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(*b*. Paris, France. 12 February 1908; *d*. La Bérarde, Isère, France, 27 July 1931)

mathematics, *logic*.

Herbrand gave early signs of his mathematical gifts, entering the École Normale Supérieure at the exceptional age of seventeen and ranking first in the entering class. He completed his doctoral dissertation in April 1929. That October he began a year of service in the French army. He then went to Germany on a Rockefeller fellowship, studying in Berlin (until May 1931) with John von Neumann, then in Hamburg (May-June) with Emil Artin, and in Göttingen (June-July) with Emmy Noether. He left Göttingen for a vacation in the Alps and a few days later was killed in a fall at the age of twenty-three.

Herbrand's contributions to mathematics fall into two domains: mathematical logic and modern algebra. He showed an early interest in mathematical logic, a subject to which French mathematicians were then paying scant attention, and published a note on a question of mathematical logic in the *Comptes rendus* of the Paris Academy of Sciences when he was hardly twenty. Herbrand's main contribution to logic vas what is now called the Herbrand theorem, published in his doctoral dissertation: it is the most fundamental result in quantification theory. Consider an arbitrary formula F of quantification theory, then delete all its quantifiers and replace the variables thus made free with constants selected according to a definite procedure. A lexical instance of F is thus obtained. Let F_i be the ith lexical instance of F, the instances being generated in some definite order. The Herbrand theorem states that F is provable in any one of the (equivalent) systems of quantification theory if and only if for some number k the disjunction

(now called the kth Herbrand disjunction) is sentential)' valid. (Herbrand's demonstration of the theorem contains a gap, discovered in 1963 by B. Dreben, P. Andrews, and S. Aanderaa.)

The Herbrand theorem establishes an unexpected bridge between quantification theory and sentential logic. Testing a formula for sentential validity is a purely mechanical operation. Given a formula F of quantification theory, one tests the kth Herbrand disjunction of F successively for $k = 1$, $k = 2$, and so on; if F is provable, one eventually reaches a number k for which the kth Herbrand disjunction is valid. If F is not provable, there is, of course, no such k; and one never learns that there is no such k (in accordance with the fact that there is no decision procedure for quantification theory). Besides yielding a very convenient proof procedure, the Herbrand theorem has many applications (a field explored by Herbrand himself) to decision and reduction problems and to proofs of consistency. Almost all the methods for proving theorems by machine rest upon the Herbrand theorem.

In modern algebra Herbrand's contributions are in class-field theory, the object of which is to gain knowledge about Abelian extensions of a given algebraic number field from properties of the field. Initiated by Leopold Kronecker and developed by Heinrich Weber, David Hilbert, Teiji Takagi, and Emil Artin, the theory received essential contributions from Herbrand in 1930–1931. He wrote ten papers in this field, simplifying previous proofs, generalizing theorems, and discovering important new results.

BIBLIOGRAPHY

Herbrand's logical writings have been reprinted in his *Écrits logiques*, Jean van Heijenoort, ed. (Paris, 1968); the pref. includes reference to the paper by B. Dreben, P. Andrews, and S. Aanderaa, "False Lemmas in Herbrand," *in Bulletin of the American Mathematical Society*, **69** (1963), 699–706, as well as further information about the gap in Herbrand's demonstration of his theorem. Also see *The Logical Writings of Jacques Herbrand*, Warren D. Goldfarb, ed. (Reidel, 1971).

The list of Herbrand's papers on class-field theory can be found in Ernest Vessiot's intro. to Helmut Hasse, *Über gewisse Ideale in einer einfachen Algebra* (Paris, 1934). Herbrand's *Le développement moderne de la théorie des corps algébriques* was published posthumously and edited by his friend Claude Chevalley (Paris, 1936).

Jeanvan Heijenoort