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(*b*. Königsberg, Prussia [now Kaliningrad, R.S.F.S.R.], 18 March 1690; *d*. Moscow, Russia, 20 November 1764)

*mathematics*.

The son of a minister, Goldbach studied medicine and mathematics at the University of Königsberg before embarking, sometime around 1710, on a series of travels across Europe. Everywhere he went, he formed acquaintances with the leading scientists of his day, laying the basis for his later success as first corresponding secretary of the Imperial Academy of Sciences in St. Petersburg. Among others, he met Leibniz in Leipzig in 1711, Nikolaus I Bernoulli and Abraham de Moivre in London in 1712, and Nikolaus II Bernoulli in Venice in 1721. At Nikolaus II's suggestion, in 1723 Goldbach initiated a correspondence with Daniel Bernoulli which continued until 1730. Back in Königsberg in 1724, Goldbach met Jakob Hermann and Georg Bilfinger on their way to participate in the formation of the Imperial Academy and decided to follow them. Writing from Riga in July 1725, he petitioned the president-designate of the new academy, L. L. Blumentrost, for a post in that body. Among his references he named General James Bruce, commander of the imperial forces, with whom he had exchanged ideas on a problem in ballistics around 1718. Although at first informed that no places were open, Goldbach soon received the position of professor of mathematics and historian of the academy at a yearly salary of 600 rubles. In the latter capacity he acted as recording secretary from the first meeting until January 1728, when he moved to Moscow.

That move resulted from Goldbach's new post as tutor to Tsarevich Peter II and his distant cousin Anna of Courland. Introduced into court circles by Blumentrost as early as 1726, Goldbach was in a position to benefit from the split between Peter II and Prince Menshikov by replacing the tutors appointed by the prince. Peter's sudden death in 1730 ended Goldbach's teaching career but not his connections with the imperial court. He continued to serve Peter's successor Anna and returned to St. Petersburg and the Imperial Academy only when she moved the court there in 1732. While in Moscow in 1729, Goldbach began the exchange of letters with Leonhard Euler that would continue regularly until 1763.

Returning to the Imperial Academy in 1732, Goldbach quickly rose to a commanding position. Under the presidency of Baron Johann-Albrecht Korf, he was first designated corresponding secretary (1732) and later named a *Kollegialrat* and, together with J. D. Schuhmacher, was charged with the administration of the Academy (1737). At the same time he rose steadily in court and government circles. The two roles began to conflict seriously in 1740, when Goldbach requested release from administrative duties at the Academy; and his promotion to *Staatsrat* in the Ministry of Foreign Affairs in 1742 ended his ties to the Imperial Academy. In 1744 his new position was confirmed with a raise in salary and (in 1746) a grant of land; in 1760 he attained the high rank of privy councilor at 3,000 rubles annually. That same year he set down guidelines for the education of the royal children that served as a model during the next century.

Coupled with a vast erudition that equally well addressed mathematics and science or philology and archaeology, and with a superb command of Latin style and equal fluency in German and French, Goldbach's polished manners and cosmopolitan circle of friends and acquaintances assured his success in an elite society struggling to emulate its western neighbors. But this very erudition and political success prevented Goldbach's obvious talent in mathematics from attaining its full promise. Unable or unwilling to concentrate his efforts, he dabbled in mathematics, achieving nothing of lasting value but stimulating others through his flashes of insight.

Goldbach's mathematical education set the pattern for his episodic career. Rather than engaging in systematic reading and study, he apparently learned his mathematics in bits and pieces from the various people he met, with the result that later he frequently repeated results already achieved or was unable to take full advantage of his insights. As he himself related in a letter to Daniel Bernoulli, he first encountered the subject of infinite series while talking to Nikolaus I Bernoulli at Oxford in 1712. Unable to understand a treatise by Jakob I Bernoulli on the subject, loaned to him by Nikolaus, he dropped the matter until 1717, when he read Leibniz's article on the quadrature of the circle in the *Acta eruditorum*. His reawakened interest led to his own article "Specimen methodi ad summas serierum," which appeared in the *Acta* in 1720. Only afterward did Goldbach discover that the substance of his article formed part of Jakob I's *Ars conjectandi*, published in 1713. In his article "De divisione curvarum..." Goldbach frankly admitted that Johann I Bernoulli had already solved the problem in question but that he could not remember the solution and so was deriving it again. Often Goldbach's mathematical knowledge showed surprising bare spots. Impressed by his solution of several cases of the Riccati equation (in "De casibus quibus integrari potest aequatio differentialis..." and "Methodus integrandi aequationis differentialis..." and in correspondence),[1] Daniel Bernoulli encouraged him to extend his results to exponential functions. Goldbach replied that he knew nothing about exponential functions and did not want to give the impression that he did.

Of Goldbach's other published articles, the two on infinite series—"De transformatione serierum" and "De terminis generalibus serierum"—and the one on the theory of equations, "Criteria quaedam aequationum...," show the greatest originality. "De transformatione serierum," read to the Imperial Academy in 1725, contains a technique for transforming one series, $A$, into another series, $B$, having the same sum, through term-by-term addition of $A$ to, or subtraction of $A$ from, a series, $C$, of which the sum is zero. Adjustment of the technique leads to a similar transformation through multiplication of the given series by a series of which the sum is one. In reply to objections that the multiplicative method may involve a divergent series as unit multiplier,[2] Goldbach defended the use of such a series provided that it leads to a convergent result. "De terminis generalibus serierum," read in 1728, continues the work begun in "Specimen methodi ad summas serierum" (1720) by addressing the problem of determining the "general term" of any sequence;[3] that is, it seeks a function (either explicit or finitely recursive) that yields the $n$th term of the sequence for a given $n$. Goldbach shows that the general term can always be expressed as an infinite series and that the problem therefore reduces to one of finding a general formula for the sum of that series. The general term of an infinite sequence proves useful, he argues, both for interpolation of missing terms and for the determination of terms for noninteger indices. Although Goldbach and Daniel Bernoulli corresponded on the specific problem of determining the general term of the sequence {n!}, neither could offer a solution (Euler later provided one).

In the article "Criteria quaedam aequationum quarum nulla radix rationalis est" Goldbach begins from results contained in "Excerpta a litteris C. G. ad * * * Regiomonte datis" and applies further some of the number-theoretical results worked out in correspondence with Euler to obtain a technique for testing quickly whether an algebraic equation has a rational root. For equations of the form $x^n = P(x)$, where P is an algebraic polynomial of degree $n—1$ or less, the technique rests basically on determining all integers $m$ for which P can be expressed in the form $mx\ R\ (x) + r$ and then ascertaining whether $r$ is an $n$th-degree residue *modulo m*. If no such residue exists, the equation in question has no rational root.

If, in the realm of analysis, Goldbach's native talent could not substitute for thorough training in the subject, that talent did come into full play in his correspondence with Euler on number theory, a field then still at a rudimentary stage of development. Here Goldbach could be provocative on a fundamental level, as "Demonstratio theorematis Fermatiani..." and "Criteria quaedam aequationum quarum nulla radix rationalis est" show. Calling attention in his correspondence to Pierre de Fermat's assertion that all numbers of the form $2^{2^n} + 1$ are prime, he stimulated Euler's disproof for the case n = 5 (Euler's memoir in fact immediately follows Goldbach's "Criteria..."). Not all of his suggestions led to such positive results. In 1742 Goldbach conjectured that all even numbers may be expressed as the sum of two primes (taking **I** as a prime where necessary). Euler agreed with the assertion but could offer no proof, nor has any proof of "Goldbach's conjecture" yet been found. Goldbach also stated that every odd number may be expressed as the sum of three primes; in the form given it by Edward Waring (which excludes **I** as a prime) this assertion also remains an unproved conjecture. The above are only the outstanding results of the prolix correspondence with Euler on number theory.[4] That correspondence as a whole marks Goldbach as one of the few men of his day who understood the implications of Fermat's new approach to the subject.

# NOTES

1. The solution includes the full conditions for the integrability of binomial differentials usually credited to Euler. See Youschkevich, *Istoria matematiki v Rossii*, p. 96.

2. I.e., the unit multiplier may not share the same domain of convergence as the resultant series.

3. Goldbach uses the same Latin term, *series*, to denote both series and sequences.

4. For details, consult Leonard E. Dickson, *History of the Theory of Numbers*, 2nd ed. (New York, 1952), I and II, *passim*.

# BIBLIOGRAPHY

I. Original Works. Goldbach's writings include "Temperamentum musicum universale," in *Acta eruditorum*, **36** (1717), 114-115; "Excerpta a litteris C[hristiani]. G[oldbachi]. ad * * * Regiomonte datis," in *Actorum eruditorum supplementum*, **6** (1718), 471-472; "Specimen methodi ad summas serierum," in *Acta eruditorum*, **39** (1720), 27-31; "Demonstratio theorematis Fermatiani, nullum numerum triangularem praeter I esse quadrato-quadratum," in *Actorum eruditorum supplementum*, **8** (1724), 483-484; "De casibus quibus integrari potest aequatio differentialis $ax^m dx + byx^p dx + cy^2 dx = dy$ observationes quaedam," in Commentarii *Academiae scientiarum imperialis Petropolitanae*, **1** (1728), 185-197; "Methodus integrandi aequationis differentialis $aydx + bx^n dx + cx^{n-1} dx + ex^{n-2} dx = dy$ ubi $n$ sit numerus integer positivus," *ibid*., 207-209; "De transformatione serierum," *ibid*., **2** (1729), 30-34; "De divisione curvarum in partes quotcunque quarum subtensae sint in data progressione," *ibid*., 174-179; "De terminis generalibus serierum," *ibid*., **3** (1732), 164-173; and "Criteria quaedam aequationum quarum nulla radix rationalis est," *ibid*., **6** (1738), 98-102.

For Goldbach's correspondence with Nikolaus II and Daniel Bernoulli and Leonhard Euler, see Paul-Henri Fuss, *Correspondance mathématique et physique de quelques célèbres géométres du* XVIIIème siècle, 2 vols. (St. Petersburg, 1843); for a more recent ed. of part of that correspondence, see *Leonhard Euler und Christian Goldbach, Briefwechsel*, 1729-1764, edited with introduction by A. P. Juškevič [Youschkevich] and E. Winter (Berlin, 1965).

II. Secondary Literature. Piotr P. Pekarskii, *Istoria imperatorskoi akademii nauk v Peterburge*, I (St. Petersburg, 1870), 155-172, contains the most complete biography of Goldbach and quotes heavily from his nonmathematical correspondence and papers now in the State Archives, Moscow. A. P. Youschkevich includes a fairly complete account of Goldbach's mathematical work in his *Istoria matematiki v Rossii* (Moscow, 1968), pp. 92-97. See also the eds. cited above and the works cited in the notes.

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