Haar, Alfréd | Encyclopedia.com

Complete Dictionary of Scientific Biography COPYRIGHT 2008 Charles Scribner's Sons
4-5 minutes

(*b*. Budapest, Hungary, 11 October 1885; *d*. Szeged, Hungary, 16 March 1933)

mathematics.

Alfréd was the son of Ignatz Haar and Emma Fuchs. While a student at the Gymnasium in Budapest, he was a collaborator on a mathematical journal for high schools and in 1903, his last year at the Gymnasium, he won first prize in the Eötvös contest in mathematics. He had started studying chemistry, but his success in the contest induced him to switch to mathematics. From 1904 he studied in Göttingen; in 1909 he took his Ph. D. degree as a student of D. Hilbert and that year became a *Privatdozent* at the University of Göttingen.

In 1912, after a short time at the Technical University of Zurich, he returned to Hungary and succeeded L. Fejér at Klausenburg University, first as extraordinary professor and then, from 1917, as ordinary professor. When Klausenburg became Rumanian he went to Budapest with his colleague F. Riesz. Together they continued their activity at Szeged University, where in 1920 they founded *Acta scientiarum mathematicarum*, a journal of great reputation. In 1931 Haar became a corresponding member of the Hungarian Academy of Sciences.

Haar did work in analysis. Although not formally abstract, it is so close to the abstract method that it still looks modern. His doctoral thesis had dealt with orthogonal systems of functions. Twenty years later he returned to the same subject. Haar first extended what was known on divergence, summation, and oscillation for the Fourier system to other orthogonal systems, in particular to solutions of Sturm-Liouville problems. He discovered a curious orthogonal system according to which every continuous function can be developed into an everywhere converging series; its elements are discontinuous functions admitting, at most, three values. Later he became interested in multiplicative relations of orthogonal systems and characterized their multiplication tables. This research led him to the character theory of commutative groups as a precursor of Pontryagin on duality.

In complex functions Haar did work on splitting lines of singularities and on asymptotics. As one of the first applications of Hilbert's integral methods in equations and of Dirichlet's principle, Haar in 1907 studied the partial differential equation $\Delta \Delta u = 0$; with T. von Kármán he put this method to use in elasticity theory. Haar also wrote a number of papers on Chebyshev approximations and linear inequalities.

Two of Haar's shorter papers (1927–1928) that greatly influenced problems and methods in partial differential equations in the 1930's concern the equation.

$F(x,y,z,p,q) = \mathrm{O}$,

which is usually dealt with by the method of characteristics. Since this method presupposes the existence of the second instead of the first derivative of the solutions, one may ask whether there exist solutions which escape the methods of characteristics. Under rather broad conditions, Haar answered this question in the negative.

Haar's most important contribution to variational calculus (1917–1919) features an analogous principle, Haar's lemma, an extension of Paul du Bois- Reymond's to double integrals: If

for all continuously differentiable *f* which vanish on the boundary of *B*, then there is a *w* such that

Haar's lemma allows one to deal with variational problems like

without supplementary assumptions on the second derivative of the unknown function *z*. He applied his lemma to variational problems like Plateau's. A multitude of papers by others show the influence this lemma exerted on the whole area of variational calculus.

The notion to which Haar's name is most firmly attached is Haar's measure on groups. In 1932 Haar showed, by a bold direct approach, that every locally compact group possess an invariant measure which assigns positive numbers to all open sets. An immediate consequence of this theorem was the analytic character of compact groups (J. Von Neumann). It was somewhat later applied to locally compact Abelian groups by Pontryagin. The theorem is now one of the cornerstones of those areas of mathematics where algebra and topology meet.

BIBLIOGRAPHY

See *Alfréd Haar: Gesammelte Arbeiten* (Budapest, 1959), B.S.-Nagy, ed.

H. Freudenthal