contrast, his mathematical work (essentially complete three decades before his death) was, and is, in the words of von Neumann, "a landmark which will remain visible far in space and time."

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publications from 1929 to 1936, was published in 1986; the second volume, containing his publications after 1937, appeared in 1988. A third volume, consisting of correspondence and previously unpublished manuscripts, is in preparation.

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