Hasse, Helmut | Encyclopedia.com

(*b* Kassel, Germany, 25 August] 1898; *d*. Ahrensburg, near Hamburg, Federal Republic of Germany, 26 December 1979)

number theory.

One of the most important mathematicians of the twentieth century, Helmut Hasse was a man whose accomplishments spanned research, mathematical exposition, teaching, and editorial work. In research his contributions permeate modern number theory; particularly noteworthy are his "local-global principle," which established Kurt Hensel's p-adic numbers as indispensable tools of number theory, and his proof of the "Riemann hypothesis" for elliptic curves. In exposition his report on class field theory in the 1920's made the work of Teiji Takagi, Philipp Furtwängler, Emil Artin, Hasse, and others available to a wide audience (and, like any good exposition, contained a great deal of Hasse's own reworking of the material). Later books and monographs confirmed Hasse's reputation as a writer who could be counted on to present the most difficult subjects with great clarity. In teaching, the long list of his students and their descriptions of his inspiring lectures give ample testimony to his continuation of "Crelle's Journal'—*Journal für die reine und angewandte Mathematik*—as one of the world's foremost mathematical periodicals during the fifty years of his editorship was largely a result of his painstaking efforts, high standards, and editorial ability.

Hasse's parents were Paul Reinhard Hasse, a judge, and Margaret ha Quentin (born in Milwaukee, Wisconsin, but raised from the age of five by an aunt in Kassel). His secondary education was in gymnasiums in the Kassel area until the family moved to Berlin, where his father had received a high judicial appointment in 1913. After two years in the FichteGymnasium there, he took the exit examination early (*a Notabitur*) in order to volunteer for the navy. Hasse had evidently decided on a career in mathematics while still at gymnasium, because while he was stationed in the Baltic, he studied, on the advice of his gymnasium teacher, the DirichletDedekind lectures on number theory. During the last year of his naval service he was stationed in Kiel, where he attended classes in mathematics under Otto Toeplitz. Upon leaving the navy in December 1918, Hasse went to Göttingen to begin his mathematical studies in earnest. The teacher at Göttingen who made the greatest impression on him was Erich Hecke, who left Göttingen to go to Hamburg in the spring of 1919. Hasse left the following year. Hasse did not, however, follow Hecke. Greatly impressed by Kurt Hensel's book *Zahlentheorie* (1913), which he had found while browsing in a Göttinge bookstore, he decided to go to Marburg to study with Hensel. What made Hensel's book special was his introduction of p-adic numbers, and it was in order to study this new tool of number theory that Hasse went to Marburg.

In October 1920 Hasse discovered his "localglobal principle," which transformed the p-adic numbers from a curiosity that the Göttingen establishment regarded as a fruitless sidetrack into a natural tool of number theory. In 1975, in a *Geleitwort* to the first volume of his mathematical papers, Hasse recounted his discovery of the local-global principle, emphasizing Hensel's role in it. At Hensel's suggestion, Hasse had investigated the necessary and sufficient conditions for a rational number to be representable by a rational quadratic form. He found the key to the answer in a reduction procedure of Lagrange for ternary quadratic forms that he had learned from his reading of Dirichlet-Dedekind. However, to his disappointment, his solution did not appear to call for the use of p-adic numbers. He communicated this to Hensel, whose reply— a postcard that Hasse kept all his life—opened Hasse's eyes, as he later said, to the true significance of what he had proved: a ternary quadratic form that represents 0 nontrivially over the padic numbers for all p (including p = ∞, which corresponds to the real numbers) represents 0 nontrivially over the rationals. That is, if there is a solution "locally" for all p, then there is a "global" (rational) solution. Hasse soon expanded this principle to a wide range of problems dealing with the equivalence of quadratic forms, work that allowed him to complete his doctoral dissertation in 1921 and his *Habilitationsschrift* in 1922.

Hasse left Marburg in 1922 to accept a paid teaching appointment as *Privatdozent* in Kiel. Hasse married Clara Ohle on 11 April 1923. They had a daughter. Jutta, and a son. Rüdiger.

In 1925 he was appointed professor at Halle, and in 1930 he returned to Marburg to assume the chair made vacant by Hensel's retirement. This succession, which was the realization of Hensel's fondest wish, was of short duration. The strong center of German mathematics, Göttingen, was demolished by the firings and resignations that followed the coming to power of the Nazis in 1933. Hasse, who appeared to be politically acceptable to the Nazis and yet was a mathematician of the highest caliber, was a natural choice as a successor to the deposed director of the Mathematics in Göttingen, Richard Courant. Even Courant, in the interest of the continuation of mathematics in Göttinger, favored Hasse's appointment. (The directorship of the institute was not linked to any one chair. Courant was still "on leave" in 1934, and his chair was not vacant. Hasse was appointed to the more prestigious chair left vacant by Hermann Weyl's resignation, the chair that had formerly been David Hilbert's.) Hasse became the director at Göttingen in 1934 and remained in that post formally—although he was on leave and

engaged in naval research in Berlin from 1937 to 1945—until he was dismissed by the British occupation authorities in September 1945.

During the years in Kiel, Hasse explored the subject of norm residue symbols and explicit reciprocity laws by means of *p*-adic techniques. The proximity of these interests to those of Emil Artin, and Artin's physical proximity in Hamburg, led to friendly collaboration and frequent meetings to exchange ideas. (A joint paper, written in 1925, says the work was drafted by "the younger" of the two authors. Few readers could have known that Hasse was younger— by less than half a year.) During this period Hasse undertook to prepare for the Deutsche Mathematiker Vereinigung a report on recent developments in class field theory, particularly the advances of the Japanese mathematician Teiji Takagi. Thereport was delivered at Danzig in the summer of 1925, but the published *Klassenköperbericht*, as it was called took longer to produce. It was published in two parts, the second part not appearing until 1930; by this time Artin had succeeded in proving his general law of reciprocity (1927), and Hasse included a full account of it in the second part of the *Bericht*.

In the early 1930's Hasse completed a thoroughgoing revision of class field theory—including reciprocity laws and norm residue symbols—in terms of the theory ofnoncommutative algebras applied to p-adic number fields, an approach that has been called "the number high point of the local-global principle."

Soon after this success, Hasse began work in another area that was to become one of the central subjects of modern number theory. Challenged by his English colleague Harold Davenport to bring his algebraic methods to bear on a problem Davenport and Louis Joel Mordell had been studying concerning the number of solutions of a congruence of the form $y^2=f(x)$ mod p for large primes p when f(x) is a fixed cubic polynomial, Hasse succeeded magnificently. He first perceived that the problem Davenport and Mordell had been considering was in fact equivalent to a problem that Artin had formulated in an altogether different form as an analogue of the Riemann hypothesis, and he then proved this analogue in the case Davenport and Mordell considered, which was the "Riemann hypothesis" in the case of a function field of an elliptic curve over a finite field. To establish connections between such apparently disparate subjects in mathematics and, more than that, to solve a problem using techniques that were developed for altogether different purposes is an achievement of the highest sort in pure mathematics. Great progress has since been made in generalizing Hasse's solution. It is now known that the "Riemann hypothesis" is true in a vast range of cases, provided that algebraic techniques apply; the actual Riemann hypothesis, which is transcendental rather than algebraic, is still an unsolved problem.

Hasse's political views and his relations with the Nazi government are not easily categorized. On the one hand, his relations with his teacher Hensel, who was unambiguously Jewish by Nazi standards, were extremely close, right up to Hensel's death in 1941, and his relations with the Hensel family remained close and warm throughout his life. One of his most important papers was a collaboration with Emmy Noether and Richard Brauer, both Jewish, published in 1932 in honor of Hensel's seventieth birthday (which occurred at the end of 1931). Also in 1932 he dedicated an extremely important paper to Emmy Noether on the occasion of her fiftieth birthday. Hasse did not compromise his mathematics for political reasons. In his years as director of the Mathematics Institute in Göttingen, he brought to Göttingen as difficult a figure as Carl Ludwig Siegel, and he struggled against Nazi functionaries who tried (sometimes successfully) to subvert mathematics to political doctrine. Hasse never published in the journal *Deutsche Mathematik*. On the other hand, he made no secret of his strongly nationalistic views and of his approval of many of Hilter's policies. As an academic administrator and, during the war, as a military officer, he was of course a participant in the regime. He did apply, in 1937, for membership in the National Socialist Party, but the application was refused because he had a remote Jewish ancestor, (However, Slegel reported in a letter to Courant in March 1939 that he had seen Hasse wearing Nazi insignia.) After the war, many emigrant mathematicians condemned Hasse's activities during the Nazi period; invitations to the United States were few, considering his scientific eminence, and what invitations there were aroused controversy.

The occupation authorities revoked Hasse's right to teach in September 1945. He refused to remain in Göttingen in a purely research capacity, on the grounds that his inability to teach would be detrimental to his research. However, after a difficult year, he finally decided to accept an appointment as a research professor at the Berlin Academy, and moved to Berlin in September 1946. During the next two years he gave private lectures to a small group of students, many of whom later had successful careers. In 1948, Hasse's rating in the "denazification" program was improved to the point where he was allowed to give public lectures at Humboldt University in Berlin, and in 1949 he was named to a professorship there. One year later he accepted an appointment as professor at the University of Hamburg. He retired in 1966, but continued to make his home in Ahrensburg, outside Hamburg, for the rest of his life. In the postwar period he remained active as a teacher, as a writer of both research and expository works, and as an editor, although in his last years his work was curtailed by ill health.

Among the many honors Hasse received were the German National Prize, First Class, for Science and Technology (1953), an honorary doctorate from the University of Kiel (1968), and the Cothenius Medal of the Academia Leopoldina in Halle (1968). He was a member of the academies of science of Berlin, Halle, Göttingen, Helsinki, Mainz, and Madrid.

BIBLIOGRAPHY

I. Original Works. *Mathematische Abhandlungen*, Heinrich Wolfgang Leopoldt and Peter Roquette, eds., 3 vols. (Berlin and New York, 1975). A complete list of Hasse's scientific publications is at the end of vol. III.

II. Secondary Literature. G. Frei, "Helmut Hasse (1898–1979)," in *Expositiones Mathematicae*, 3 (1985), 55–69; Heinrich Wolfgang Leopoldt, "Zum wissenschaftlichen Werk von Helmut Hasse," in *Journal für die reine und angewandte Mathematik*, **262/263** (1973), 1–17, and "Helmut Hasse," in *Journal of Number Theory*, **14** (1982), 188–120; and S. L. Segal, "Helmut Hasse in 1934," in *Historia Mathematica*, 7 (1980), 46–56.

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