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(*b*. Dieuze, Lorraine, France, 24 December 1822; *d*. Paris, France, 14 January 1901)

mathematics.

Hermite was the sixth of the seven children of Ferdinand Hermite and the former Madeleine Lallemand. His father, a man of strong artistic inclinations who had studied engineering, worked for a while in a salt mine near Dieuze but left to assume the draper's trade of his in-laws—a business he subsequently entrusted to his wife in order to give full rein to his artistic bent. Around 1829 Charles's parents transferred their business to Nancy. They were not much interested in the education of their children, but all of them attended the Collège of Nancy and ived there. Charles continued his studies in Paris, first it the Collège Henri IV, where he was greatly influenced by the physics lessons of Despretz, and then, in 1840–1841, at the Collège Louis-le-Grand: his mathematics professor there was the same Richard who fifteen years earlier had taught Evariste Galois. Instead of seriously preparing for his examination Hermite read Euler, Gauss's *Disquisitiones arithmeticae*, and Lagrange's *Traité sur la résolution des équations numériques*, thus prompting Richard to call him *un petit Lagrange*.

Hermite's first two papers, published in the *Nouvelles annales de mathématiques*, date from this period. Still unfamiliar with the work of Ruffini and Abel, he tried to prove in one of these papers the impossibility of solving the fifth-degree equation by radicals. Hermite decided to continue his studies at the École Polytechnique; during the preparation year he was taught by E. C. Catalan. In the 1842 contest of the Paris colleges Hermite failed to win first *prix de mathématiques spéciales* section but received only first "accessit." He was admitted to the École Polytechnique in the fall of 1842 with the poor rank of sixty-eighth. After a year's study at the École Polytechnique, he was refused further study, because of a congenital defect of his right foot, which obliged him to use a cane. Owing to the intervention of influential people the decision was reversed, but under conditions to which Hermite was reluctant to submit. At this time, Hermite—a cheerful youth who, according to some, resembled a Galois resurrected—was introduced into the circle of Alexandre and Joseph Brtrand. Following the example of others, he declined the paramount honor of graduating from the École Polytechnique, contenting himself with the career of *professeur*. He took his examinations for the *baccalauréat and licence* in 1847.

At that time Hermite must have become acquainted with the work of Cauchy and Liouville on general function theory as well as with that of C. G. J. Jacobi on elliptic and hyperelliptic functions. Hermite was better able than Liouville, who lacked sufficient familiarity with Jacobi's work, to combine both fields of thought. In 1832 and 1834 Jacobi had formulated his famous inversion problem for hyperelliptic integrals, but the essential properties of the new ranscendents were still unknown and the work of A. Göpel and J. G. Rosenhain had not yet appeared. Through his first work in this field, Hermite placed himself, as Darboux says, in the ranks of the first analysts. He generalized Abel's theorem on the division of the argument of elliptic functions to the case of hyperelliptic ones. In January 1843, only twenty years old, he communicated his discovery to Jacobi, who did not conceal his delight. The correspondence continued for at least six letters; the second letter, written in August 1844, was on the transformation of elliptic functions, and four others of unknown dates (although before 1850) were on number theory. Extracts from these letters were inserted by Jacobi in *Crelle's Journal* and in his own *Opuscula*. and are also in the second volume of Dirichlet's edition of Jacobi's work. Throughout his life Hermite exerted a great scientific influence by his correspondence with other prominent mathematicians. It is doubtful that his *Oeuvres* faithfully reflects this enormous activity.

In 1848 Hermite was appointed a *répétiteur* and admissions examiner at the École Polytechnique. The next ten years were his most active period. On 14 July 1856 he was elected a member of the Académie des Sciences, receiving forty out of forty-eight votes.

In 1862, through Pasteur's influence, a position of *maître de conférence* was created for Hermite at the École Polytechnique; in 1863 he became an *examinateur de sortie et de classement* there. He occupied that position until 1869, when he took over J. M. C. Duhamel's chair as professor of analysis at the École Polytechnique and at the Faculté des Sciences, first in algebra and later in analysis as well. His textbooks in analysis became classics, famous even outside France. He resigned his chair at the École Polytechnique in 1876 and at the Faculté in 1897. He was an honorary member of a great many academies and learned societies, and he was awarded many decorations. Hermite's seventieth birthday gave scientific Europe the opportunity to pay homage in a way accorded very few mathematicians.

Hermite married a sister of Joseph Bertrand; one of his two daughters married Émile Picard and the other G. Forestier. He lived in the same building as E. Boumoff at Place de l'Odéon, and it was perhaps his acquaintance with this famous philologist that led him to study Sanskrit and ancient Persian. Hermite was seriously ill with smallpox in 1856, and under Cauchy's influence became a devout Catholic. His scientific work was collected and edited by Picard.

From 1851–1859 Europe lost four of its foremost mathematicians, Gauss, Cauchy, Jacobi, and Dirichlet. Nobody, except Hermite himself, could guess the profoundness of the work of Weierstrass and Riemann on Abelian functions and of Kronecker and Smith on the mysterious relations between number theory and elliptic functions. Uncontested, the scepter of higher arithmetic and analysis passed from Gauss and Cauchy to Hermite who wielded it until his death, notwithstanding the admirable discoveries of rivals and disciples whose writings have tarnished the splendor of the most brilliant performance other than his [unspecified quotation by P. Mansion].

Throughout his lifetime and for years afterward Hermite was an inspiring figure in mathematics. In today's mathematics he is remembered chiefly in connection with Hermitean forms, a complex generalization of quadratic forms, and with Hermitean polynomials (1873), both minor discoveries. Specialists in number theory may know that some reduction of quadratic forms is owed to him; his solution of the Lamé differential equation (1872, 1877) is even less well known. An interpolation procedure is named after him. His name also occurs in the solution of the fifth-degree equation by elliptic functions (1858). One of the best-known facts about Hermite is that he first proved the transcendence of e (1873). In a sense this last is paradiematic of all of Hermite's discoveries. By a slight adaptation of Hermite's proof, Felix Lindemann, in 1882, obtained the much more exciting transcendence of π. Thus, Lindemann, a mediocre mathematician, became even more famous than Hermite for a discovery for which Hermite had laid all the groundwork and that he had come within a gnat's eye of making. If Hermite's work were scrutinized more closely, one might find more instances of Hermitean preludes to important discoveries by others, since it was his habit to disseminate his knowledge lavishly in correspondence, in his courses, and in short notes. His correspondence with T. J. Stieltjes, for instance, consisted of at least 432 letters written by both of them between 1882 and 1894. Contrary to Mansion's statement above, Hermite's most important results have been so solidly incorporated into more general structures and so intensely absorbed by more profound thought that they are never attributed to him. Hermite's principle, for example, famous in the nineteenth century, has been forgotten as a special case of the Riemann-Roch theorem. Hermite's work exerted a strong influence in his own time, but in the twentieth century a few historians, at most, will have cast a glance at it.

In Hermite's scientific activity, shifts of emphasis rather than periods can be distinguished; 1843–1847, division and transformation of Abelian and elliptic functions; 1847–1851, arithmetical theory of quadratic forms and use of continuous variables; 1854–1864, theory of invariants; 1855, a connection of number theory with theta functions in the transformation of Abelian functions; 1858–1864. fifth-degree equations, modular equations, and class number relations: 1873, approximation of functions and transcendence of e; and 1877–1881, applications of elliptic functions and Lamé's equation.

In the 1840's, and even in the early 1850's, the inversion of integrals of algebraic functions was still a confusing problem, mainly because of the paradoxical occurrence of more than two periods. Jacobi reformulated the problem by simultaneously inverting p integrals—if the irrationality is a square root of a polynomial of the $(2p - 1)$th or 2 pth degree. In the early 1840's the young Hermite was one of the very few mathematicians who viewed Abelian functions clearly, owing to his acquaintance with Cauchy's and Liouville's ideas on complex functions. To come to grips with the new transcendents, he felt that one had to start from the periodicity properties rather than from Jacobi's product decomposition. This new approach proved successful in the case of elliptic functions, when Hermite introduced the theta functions of n th order as a means of constructing doubly periodic functions. In the hyperelliptic case he was less successful, for he did not find the badly needed theta functions of two variables. This was achieved in the late 1840's and early 1850's by A. Göpel and J. G. Rosenhain for $p = 2$; the more general case was left to Riemann. In 1855 Hermite took advantage of Göpel's and Rosenhain's work when he created his transformation theory (see below).

Meanwhile, Hermite turned to number theory. For definite quadratic forms with integral coefficients, Gauss had introduced the notion of equivalence by means of unimodular integral linear transformations; by a reduction process he had proved for two and three variables that, given the determinant, the class number is finite. Hermite generalized the procedure and proved the same for an arbitrary number of variables.

He applied this result to algebraic numbers to prove that given the discriminant of a number field, the number of norm forms is finite. By the same method he obtained the finiteness of a basis of units, not knowing that Dirichlet had already determined the size of the basis. Finally, he extended the theorem of the finiteness of the class number to indefinite quadratic forms, and he proved that the subgroup of unimodular integral transformations leaving such a form invariant is finitely generated.

Hermite did not proceed to greater depths in his work on algebraic numbers. He was an algebraist rather than an arithmetician. Probably he never assimilated the much more profound ideas that developed in the German school in the nineteenth century, and perhaps he did not even realize that the notion of algebraic integer with which he had started was wrong. Some of his arithmetical ideas were carried on with more success by Hermann Minkowski in the twentieth century.

In the reduction theory of quadratic and binary forms Hermite had encountered invariants. Later he made many contributions to the theory of invariants, in which Arthur Cayley, J. J. Sylvester, and F, Brioschi were active at that time. One of his most important contributions to the progress of the theory of invariants was the "reciprocity law," a one-to-one relation between the covariants of fixed degree of irder p of an mth-degree binary form and those of order m of a pth-degree binary form. One of his nvariant theory subjects was the fifth-degree equaion, to which he later applied elliptic functions.

Armed with the theory of invariants, Hermite returned to Abelian functions. Meanwhile, the badly needed theta functions of two arguments had been found, and Hermite could apply what he had learned ibout quadratic forms to understanding the trans-ormation of the system of the four periods. Later, Hermite's 1855 results became basic for the transformation theory of Abelian functions as well as for Camille Jordan's theory of "Abelian" groups. They also led to Herrnite's own theory of the fifth-degree equation and of the modular equations of elliptic functions. It was Herrnite's merit to use ω rather than Jacobi's $q = e^{\pi i \omega}$ as an argument and to prepare the present form of the theory of modular functions. He again dealt with the number theory applications of his theory, particularly with class number relations or quadratic forms. His solution of the fifth-degree equation by elliptic functions (analogous to that of third-degree equations by trigonometric functions) was the basic problem of this period.

In the 1870's Hermite returned to approximation problems, with which he had started his scientific career. Gauss's interpolation problem, Legendre functions, series for elliptic and other integrals, con-nued fractions, Bessel functions, Laplace integrals, and special differential equations were dealt with in this period, from which the transcendence proof for *e* and the Lamé equation emerged as the most remarkable results.

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II. Secondary Literature. The biographical data of this article are taken from G. Darboux's biography in *La revue du mois*, **1** (1906), 37–58, the most accurate and trustworthy source. Other sources are less abundant; the exception is P. Mansion and C. Jordan, "Charles Hermite (1822–1901)," in *Revue des questions scientifiques*, 2nd ser., **19** (1901), 353–396, and **20** (1901), 348–349; unfortunately, Mansion did not sufficiently account for the sources of his quotations.

An excellent analysis of Hermite's scientific work is M. Noether, "Charles Hermite," in *Mathematische Annalen*, **55** (1902), 337–385. Others, most of them superficial *éloges*, can be retraced from *Jahrbuch über die Fortschritte der Mathematik*, **32** (1901), 22–28; **33** (1902), 36–37; and **36** (1905), 22.

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