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(*b*. Breslau, Germany, 8 November 1868; *d*. Bonn, Germany, 26 January 1942),

mathematics, philosophy, literature.

Hausdorff studied at Leipzig, Freiburg, and Berlin between 1887 and 1891 and started research in applied mathematics. After his *habilitation* (1895) he taught at the University of Leipzig and a local commercial school. He moved in a milieu of Leipzig intellectuals and artists, strongly influenced by the early work of Friedrich Nietzsche (1844–1900), striving for a cultural modernization of late-nineteenth-century Germany.

Between 1897 and 1910 Hausdorff published two philosophical books, a poem collection, and a satirical theater play under the pseudonym Paul Mongré. He regularly contributed cultural critical essays to the *Neue Deutsche Rundschau*, a leading German intellectual journal of the time. In his second book, *Das Chaos in kosmischer Auslese* (1898; Chaos in cosmic selection), he radicalized Kantian transcendental idealism by a Nietzschean perspective and attempted to dissolve the belief in any determined a priori form of transcendent reality. In particular he reviewed the contemporary discussion on space and time, employing transformations of increasing generality. In the end he was led to considering transfinite Cantorian sets and their general set transformations as a mathematical expression for a completely unstructured reality, and thus to ‘‘transcendent nihilism.”

During this period, Hausdorff reoriented his mathematical work toward the new field of transfinite set theory. He gave one of the first lecture courses on this topic in summer 1901 and contributed important results to it, among others the *Hausdorff recursion* for aleph exponentiation (1904) and deep methods for the classification of order structures (confinality, gap types, general ordered products, and eta-alpha sets; 1906–1907). He employed a “naive” concept of set, but even so achieved an exceptionally high precision of argumentation. He contributed crucial insights into foundational questions, most importantly his maximal chain principle (related to but different from Zorn’s lemma), a characterization of weakly inaccessible cardinals (in present terminology), and the universality property for order structures of what he called “eta-alpha sets.” The latter became one of the roots of *saturated structures* in model theory of the 1960s. Moreover, Hausdorff hit on the importance of the *generalized continuum hypothesis* in these studies.

In summer 1910 Hausdorff started teaching at the University of Bonn and broadened his perspective on set theory as a general basis for mathematics. In early 1912 he found an axiomatic characterization of topological spaces by neighborhood systems and started to compose a monograph on “basic features of set theory” (*Grundzüge der Mengenlehre*). It was finished two years later (1914b), after he was called in 1913 to a full professorship at the University of Greifswald.

In this book, Hausdorff showed how set theory could be used as a working frame for mathematics more broadly. While he introduced set theory in a nonaxiomatic style, although with extraordinary precision, topological spaces and measure theory were given an axiomatic presentation. In part two Hausdorff published his neighborhood axioms found two years earlier, introduced separation and countability axioms, and studied connectivity properties. This part of the book contained the first comprehensive treatment of the theory of metric spaces, initiated by Maurice-René Fréchet in 1906. It laid the basis for an important part of general topology of the twentieth century.

In part three he provided a lucid introduction to measure theory, building on the work of Émile Borel and Henri-Léon Lebesgue. In a paper published shortly before the book (also in an appendix to it) Hausdorff gave a negative answer to Lebesgue’s question, whether a (finitely) additive content function invariant under congruences can be defined on *all* subsets of Euclidean three-space (1914a). His peculiar use of the axiom of choice became the starting point for the later paradoxical constructions of measure theory by Stefen Banach and Alfred Tarski.

The *Grundzüge* became influential only after World War I, most strongly in the rising schools of modern mathematics in Poland, around the journal *Fundamenta Mathematicae*, and in the Soviet Union mainly among Nikolai N. Luzin’s students around Pavel Alexandrov. The *Grundzüge* became one of the founding documents of mathematical modernism in the 1920 and 1930s.

Already in the *Grundzüge* Hausdorff had started to study Borel sets. In 1916 he, and independently Alexandrov, could show that any infinite Borel set in a separable metrical space is countable or of cardinality of the continuum. That was an important step forward for a strategy proposed by Georg Cantor to clarify the continuum hypothesis. Although this goal could not be achieved along this road, it led to the development of an extended field of investigation on the border region between set theory and analysis, now dealt with in descriptive set theory.

On the other side, Hausdorff took up questions in real analysis, now informed by the new basic features of general set theory. His introduction of what came to be called Hausdorff measure and Hausdorff dimension (1919) became of long-lasting importance in the theory of dynamical systems, geometrical measure theory, and the study of "fractals."

Other important technical contributions dealt with summation methods of infinite divergent series and a generalization of the Riesz-Fischer theorem, which established the well-known relation between function spaces and series of Fourier coefficients (1923). It opened the path for later developments in harmonic analysis on topogical groups. In a lecture course in 1923 Hausdorff introduced an axiomatic basis for probability theory, which anticipated Andrey Kolmogorov's axiomatization of 1933.

When Hausdorff revised his magnum opus for a second edition, he rewrote the parts on descriptive set theory and topological spaces completely, extending the first part considerably and concentrating the second one on metrical spaces. As other books on general set and general topology had appeared in the meantime, he omitted these parts for the so-called second edition, which became essentially a new book (1927).

In 1921 Hausdorff had returned to Bonn, now as a full professor and colleague of Eduard Study and (some years later) Otto Toeplitz. While he was still regularly emeritated in early 1935, general life and working conditions deteriorated drastically for Hausdorff and other people of Jewish origin, after the rise to power of the Nazi regime. His attempt at emigration came too late. When Hausdorff, his wife Charlotte, and a sister of hers were ordered to leave their house for a local internment camp in January 1942, they opted for suicide rather than suffering further persecution. His scientific *Nachlass* was handed over to a local friend. It survived the end of the war with only minor damage.

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