

Hipparchus (astronomer) | Encyclopedia.com

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(b. Nicaea, Bithynia [now Iznik, Turkey], first quarter of second century B.C.; d. Rhodes[?], after 127 B.C.)

astronomy, mathematics geography.

The only certain biographical datum concerning Hipparchus is his birthplace, Nicaea, in northwestern [Asia Minor](#). This is attested by several ancient sources¹ and by Nicaean coins from the second and third centuries of the Christian era that depict a seated man contemplating a globe, with the legend ΙΙΙΙΑΡΧΟΣ. His scientific activity is dated by a number of his astronomical observations quoted in Ptolemy's *Almagest*. The earliest observation indubitably made by Hipparchus himself is of the autumnal equinox of 26/27 September 147 b.c.² The latest is of a lunar position on 7 July 127 b.c.³ Ptolemy reports a series of observations of autumnal and vernal equinoxes taken from Hipparchus, ranging from 162 to 128 b.c., but it is not clear whether the earliest in the series had been made by Hipparchus himself or were taken from others; Ptolemy says only that "they seemed to Hipparchus to have been accurately observed." We can say, then, that Hipparchus' activity extended over the third quarter of the second century b.c., and may have begun somewhat earlier. This accords well with the calculations of H. C. F. C. Schjellerup and H. Vogt concerning the epoch of the stellar positions in Hipparchus' commentary on Aratus⁴.

It is probable that Hipparchus spent the whole of his later career at Rhodes: observations by him ranging from 141 to 127 b.c. are specifically attributed to Rhodes by Ptolemy. In Ptolemy's *Phases of the Fixed Stars*, however, it is stated that the observations taken from Hipparchus in that book were made in Bithynia (presumably Nicaea). We may infer that Hipparchus began his scientific career in Bithynia and moved to Rhodes some time before 141 b.c. The statement found in some modern accounts that he also worked in Alexandria is based on a misunderstanding of passages in the *Almagest* referring to observations made at Alexandria and used by or communicated to Hipparchus.⁵ The only other biographical information we have is an utterly untrustworthy anecdote that Hipparchus caused amazement by sitting in the theater wearing a cloak, because he had predicted a storm.⁶

Hipparchus is a unique figure in the history of astronomy in that, while there is general agreement that his work was of profound importance, we are singularly ill-informed about it. Of his numerous works only one (the commentary on Aratus) survives, and that a comparatively slight one (although valuable in the absence of the others). We derive most of our knowledge of Hipparchus' achievements in astronomy from the *Almagest*; and although Ptolemy obviously had studied Hipparchus' writings thoroughly and had a deep respect for his work, his main concern was not to transmit it to posterity but to use it and, where possible, improve upon it in constructing his own astronomical system. Most of his references to Hipparchus are quite incidental; and some of them are obscure to us, since we cannot consult the originals. Some supplementary information can be gathered from remarks by other ancient writers; but for the most part they were not professional astronomers, and they frequently misunderstood or misrepresented what Hipparchus said (a typical example is the elder Pliny). Since the evidence is so scanty, the following account necessarily contains much that is uncertain or conjectural. Painstaking analysis of that evidence, however, which has begun only in recent years,⁷ has revealed, and will reveal, a surprising amount. It also has demonstrated the groundlessness of the assumption, stated or tacit, of most modern accounts of Hipparchus; that everything in the *Almagest* that Ptolemy does not expressly claim as his own work is derived from Hipparchus. The truth is more complicated and more interesting. A further difficulty is that although we know the titles of a good many of Hipparchus' works, much of our information on his opinions and achievements cannot be assigned with certainty to any known title. Moreover, while we can make some inferences about the chronology of his work (it is certain, for instance, that his discovery of precession belongs to the end of his career), in general the dates and order of composition of his works are unknown. This discussion is therefore arranged by topics rather than by titles (the latter are mentioned under the topics that they are known or conjectured to have treated).

Mathematical Methods . In Greek astronomy the positions of the heavenly bodies were computed from geometrical models to which numerical parameters had been assigned. An [essential element](#) of the computation was the solution of plane triangles; Greek trigonometry was based on a table of chords. We are informed that Hipparchus wrote a work on chords,⁸ and we can reconstruct his chord table. It was based on a circle in which the circumference was divided, in the normal (Babylonian) manner, into 360 degrees of 60 minutes, and the radius was measured in the same units; thus R , the radius, expressed in minutes, is

This function is related to the modern sine function (for α in degrees) by

Hipparchus computed the function only at intervals of $1/48$ of a circle ($7-1/2^\circ$), using linear interpolation between the computed points for other values. Thus he was able to construct the whole table on a very simple geometrical basis: it can be computed

from the values of Crd 60° (=R), Crd 90°(=), and the following two formulas (in which d is the diameter of the base circle and s is the chord of the angle α):

(1)

(2)

The first is a trivial application of Pythagoras' theorem, and the second was already known to Archimedes.

This chord table survives only in the sine table commonly found in Indian astronomical works, with $R=3438'$ and values computed at intervals of $3-3/4^\circ$, which is derived from it. But its use by Hipparchus can be demonstrated from calculations of his preserved in *Almagest* IV,11. Otherwise, apart from a couple of stray occurrences of its use,⁹ it vanishes from Greek astronomy, being superseded by Ptolemy's improved chord table based on the unit circle ($R=60=1,0$ in Ptolemy's sexagesimal system) and calculated to three sexagesimal places at intervals at $1/2^\circ$. The results of trigonometrical calculations based on Hipparchus' chord table, although less accurate than those based on Ptolemy's, are adequate in the context of ancient astronomy. The main disadvantage of its use, in contrast with Ptolemy's, is the constant in trusion of the factor 3438 in the calculations. It has the compensating advantage, however, that for small angles (up to $7-1/2^\circ$), the chord can be replaced by the angle expressed in minutes (in this respect it is analogous to modern radian measure), which greatly simplifies computations (for an example, see below on the distances of the sun and moon).

Given the chord function, Hipparchus could solve any plane triangle by using the equivalent of the modern sine formula:

No doubt, like Ptolemy, he usually computed with right triangles, breaking down other triangles into two right triangles. In the absence of a tangent function, he had to use the chord function combined with Pythagoras' theorem; but his methods were as effective as, if more cumbersome than, those of modern trigonometry. Particular trigonometrical problems had been solved before Hipparchus by [Aristarchus of Samos](#) (early third century B.C.) and by Archimedes, using approximation methods; but it seems highly probable that Hipparchus was the first to construct a table of chords and thus provide a general solution for trigonometrical problems.¹⁰ A corollary of this is that, before Hipparchus, astronomical tables based on Greek geometrical methods did not exist. If this is so, Hipparchus was not only the founder of trigonometry but also the man who transformed Greek astronomy from a purely theoretical into a practical, predictive science.

In Greek astronomy most problems arising from computations of the positions of the heavenly bodies were either problems in plane trigonometry or could be reduced to such by replacing the small spherical triangles involved by plane triangles. The principal exception was those problems in which the earth is no longer treated as a point—that is, those in which the position of the observer on the earth must be taken into account, notably those concerned with parallax and rising times. Exact mathematical treatment of these requires spherical trigonometry. Since Hipparchus did treat these subjects, the question arises whether he used spherical trigonometry. In the *Almagest* spherical trigonometry is based on a theorem of Menelaus (late first century of the Christian era),¹¹ and there is no evidence for the existence of the trigonometry of the surface of the sphere before Menelaus. It was possible for Hipparchus to solve the problems he encountered in other ways, however, and there is considerable evidence that he did so.

The problem of the rising times of arcs of the ecliptic at a given latitude was usually connected in ancient astronomy with the length of daylight. The usual method of reckoning time in antiquity was to divide both the daylight and the nighttime into twelve hours. These "seasonal hours" (*ὥραι καιρικαί*) varied in length throughout the year, depending on the season and the latitude of the place. In order to convert them into hours of equal length ("equinoctial hours," *ὥραι ἰσημεριναί*), one needs to know the length of daylight on the date and at the place in question. This is given by the time it takes the 180° of the ecliptic following the longitude of the sun on that date to cross the horizon at that latitude (or, in spherical terms, the arc of the equator that rises with those 180°). For most purposes it is sufficient to know the rising times of the individual signs of the zodiac. This problem was solved in Babylonian astronomy by a simple arithmetical scheme that, although only approximately correct, produced remarkably good results.

If we number the rising of the signs of the ecliptic, beginning with Aries, $\alpha_1, \alpha_2, \dots, \alpha_{12}$ then, for the first six signs, it is assumed that the increment d between the rising time of a sign and the preceding sign is constant: $\alpha_2 = \alpha_1 + d, \alpha_3 = \alpha_2 + d = \alpha_1 + 2d$, and so on. The rising times of the signs of the second half of the ecliptic are equal to those of the corresponding signs of the first half according to the symmetry relations $\alpha_7 = \alpha_6, \alpha_8 = \alpha_5$, and so on. The length of the longest daylight, M , is the sum of the six arcs α_4 to α_3 inclusive; that of the shortest day, m , is the sum of the remaining six arcs. Thus, if the ratio M/m is given, the value of α_1 and d —and hence of all the rising times—can be computed arithmetically. For, in degrees of the equator, $M + m = 360^\circ, M = 6\alpha_1 + 24d, m = 6\alpha_1 + 6d$. In Babylonian astronomy the ratio M/m was always taken as $3/2$, which is approximately correct for Babylon. A contemporary of Hipparchus, [Hypsicles of Alexandria](#), in his extant work *'Αναφορικός* ("On Rising-times"), expounds exactly the same method, but for Alexandria, taking M/m as $7/5$. It seems that Hipparchus extended the scheme to a number of different latitudes (probably the seven standard "climata" characterized by longest daylights extending from thirteen hours to sixteen hours at half-hour intervals, which Hipparchus used in his geographical treatise), for Pappus mentions a work by Hipparchus entitled "On the Rising of the Twelve [Signs of the Zodiac](#)" (*Ἐν τῷ Περί τῆς τῶν ἡβ' ζῳδίων ἀναφορᾶς*) in which he proved a certain proposition "arithmetically" (*δι' ἀριθμῶν*).¹²

Other examples of the employment of arithmetical schemata for problems that would require spherical trigonometry if solved strictly can be inferred from Strabo's quotations from Hipparchus' geographical treatise. For instance, Hipparchus gave the following information: for the region where the longest daylight is sixteen hours, the maximum altitude of the sun above the horizon at winter solstice is nine cubits; for M =seventeen hours, it is six cubits, for M =eighteen hours, four cubits, and for M =nineteen hours, three cubits.¹³ The altitudes form a series with constant second-order difference. Strabo also excerpted from Hipparchus' treatise the distance in stades between Parallels with a given longest daylight. Here too the relationship is based on a constant difference, of third order.¹⁴ Thus problems involving the relationship between geographical latitude and the length of daylight, which Ptolemy solved by spherical trigonometry at *Almagest* III, 2-5, were solved arithmetically by Hipparchus in every case for which there is evidence.

It does not seem possible, however, that Hipparchus solved arithmetically every problem that would normally require spherical trigonometry. Moreover, he himself said that he had written a work enabling one to determine "for almost every part of the inhabited world" which fixed stars rise and set simultaneously.¹⁵ Theoretically one could solve such problems approximately by suitable manipulation of a celestial globe (see below on instruments). Hipparchus, however, stated that he had solved a particular problem of this type "in the general treatises we have composed on this subject [presumably the same as the above] geometrically" (*διὰ τῶν γογαμμῶν*)¹⁶. It is a plausible conjecture that at least one of the methods Hipparchus used for the solution of such problems was that known in antiquity as "analemma" and in modern times as descriptive geometry. This is best explained by an actual example, in which a numerical result given by Hipparchus is recomputed.¹⁷ In the commentary on Aratus he said that a certain star is $27-1/3^\circ$ north of the equator, and therefore $(15/24-1/20.1/24)$ of the parallel circle through that star is above the horizon.¹⁸ The horizon in question is Rhodes, with latitude $\phi=36^\circ$ the star's declination.

δ , is $27-1/3^\circ$. In Figure 1 the circle $ACDB$ represents the meridian at latitude 36° OT the trace of the local horizon, AB the trace of the equator, and CD the trace of the parallel circle on which the star lies. Half of the parallel circle CSD is then drawn (rotated through 90° to lie in the plane of the meridian). It is required to find arc $SC(=90^\circ+\omega)$, which is the portion of the parallel circle lying between meridian and horizon. The radius of the parallel circle, r_p , is computed by

and,

$$OQ = \text{Crd } 2\delta$$

Then

and in triangle STQ ,

giving $2\omega = 44-1/8^\circ$ and the part of the circle above the horizon as $224-1/8^\circ$ (Hipparchus' result was equivalent to $224-1/4^\circ$).

The method we assume here is not attested for Hipparchus, but his use of small circles (such as the parallel circle here) is alien to the spherical astronomy of the *Almagest*, which uses only great circles, while it is exactly parallel to the methodology of Indian astronomical texts in which the analemma is implicit.¹⁹ On the other hand, for computation of simultaneous risings and settings of fixed stars, stereographic projection would be more convenient; and we shall see, in connection with the astrolabe, that there is evidence that Hipparchus used stereographic projection too.

Theory of the Sun and Moon. Geometrical models that would in principle explain the anomalistic motion of the heavenly bodies had been developed before Hipparchus. Ptolemy attests the use of both epicycle and eccentric by Apollonius (*ca.* 200 B.C.)²⁰ and it is clear from his discussion that Apollonius was fully aware of the equivalence between the two models. Hipparchus used both models, and is known to have discussed their equivalence. What was new was (in all probability) his attempt to determine numerical parameters for the models on the basis of observations. We are comparatively well-informed about his solar theory, since Ptolemy adopted it virtually unchanged. Having established the lengths of the four seasons, beginning with the spring equinox, as $94-1/2$, $92-1/2$, $88-1/8$, and $90-1/8$ days, respectively, and assuming a single anomaly (in Greek terms, an eccentric with fixed apogee or the equivalent epicycle model), he was able to determine the eccentricity ($1/24$) and the position of the apogee (Gemini $5-1/2^\circ$) from the first two season lengths combined with a value for the length of the year.²¹ We do not know what value Hipparchus adopted for the latter (his treatise "On the Length of the Year," in which he arrived at the value $365-1/4-1/300$ days, belongs to the end of his career, after his discovery of precession, whereas he must have established a solar theory much earlier), but for this purpose the approximate value of $365-1/4$ days (established by Callippus in the fourth century b.c.) is adequate. Hipparchus presumably then constructed a solar table similar to (but not identical with) that at *Almagest* III, 6, giving the equation as a function of the anomaly; the astrological writer Vettius Valens claimed to have used Hipparchus' table for the sun.²²

In attempting to construct a lunar theory Hipparchus was faced with much greater difficulties. Whereas (in ancient theory) the sun has a single anomaly, the period of which is the same as that of its return in longitude (one year), for the moon one must distinguish three separate periods: the period of its return in longitude, the period of its return to the same velocity ("anomalistic month"), and the period of its return to the same latitude ("draconic month"). Related to the return in longitude and the solar motion is the "synodic month" (the time between successive conjunctions or oppositions of the sun and moon). Hipparchus enunciated the following relationships: (1) In 126,007 days, 1 hour, there occur 4,267 synodic months. 4,573

returns in anomaly, and 4,612 sidereal revolutions, less $7-1/2^\circ$ (hence the length of the mean synodic month is 29; 31, 50, 8, 20 days); (2) In 5,458 synodic months there occur 5,923 returns in latitude.

According to Ptolemy, *Almagest* IV, 2, Hipparchus established these relationships “from Babylonian and his own observations.” Had he in fact arrived at these remarkably accurate periods merely by comparison of eclipse data, he would indeed be worthy of our marvel. In fact, as F. X. Kugler showed,²³ all the underlying parameters can be found in Babylonian astronomical texts. The second relationship can be derived directly from those texts, while the first is a result of purely arithmetical manipulation of the following Babylonian parameters: (a) 251 synodic months= 269 anomalistic months; (b) 1 mean synodic month = 29; 31, 50, 8, 20 days; (c) 1 year= 12; 22. 8 synodic months. One finds the first relationship by

multiplying relationship (a) by 17. Hipparchus did this because he wanted to produce an eclipse period; and 17 is the smallest multiplier of (a) that will generate a period in which the moon can be near a node and the sun is in approximately the same position, at both beginning and end.²⁴ He wanted an eclipse period because he wished to confirm the Babylonian parameters by comparison of Babylonian and his own eclipse data; we can identify some of the eclipses that he used for this observational confirmation. Since one is the eclipse of 27 January 141 b.c., we have a *terminus post quem* for the establishment of the lunar theory.

The revelation of Hipparchus’ dependence on Babylonian sources raises the question of what material was available to him, and in what form. We can infer that he possessed a complete or nearly complete list of lunar eclipses observed at Babylon since the reign of Nabonassar (beginning 747 B.C.). This list was available to Ptolemy. Hipparchus also had a certain number of Babylonian observations of planets (see below). Furthermore, he had access to some texts (apparently unknown to Ptolemy) that gave the fundamental parameters listed above. It seems highly improbable that he derived them from the type of texts in which they have come down to us, the highly technical lunar ephemerides. Rather, they must have been excerpted and translated by someone in Mesopotamia who was well acquainted with Babylonian astronomical methods. But when and how the transmission occurred is unknown. Hipparchus is the first Greek known to have used this material. Without it his lunar theory, and hence his eclipse theory, would not have been possible.

To represent the lunar anomaly Hipparchus used a simple epicycle model in which the center of the epicycle C moves at constant distance about the earth O with the mean motion in longitude, while the moon M moves about the center of the epicycle with the mean motion in anomaly. To determine the parameters of this model (that is, the size of the radius of the epicycle r relative to the radius of the deferent R), he devised an ingenious method involving only the observation of the times of three lunar eclipses. By calculating the position of the sun at the three eclipses (presumably from his solar table), he found the true longitude of the moon at the middle of each eclipse (180° away from the true sun). From the time intervals between the three eclipses he found the travel in mean longitude and mean anomaly between the three points. In Figure 2 points M_1, M_2, M_3 represent the positions of the moon on the epicycle at the three eclipses. The angles α at the center of the epicycle are given by the travel in mean anomaly (modulo 360°); the angles δ at the earth are the equational differences, found by comparing the intervals in mean longitude with the intervals in true longitude. Then, by solving a series of triangles, he found, first r in terms of $OB = s$, then M_2B in terms of s and finally r in terms of R from $R^2 - r^2 = (R+r)(R-r) = s \cdot M_2O = s(s - M_2B)$.

Hipparchus performed this calculation twice, with two different sets of eclipses, using the epicycle model for one and the equivalent eccentric model for the other. Unfortunately the elegance of his mathematical approach was not matched by the accuracy of his calculations, so that carelessness in computing the intervals in time and longitude produced widely differing results from the two calculations, as Ptolemy demonstrates at *Almagest* IV, 11. The parameters that he found were $R:r = 3122-1/2:247-1/2$ for the epicycle model, and $R:e = 3144:327-2/3$ for the eccentric model.²⁵ We do not know how Hipparchus dealt with the discrepancy (he may have considered the possibility of a variation in the size of the epicycle), but we do know that he adopted the value $3122-1/2:247-1/2$ (which is distinctly too small for the epicycle) in his work “On Sizes and Distances.”

Ptolemy shows (*Almagest* V, 1-5) that a lunar model of the type developed by Hipparchus, with a single anomaly, works well enough at syzygies (oppositions and conjunctions) but that for other elongations of the moon from the sun, one must assume a second anomaly that reaches its maximum near quadrature (elongation of 90° or 270°). We can infer from material used here by Ptolemy that, toward the end of his career, Hipparchus had at least an inkling that his lunar theory was not accurate outside syzygies, and that he was systematically making observations of the moon at various elongations. Three of his latest observations that Ptolemy quotes (5 August 128 b.c., 2 May 127 b.c., and 7 July 127 b.c.) are of elongations of about 90° , 315° , and 45° respectively.²⁶ Hipparchus, however, seems never to have reached any firm conclusions as to the nature of the discrepancies with theory that he must have found; and it was left to Ptolemy to devise a model that would account for them mathematically.

In order to construct a theory of eclipses (one of the ultimate goals of his theory of the moon), Hipparchus had to take account of its motion in latitude. We have seen that he accepted a Babylonian parameter for the mean motion in latitude. He confirmed this by comparing two eclipses at as great an interval as possible.²⁷ He established the maximum latitude of the moon as 5° (*Almagest* V, 7) and devised a method of finding its epoch in latitude from an observation of the magnitude of an eclipse, combined with the data (obtained by measurement with the diopter; see below) that the apparent diameter of the moon at mean distance is $1/650$ of its circle (that is, $360^\circ/650$) and $2/5$ of the shadow.

Sizes, Distances, and Parallax of the Sun and Moon. These last data are connected with another topic on which Hipparchus wrote. In order to predict the circumstances of a solar eclipse, one must know the relative sizes and distances of the bodies

concerned: sun, moon, and earth (for lunar eclipses it suffices to know the apparent sizes; but in solar eclipses parallax, which depends on the distances of the moon and sun, is very important). Hipparchus devoted a treatise “On Sizes and Distances” (*Περὶ μεγέθων καὶ ἀποστημάτων*)²⁸ in two books, to the topic. By combining the remarks of Ptolemy and Pappus,²⁹ we can infer that Hipparchus proceeded as follows. By measurement with the diopter he had established the following data:

- (1) The moon at mean distance measures its own circle 650 times.
- (2) The moon at mean distance measures the earth’s shadow (at the moon) 2-1/2 times.
- (3) The moon at mean distance is the same apparent size as the sun.

He also had established by observation that the sun has no perceptible parallax. But from this he could deduce only that the sun’s parallax was less than a certain amount, which he set at seven minutes of arc.

In book I, Hipparchus assumed that the solar parallax was the least possible –that is, zero. He then derived the lunar distance from two observations of a solar eclipse (which can be identified as the total eclipse of 14 March 190 b.c.),³⁰ in which the sun’s disk was totally obscured near the Hellespont and four-fifths obscured at Alexandria. The assumption that the sun has zero parallax means that we can take the whole shift in the obscured amount of the sun’s disk (a fifth of its diameter) as due to lunar parallax. Assuming that the eclipse was in the meridian at both Alexandria and the Hellespont, we have the situation of Figure 3, where M represents the center of the moon, O the earth’s center, H the observer at the Hellespont, A the observer at Alexandria, Z the direction of the zenith at the Hellespont, and POQ the equator of the earth, the radius of which is r_{\oplus} . Since Hipparchus knew ϕ_H and ϕ_A , the latitudes of the Hellespont and Alexandria (about 41° and 31° respectively), and could find the moon’s declination δ at the time of the eclipse (about -3°) from his tables, he

could calculate the distance of the moon (OM in Figure 3):

The zenith distance of the moon at H , ζ , is approximately equal to ζ : and $\zeta = \phi_H - \delta \approx 44^\circ$.

μ is one-fifth of the apparent diameter of the sun, or $21600 / (5 \cdot 650)$ (from [1] and [3] above). From AH , ϑ , and μ the triangle AHM is determined (in terms of r): and we find

This is the distance of the moon at the time of the eclipse. To find the least distance of the moon, we have to reduce it by one or two earth radii. Hipparchus found 71 as the least distance. The small discrepancy is no doubt due to the approximations (in ϕ_H, ϕ_A , and ζ) made above. By applying the ratio $R:e = 3122-1/2:247-1/2$, derived from his lunar model, Hipparchus found the greatest distance as 83. The assumption that the eclipse took place in the meridian (which Hipparchus knew to be false) implies, however, that the distances must be greater than those computed (for as the moon moves away from the meridian, the angle ϑ , and hence D' , increases), so that 71 represents the minimum possible distance of the moon.

Whereas in book I, Hipparchus had assumed that the solar parallax was the least possible, in book II he assumed that it was the greatest possible (consistent with the fact that it was not great enough to be observed) –that is, $7'$. This immediately gives the solar distance, for since the angle is small, we can substitute the angle for the chord and say that the sun’s distance is $3438/7 \approx 490$. In Figure 4, S, M, O, U are the centers of the sun, moon, earth, and shadow, respectively, and $OU = OM$. From the similar triangles with bases UA, OB , and MD , it follows that

$$MD = 2OB - UA.$$

From (2) $UA = 21/2MC$.

Therefore $CD = MD - MC = 2OB - 31/2MC$

and $OB - CD = 31/2MC - OB$.

From (1), and substituting angles for chords,

And from the similar triangles OMC, OSE and OBE, CDE

Since $OS = 490 OB$.

Hipparchus found the mean distance of the moon as $67-1/3r$, and the distance of the sun as 490. These too were computed under an external assumption however: that the solar parallax was the maximum possible (the solar distance was the minimum possible). This in turn implies that the resultant lunar distance is a maximum, for it is easily seen from equation (4) that as the sun’s distance (490 in the expression) increases, the moon’s distance decreases. Moreover, it does not decrease indefinitely, for

as the sun's distance tends to infinity, the expression in (4) tends to the limit $(2 \cdot 650 \cdot 3438)/(3-1/2 \cdot 21600) \approx 59$. Although it is not explicitly attested, there is little doubt that Hipparchus realized this, and stated that the mean distance of the moon lay between the bounds $67-1/3$ and 59 earth radii³¹. He thus arrived at a value for the lunar mean distance that was not only greatly superior to earlier estimates but was also stated in terms of limits that include the true value (about 60 earth radii). His methods must be regarded as a tour de force in the use of crude and scanty observational data to achieve a result of the right order of magnitude.

Ptolemy criticized Hipparchus' procedure (unjustifiably, if the above reconstruction is correct), on the ground that it starts from the solar parallax, which is too small to be observed³². He himself used Hipparchus second method (with different data) but started from the lunar distance (derived from an observation of the moon's parallax), whence he found the solar distance. This modified procedure became standard in later astronomical works, and was used by Copernicus.

Hipparchus also computed the true sizes of sun and moon (relative to the earth), which are easily found from the distances and apparent diameters. Theon of Smyrna stated that Hipparchus said that the sun was about $1,880$ times the size of the earth, and the earth 27 times the size of the moon.³³ Since a lunar distance of about 60 earth radii implies that the earth's diameter is about 3 times the size of the moon's, Hipparchus must have been referring to comparative volumes. Hence the sun's diameter was, according to him, about $12-1/3$ times the earth's; and therefore (by the method of book II) the sun's distance was about $2,500$ earth radii and the moon's about $60-1/2$ earth radii. These may have been the figures he finally decided on for the purposes of parallax computation.

In any case, his investigation of the distances furnished Hipparchus with a horizontal parallax that, for the moon, was approximately correct. For computation of the circumstances of solar eclipses, and to correct observations of the moon with respect to fixed stars, he had to find the lunar parallax for a given lunar longitude and terrestrial latitude and time. This is a very unpleasant problem in spherical astronomy. We know that Hipparchus solved it; but we know very little about his solution, although it is certain that it was not carried out with full mathematical rigor. We can infer from a criticism made by Ptolemy³⁴ that Hipparchus wrote a work on parallax in at least two books. The details of Ptolemy's criticism are quite obscure to me; and the only safe inference is that in converting the total parallax into its longitudinal and latitudinal components, Hipparchus treated spherical triangles as plane triangles (some of these triangles are too large for the procedure to be justifiable, but one can apply the same criticism to Ptolemy).³⁵ The actual corrections for parallax that Hipparchus is known to have applied to particular lunar observations are, however, reasonably accurate.³⁶ One might guess that the methods of computing parallax found in Indian astronomical texts (which are quite different from Ptolemy's) are related to Hipparchus' procedures, but at present this is mere speculation.

Eclipses. Mastery of the above topics enabled Hipparchus to predict eclipses of both sun and moon for a given place: and we may presume, although there is no specific evidence, that he compiled eclipse tables for this purpose. The elder Pliny tells us that he predicted eclipses of the sun and moon for a period of 600 years³⁷; and this has been taken seriously by some modern scholars, who envisage Hipparchus producing a primitive "Oppolzer." Although utterly incredible as it stands, the story must have some basis; and O. Neugebauer makes the plausible suggestion that what Hipparchus did was to arrange the eclipse records available to him from Babylonian and other sources (which, as we have seen, covered a period of 600 years from Nabonassar to his own time) in a form convenient for astronomical use. If this was so, we can explain more easily how Ptolemy was able to select eclipses with very special circumstances to suit a particular demonstration³⁸: he was using material already digested by Hipparchus.

Pliny provides another valuable piece of information about Hipparchus' work on eclipses. He says that Hipparchus discovered that lunar eclipses can occur at five-month intervals, solar eclipses at seven-month intervals, and solar eclipses at one-month intervals—but in the last case only if one eclipse is seen in the northern hemisphere and the other in the southern.³⁹ Hipparchus, then, had discussed eclipse intervals; and we see from Ptolemy's discussion of the topic at *Almagest* VI, 5-6, how Hipparchus must have approached the problem. Babylonian astronomers were aware that lunar eclipses could occur at intervals of five synodic months as well as the usual six. Ptolemy showed that five-month intervals are possible for lunar eclipses but seven-month intervals are not, that both five- and seven-month intervals are possible for solar eclipses, and that one-month intervals are not possible for solar eclipses "in our part of the world." Hipparchus must have proved all this, and also that solar eclipses at one-month intervals are possible for different hemispheres. The essential elements in the proof (besides the apparent sizes of sun, moon, and shadow) are parallax (for solar eclipses) and the varying motion of the moon in latitude in a true synodic month. It is to this topic that we must refer the work "On the Monthly Motion of the Moon in Latitude" (*Περί τῆς κατά Πλάτος μηνιαίας τῆς σελήτ κινήσεως*), the title of which is preserved in the *Suda*.⁴⁰ Presumably Achilles Tatius is referring to Hipparchus' work on eclipse intervals when he names him as one of those who have written treatises on solar eclipses in the seven "climata,"⁴¹ for the geographical latitude is one of the most important elements in the discussion.

Fixed Stars. Hipparchus devoted much time and several works to topics connected with the fixed stars. His observations of them must have extended over many years. On these points there is general agreement among modern scholars. On all other points confusion reigns, although Hipparchus' sole surviving work, "Commentary on the Phaenomena of Aratus and Eudoxus" (*Τὼν Ἀράτων καὶ Εὐδόξου φαινομένων ἐξηγησις*), in three books, is concerned with the fixed stars. Nevertheless, it is possible, if we look at the evidence without formulating prior hypotheses, to come to some conclusions.

In the mid-fourth century B.C., Eudoxus wrote a pioneering work naming and describing the constellations. This is now lost (Hipparchus is the main source of our knowledge of it). In the early third century B.C., Aratus wrote a poem based on

Eudoxus, called “Phaenomena,” which became immensely popular and is still extant. Not long before Hipparchus a mathematician, Attalus of Rhodes, wrote a commentary on Aratus (now lost). Hipparchus’ treatise is a critique of all three works. None of the three contained any mathematical astronomy, only descriptions of the relative positions of stars, simultaneous risings and settings, and the like; and much of Hipparchus’ criticism in books 1 and 2 is of the same qualitative kind. But even from this, one can see that he had fixed the positions of a number of stars according to a mathematical system (he incidentally notes some polar distances, declinations, and right ascensions). The last part of book 2 and the whole of book 3 are devoted to his own account of the risings and settings of the principal constellations for a latitude where the longest day is 14-1/2 hours. Whereas his predecessors had merely reported the stars or constellations that rise and set together with a given constellation, Hipparchus gave the corresponding degrees of the ecliptic (for risings, settings, and culminations). At the end of book 3 is a list of bright stars that lie on or near twenty-four-hour circles, beginning with the hour circle through the summer solstice. Hipparchus says that the purpose of this is to enable one to tell the time at night when making astronomical observations.

In this treatise Hipparchus indicates the position of a star in various ways. We have already mentioned declination (which he calls “distance from the equator along the circle through the pole”) and polar distance (the complement of declination). He, frequently uses an odd form of right ascension. Thus he says that a star “occupies three degrees of Leo along its parallel circle” This means that each small circle parallel to the equator is divided into twelve “signs” of 30°, and thus the right ascension of the star is 123°. The assignment of the stars to the hour circles at the end of book 3 is also a form of right ascension. Besides these equatorial coordinates, we find in the section on simultaneous risings and settings a mixture of equatorial and ecliptic coordinates: Hipparchus names the point of the ecliptic that crosses the meridian together with a given star. In other words, he gives the point at which the declination circle through the star cuts the ecliptic. It is significant that this, the “polar longitude,” is one of the standard coordinates for fixed stars in Indian astronomical texts. There are no purely ecliptic coordinates (latitude and longitude) in Hipparchus’ treatise.

Far from being a “work of his youth” as it is frequently described, the commentary on Aratus reveals Hipparchus as one who has already compiled a large number of observations, invented methods for solving problems in spherical astronomy, and developed the highly significant idea of mathematically fixing the positions of the stars (Aristyllus and Timocharis recorded a few declinations in the early third century, but we know of nothing else before Hipparchus). There is no hint of a knowledge of precession in the work; but, as we shall see, that discovery falls at the end of his career. The treatise is probably subsequent to Hipparchus’ catalog of fixed stars.

The nature of this catalog (presumably “On the Compilation of the Fixed Stars and of the [?] Catasterisms”) to which the *Suda* refers⁴² is puzzling. The worst excess of the modern notion of the strict dependence of the *Almagest* on Hipparchus is the belief that we can obtain Hipparchus’ catalog simply by taking Ptolemy’s catalog in *Almagest* VII and VIII and lopping 2-2/3° (to account for precession) off the longitudes. This was conclusively refuted by Vogt, who showed by a careful analysis of the coordinates of 122 stars derived from the commentary on Aratus that in almost every case there was a significant difference between Hipparchus’ and Ptolemy’s data. The most explicit statement about Hipparchus’ star catalog is found in Pliny, who says that Hipparchus noticed a new star and, because it moved, began to wonder whether other fixed stars move; he therefore decided to number and name the fixed stars for posterity, inventing instruments to mark the positions and sizes of each.⁴³ This confused notice could be a description of a star catalog like Ptolemy’s, but it could equally well refer to an account of the number and relative positions of the stars in each constellation (without coordinates). In fact Ptolemy, wishing to show that the positions of the fixed stars relative to each other had not changed since the time of Hipparchus, reported a number of star alignments that Hipparchus seemed to have recorded for just the purpose alleged by Pliny; to allow posterity to determine whether the fixed stars have a [proper motion](#).⁴⁴ Naturally no coordinates were given.

Apparent excerpts from Hipparchus’ star catalog that are found in late Greek and Latin sources merely give the total number of stars in each constellation. These suggest that Hipparchus counted a large number of stars,⁴⁵ but they do not prove that he assigned coordinates to all—or, indeed, to any. Ptolemy quoted the declinations of a few bright stars mentioned by Hipparchus: but in every case he compared the declination with that recorded by Aristyllus and Timocharis, in order to determine the precession. These declinations may well have been taken not from Hipparchus’ catalog but from one of his works on precession. There is one text, however, that does appear to preserve coordinates from Hipparchus’ star catalog. In late Latin scholia on Aratus⁴⁶ there are found, for the circumpolar constellations, polar distances and what must be interpreted as polar longitudes, which are approximately correct for the time of Hipparchus. Whether Hipparchus gave such coordinates for every star in the catalog or only for some selected stars remains uncertain. The only thing we know for certain about the catalog is that the coordinates employed were not latitude and longitude: Ptolemy (*Almagest* VII, 3) would not have chosen such a roundabout way (conversion of declinations) to prove that latitudes of fixed stars remain constant, had he been able to cite the latitudes from Hipparchus’ catalog. It is a plausible conjecture that the star coordinates given in the commentary on Aratus are taken directly from the catalog; at the very least, they are a good indication of the ways in which the position of a star was described in that work.

Precession and the Length of the Year . Hipparchus is most famous for his discovery of the “[precession of the equinoxes](#),” the slow motion of the solstitial and equinoctial points from east to west through the fixed stars. This topic is intimately connected with the length of the year, for precession implies both that the coordinates of the fixed stars (such as right ascension and declination) change over a period of time, and also that the tropical year (return of the sun to the same equinox or solstice) is shorter than the sidereal year (return of the sun to the same star). We do not know what phenomenon first led Hipparchus to suspect precession, but we do know that he confirmed his suspicion by both the above approaches. According to Ptolemy, the

first hypothesis that he suggested was that only the stars in the zodiac move.⁴⁷ Later, in “On the Change in Position of the Solstitial and Equinoctial Points” (*Περί τῆς καταπλάτος μηνιαίας τῆς σελησημείων*) he formulated the hypothesis that all the fixed stars move with respect to the equinoxes (or, rather, vice versa). He supported this hypothesis in two ways. First he compared the distance of the star Spica (αVirginis) from the autumnal equinox in the time of Timocharis (observations of 294 and 283 B.C.) and his own time. Unfortunately this involved considerable uncertainty, since Timocharis’ observations were simply occultations of Spica by the moon, which had to be reduced to longitudes by means of the lunar theory; and Hipparchus’ own observations of elongations of Spica from the moon at two lunar eclipses led to results differing by 3/4°. ⁴⁸ He concluded that the longitudes of Spica had increased by about 2° in the 160-odd years since Timocharis, but he was well aware that his data were too shaky to allow any confidence in the precise amount.

Hipparchus’ other approach was to try to find the length of the tropical year. For this purpose he set out a series of equinox observations (listed by Ptolemy at *Almagest* III. 1) ranging from 162 to 128 b.c. (the latter date is probably close to that of the composition of the book, as is confirmed by Hipparchus’ observation of the longitude of Regulus in the same year [*Almagest* VII, 2]). Unfortunately, these seemed to indicate a variation in the length of the year, which Hipparchus evidently was prepared to consider at this point. We do not know what conclusion, if any, he reached in this work; but he must already have assumed some value for the length of the sidereal year (see below), for otherwise there would have been no point in investigating the length of the tropical year in order to determine the amount of precession. Hipparchus reverted to the topic in a later work. “On the Length of the Year” *Περί ἐνιαυσίου μεγέθους*. In this he came to more definite conclusions. He determined that the tropical and equinoctial points move at least 1 /100° a year backward through the signs of the ecliptic, and that the length of the tropical year is 365-1/4 days, less at least 1/300 of a day.⁴⁹ These two formulations are of course related. The basis of his proof was comparison of the solstice observed by himself in 135 b.c. with those observed by Aristarchus in 280 b.c. and Meton in 432 b.c. Having thus found a (maximum) value for the tropical year, Hipparchus subtracted it from his value for the sidereal year, thus producing a (minimum) value for precession.⁵⁰ From his figures for the tropical year and precession we can deduce the value he assigned to the sidereal year: 365-1/4 + 1/144 days, a very accurate estimate. There is independent evidence that he used this value, and that it is of Babylonian origin. In “On the Length of the Year” Hipparchus also assumed (correctly) that precession takes place about the poles of the ecliptic, and not of the equator, but still expressed uncertainty on that matter. He also realized that one must define the solar year as the tropical year.⁵¹

Having established the length of the tropical year, Hipparchus wrote “On Intercalary Months and Days” (*Περί εμβολίμων μηνῶν τε καίημερῶν*) in which he proposed a lunisolar intercalation cycle that was a modification of the Callippic cycle:⁵² it contained 304 tropical years, 112 with 13 synodic months and 292 with 12, and a total of 111,035 days. This produces very good approximations to Hipparchus’ values for both the tropical year and the mean synodic month. There is no evidence that it was ever used, even by astronomers.

Planetary Theory. Ptolemy tells us (*Almagest* IX.,2) that Hipparchus renounced any attempt to devise a theory to explain the motions of the five planets, but contented himself with showing that the hypotheses of the astronomers of his time could not adequately represent the phenomena. We may infer from Ptolemy’s discussion that Hipparchus showed that the simple epicycle theory propounded by Apollonius produced constant arcs of retro-gradation, whereas observation showed that they vary in length. Ptolemy further informs us that Hipparchus assembled observations of the planets “in a more convenient form.” He mentions Hipparchus specifically in connection with an observation of Mercury of 262 B.C. according to the strange “Dionysian era,”⁵³ and it is probable that all observations in that era preserved in the *Almagest* are derived from his collection; the same may be true of Babylonian observations of Mercury and Saturn according to the Seleucid era. Ptolemy mentions no planetary observations made by Hipparchus himself, and it is probable that he made few. Ptolemy does, however, ascribe to him the following planetary period relations (*Almagest* IX, 3);

Saturn in 59 years makes 57 revolutions in anomaly and 2 in the ecliptic.

Jupiter in 71 years makes 65 revolutions in anomaly and 6 in the ecliptic.

Mars in 79 years makes 37 revolutions in anomaly and 42 in the ecliptic.

Venus in 8 years makes 5 revolutions in anomaly and 8 in the ecliptic.

Mercury in 46 years makes 145 revolutions in anomaly and 46 in the ecliptic.

These are well-known relations in Babylonian astronomy, occurring in the so-called “goal-year” texts.

The only other indication of Hipparchus’ interest in the planets is a passage in Ptolemy’s *Planetary Hypotheses*⁵⁴ reporting an attempt to determine minimum values for the apparent diameters of the heavenly bodies. Hipparchus stated that the apparent diameter of the sun is thirty times that of the smallest star and ten times that of Venus.

Instruments. The only observational instrument that Ptolemy specifically ascribes to Hipparchus is the “four-cubit diopter” with which he observed the apparent diameters of the sun and moon and arrived at the estimates given above (*Almagest* V. 14). The instrument is described by Pappus and Proclus.⁵⁵ It consisted of a wooden rod of rectangular cross section some six feet in length. The observer looked through a hole in a block at one end of the rod, and moved a prism that slid in a groove along the

top of the rod until the prism exactly covered the object sighted. The ratio of the breadth of the prism to its distance from the sighting hole gave the chord of the apparent diameter of the object.

As J. B. J. Delambre remarked,⁵⁶ certain observations of Hipparchus' seem to imply use of the "armillary astrolabe" described by Ptolemy at *Almagest* V, 1, and used by him to observe the ecliptic distance between heavenly bodies, particularly between sun and moon. These are observations of the elongation of the moon from the sun reported in *Almagest* V, 5. In particular, the observation of 7 July 127 B.C.⁵⁷ seems to indicate that Hipparchus observed not merely the elongation but also, and at the same time, the ecliptic position of the sun (which is given by the shadow on the armillary astrolabe). It seems odd, however, that Ptolemy should describe the instrument so carefully (and claim it as his own) if in fact it were old and well known. Furthermore, for one of the observations Ptolemy says Hipparchus used instruments (plural).⁵⁸ It is possible that Hipparchus used one instrument to determine the sun's position and another to measure the elongation, but use of the astrolabe cannot be excluded.

Hipparchus determined the times of equinox and solstice, and perhaps measured the inclination of the ecliptic, confirming Eratosthenes' estimation that the distance between the solstices was 11/83 of the circle.⁵⁹ For these observations he must, like Ptolemy, have measured the altitude or zenith distance of the sun at meridian transit. Much of his data on fixed stars also seem to imply observation of the meridian altitude. We can only speculate about the instrument(s) he used. One might conjecture that one was a primitive form of the diopter described by Hero, which could be used for both meridian and elongation observations.

An incidental remark of Ptolemy's implies that Hipparchus (like Ptolemy at *Almagest* VIII, 3) gave instructions for constructing a celestial globe and marking the constellations on it.⁶⁰ Such globes existed before Hipparchus, probably as early as the fourth century B.C., but more as artistic than scientific objects. Presumably Hipparchus mentioned the globe in connection with his star catalog. If the stars were marked on it according to a coordinate system, and if, like Ptolemy's, it were furnished with a ring indicating the local horizon to which its axis of rotation was inclined at the correct angle, it could be used not merely for demonstration purposes but also to read off simultaneous risings and settings directly.

There is evidence that Hipparchus invented the plane astrolabe. Our authority for this, Synesius, is late (a.d. 400) and his account confused. But the whole necessary theory of stereographic projection is set out in Ptolemy's *Planisphaerium*; and use of such projection would explain, among other things, how Hipparchus computed simultaneous risings and settings. The balance of probability is that Hipparchus used (and perhaps invented) stereographic projection. If that is so, there is no reason to deny his invention of the plane astrolabe, which provides an easy solution to the problems of the rising times of arcs of the ecliptic and simultaneous risings of stars at a given latitude.

Geography . Hipparchus wrote a work in (at least) three books entitled "Against the Geography of Eratosthenes" (*Πρός την Ἐρατοσθένους γεωγραφίαν*). In the mid-third century Eratosthenes had given a description of the known world, in which he attempted to delineate crudely the main outlines of a world map by means of imaginary lines (including, but not restricted to, meridians and parallels of latitude) drawn through fixed points, and to establish distances along such lines in absolute terms (stadia). This was related to astronomical geography in the sense that Eratosthenes had made an estimate of the circumference of the earth (252,000 stadia), and had taken some account of the ratios of the shadow to the gnomon at various latitudes. Hipparchus' work was a detailed criticism of Eratosthenes' data, showing that the distances and relationships given were inconsistent with each other and with other geographical data. He evidently supplied numerous corrections and supplements to Eratosthenes; but, as we have seen, his astronomical geography is based on simple arithmetical schemes, and although he enunciated the principle that longitudes should be determined by means of eclipses,⁶¹ this was merely an expression of an ideal. The only evidence that Hipparchus used astronomical observations to improve geographical studies is the statement of Ptolemy (*Geography* 1, 4) that he gave the latitudes of "a few cities." Nor does he seem to have contributed anything to mathematical geography, which emerges from infancy only with the work of Marinus of Tyre and Ptolemy.

Other Works . Extant astrological works refer to Hipparchus as an authority on astrology; but we know almost nothing about the content of his writings on this subject, except that the topics attested are all well-worn astrological themes, such as "astrological geography," in which the various regions of the world are assigned to the influence of a zodiacal sign or part of a sign.⁶² His work on weather prognostication from the risings and settings of fixed stars, known from numerous citations in Ptolemy's "Phases of the Fixed Stars," also belonged to a traditional genre of Greek "scientific" literature. Chance quotations mention a work on optics,⁶³ in which he endorsed the theory of vision that "visual rays" emanate from the eye; a book "On Objects Carried Down by Their Weight" (*Περί τῶν διάβαρνητα κατωφερομένων*)⁶⁴ and a work on combinatorial arithmetic. The last is almost the only reference we have to the topic in antiquity, but this merely illustrates how little we know of Greek mathematics outside the "classical" domain of geometry.

Both in antiquity and in modern times Hipparchus has been highly praised and misunderstood. Since Delambre modern scholarship has tended to treat Hipparchus as if he had written a primitive *Almagest*, and to extract his "doctrine" by discarding from the extant *Almagest* what are thought to be Ptolemy's additions. This unhistorical method has obscured Hipparchus' real achievement. Greek astronomy before him had conceived the idea of explaining the motions of the heavenly bodies by geometrical models, and had developed models that represented the motions well qualitatively. What Hipparchus did was to transform astronomy into a quantitative science. His main contributions were to develop mathematical methods enabling one to use the geometrical models for practical prediction, and to assign numerical parameters to the models. For the second, his use of observations and of constants derived from Babylonian astronomy was crucial (without them his lunar theory would not

have been possible). His own observations were also important (he and Ptolemy are the only astronomers in antiquity known to have observed systematically). Particularly remarkable is his open-mindedness, his willingness to abandon traditional views, to examine critically, and to test by observation his own theories as well as those of others (Ptolemy's most frequent epithet for him is, *(φιλαληθης)* "lover of truth"). Remarkable too, in antiquity, was his attitude toward astronomy as an evolving science that would require observations over a much longer period before it could be securely established. Related to this were his attempts to assemble observational material for the use of posterity.

Hipparchus acquired a high reputation in antiquity (he was extravagantly praised by the elder Pliny, and later was depicted on the coins of his native city), but it seems likely that his works were little read. From Ptolemy's citations, one can see why. They were in the form of numerous writings discussing different, often highly specialized, topics. It seems that Hipparchus never gave an account of astronomical theory, or even part of it, starting from first principles (Ptolemy often inferred Hipparchus' opinion on a topic from incidental remarks). Hipparchus did not, then, construct an astronomical system: he only made such a system possible and worked out some parts of it. When Ptolemy, using Hipparchus' work as one of his essential foundations, did construct such a system, which became generally accepted, interest in Hipparchus' works declined further. It is not surprising that all are lost except the commentary on Aratus (which survived because of the popularity of Aratus' poem). Pappus, in the fourth century, still knew some works by Hipparchus, but there is no certain instance of firsthand knowledge of the lost works after him (the astrological treatise by Hipparchus referred to in Arabic sources was certainly pseudonymous). In Western tradition, then the influence of Hipparchus was channeled solely through the *Almagest*. But there is much evidence that Indian astronomy of the *Siddhāntas* preserves elements of Hipparchus' theories and methods. These presumably come from some pre-Ptolemaic Greek astronomical work(s) based in part on Hipparchus (there are elements in Indian astronomy, notably in planetary theory, that are also of Greek origin but cannot have been derived from Hipparchus). The task of analyzing the Indian material from this point of view has hardly begun.

NOTES

1. For instance. *Suidae Lexicon*, Ada Adler, ed., II (Leipzig, 1931), 657, no. 521; Aelian, *De natura animalium*, R. Hercher, ed. (Leipzig, 1864), 175.

2. *Almagest* III, I, Manitius ed., I, 142.

3. *Ibid.*, V, 5, Manitius ed., I, 274.

4. H.C.F.C. Schjellerup, "Recherches sur l'astronomie..." 38-39 (-140); H. Vogt, "Versuch einer Wiederherstellung..." cols. 25, 31-32 (different dates from different data: -138, -150, -130, -156).

5. The three eclipses of 201-200 B.C. (*Almagest* IV, II, Manitius ed., I, 251-252) are obviously too early to have been seen by Hipparchus. For the equinox of 24 March 146 B.C. (*Almagest* III, I, Manitius ed., I, 135), the observation made on the ring in Alexandria is stated to have differed by five hours from that made by Hipparchus himself.

6. See note 1. The text as printed by Hercher says that it was "Hieron the tyrant" who was amazed; presumably this is meant to be the tyrant of Syracuse in the early fifth century. This is, however, Valesius' emendation of "Nero the tyrant," which is equally impossible chronologically.

7. See the contributions of A. Aaboe, O. Neugebauer, N. Swerdlow, and G. J. Toomer in the bibliography. A notable exception from an earlier period is the article by vogt in the bibliography.

8. A. Rome, *Commentaires... sur l'Almageste*, (Rome, 1931), 451. The expression (*η Πραγματεία τῶν ἐν κκλῶ ἐνθεῶν*) (treatment of straight lines in a circle": that is, chords) is not the title of the work, as is commonly supposed, but Theon's description of its contents, repeating verbatim the words Ptolemy applied to his own treatment (*Almagest* I, 9, Heiberg ed., I, 31, II, 4-5). For a conjecture about the origin of Theon's incredible statement that Hipparchus' treatment was in twelve books see G. J. Toomer, "The Chord Table of Hipparchus..." 19-20.

9. Ptolemy, *Geography*, I, 20, C. F. A. Nobbe, ed., I (Leipzig, 1843), 42, II. 26-30; Pappus, *Collectio* VI, F. Hultsch, ed., II (Berlin, 1877), 546, II. 24-27. On this see G.J. Toomer, in *Zentralblatt für Mathematik*, **274** (1974), 7 (author's summary of article, "The Chord Table of Hipparchus...").

10. For a refutation of the argument of J. Tropfke that Archimedes had already constructed a chord table, see Toomer. "The Chord Table of Hipparchus..." 20-23.

11. See G.J. Toomer. "Ptolemy," *DSB*. XI. 189.

12. Pappus, *Collectio* VI, Hultsch ed., 600, II. 10-11. The phrase "arithmetically" is implicitly opposed to "geometrically" (*δια τῶν γοραμμῶν*), and in this context implies that Hipparchus was using not Euclidean geometry but Babylonian methods.

13. Strabo. *Geography* 75 and 135, H. L. Jones, ed., I, 282, 514-516. A “cubit” in the astronomical sense is 2°; it is known from cuneiform texts, and its use here and elsewhere by Hipparchus is another indication of his heavy debt to Babylonian astronomy.
14. See O. Neugebauer, *History of Ancient Mathematical Astronomy*, I , 304-306.
15. *Commentary on Aratus* II , 4, Manitius ed., 184. Ptolemy explains how to calculate simultaneous risings and settings trigonometrically at *Almagest* VIII , 5.
16. *Ibid.*, 2, Manitius ed., 150, II. 14-17.
17. See Neugebauer, *op. cit.*, 301-302.
18. *Commentary on Aratus* II , 2, Manitius ed., 148-150.
19. See, for instance, O. Neugebauer and D. Pingree, *The Pancasiddhāntikā of Varāhamihira (Kongelige Danske Videnskabernes Selskab. Historisk-Filosofiske Skrifter*, 6, no.1), II (Copenhagen, 1970), 41-42, 85. Use of the analemma is not attested before the first century B.C. (Diodorus of Alexandria) and is mainly, but not exclusively, associated with sundial theory in extant Greek texts (notably Ptolemy’s *Analemma*). I believe that it was invented before Hipparchus for purely graphical purposes in sundial theory, and adapted by him to numerical solutions in spherical astronomy (whence it is found in Indian astronomy). But it must be admitted that this is all conjectural.
20. *Almagest* XII, 1. For a detailed discussion of the theories and their equivalence see Toomer, “Ptolemy,” 190.
21. *Ibid.*, 190-191. It is likely that Hipparchus found only the first two season lengths by observation: the last two were then derived from the eccentric model after he had calculated the eccentricity.
22. Vettius Valens, *Anthologiae*, W . Kroll, ed. (Berlin, 1908), 354.
23. Kugler, *Die Babylonische Mondrechnung*, 21, 24, 40, 46, 108.
24. On all this see A. Aaboe, “On the Babylonian Origin of Some Hipparchian Parameters.”
25. For details see Toomer, “The Chord Table of Hipparchus...,” 7-16.
26. *Almagest* V.3, Manitius ed., I, 266; V, 5, Manitius ed., I, 271, 274.
27. *Ibid.*, VI, 9, Manitius ed., I. 394-395.
28. On the exact title of the work see G.J. Toomer, “Hipparchus on the Distances of the Sun and Moon,” 126 n.l.
29. *Almagest* V, 11, Manitius ed., I, 294-295; Rome, *op. cit.*, 67-68
30. For the identification see Toomer, “Hipparchus on the Distances of the Sun and Moon,” 131-138.
31. If so, it can hardly be a coincidence that Ptolemy establishes the mean distance of the moon as exactly 59 . See *ibid.*, 131, n. 25.
32. *Almagest* V.11. Manitius ed., I, 294-295.
33. Theon of Smyrna, E. Hiller, ed. (Leipzig, 1878), 197.
34. *Almagest* V. 19, Manitius ed., I, 329-330.
35. Pappus’ “explanation” of this passage, Rome *op cit.*, 150-155, appears to be a completely fictitious reconstruction. I therefore ignore it here and in the account of Hipparchus’s trigonometry (it is not credible that Hipparchus took the sides of large spherical triangles proportionate to the opposite angles).
36. Notably that for his observation of 2 May 127 B.C. (*Almagest* V, 5, Manitius ed., I, 271).
37. Pliny, *Naturalis historia* II , 54, Beaujeu ed., 24.

38. For instance. *Almagest* **IV** , 9; **VI** , 5.
39. Pliny, *op. cit.*, **II** . 57, Beaujeu ed., 25.
40. See note 1.
41. Maass, *Commentariorum in Aratum reliquiae*, 47.
42. See note 1. The Greek title is (*Περὶ τηστῶν ἀπλανῶνα Πλανῶν καὶ τὸν κατμαυριγμον*) (the last word should perhaps be emended to *καταστευασμον*).
43. Pliny, *op. cit.*, 11, 95. Beaujeu ed., 41-42. The “new star” is commonly identified with a nova Scorpii of 134 B.C., alleged from Chinese records. The identification is of the utmost uncertainty (Pliny seems rather to refer to a comet).
44. *Almagest* **VII**, **11**. Manitius ed., **II**. 5-8.
45. About 850, according to F. Boll, “Die Sternkataloge des Hipparch...,” 194; but his calculations rest on no source basis.
46. Maass, *op. cit.*, 183-189.
47. *Almagest* **VII**, 1, Manitius ed., **II**. 4. We do not know to what work of Hipparchus he refers.
48. *ibid* **III**, **I**, Manitius ed., **I**. 137. Compare *Ibid.*, **VII**. 2. Manitius ed., **II**. 12. where Hipparchus seems to have settled on a mean value between his two discrepant results.
49. *Ibid.*, **III**. **I**, Manitius ed., **I**. 145 (this quotation is in fact from the later work “On Intercalary Months and Days”). and **VII**. 2. Manitius ed., **II**. 15.
50. That he did so is certain from his calculation of the precession over 300 years (that is, between his time and Meton’s), which Ptolemy reports from this work at *Almagest* **VII**. 2. Manitius ed., **II**. 15.
51. Very interesting verbatim quotation of Hipparchus at *Almagest* **III**. **I**. Manitius ed., **I**. 145-146.
52. See G. J. Toomer. “Meton,” DSB IX, 337-338. The details of the cycle are known from Censorinus. *De die natali* 18. O Jahn. ed. (Berlin. 1845). 54.
53. *Almagest* **IX**. 7. Manitius ed., **II** . 134
54. Goldstein ed., 8. It is not clear whether the values for the apparent diameters of the other planets that Ptolemy adopts are also taken from Hipparchus.
55. Proclus, *Hypotyposis*..., Karl Manitius. ed. (Leipzig, 1909). 126- 130; Rome, *op. cit.*, 90-92. The latter appears more reliable.
56. *Histoire de l’astronomie ancienne* .**I**, 117, 185; **II** . 185.
57. *Manitius ed.*, 1.274-275.
58. That of 2 May 127 B.C.. Manitius. ed., **I**. 271.
59. *Almagest* **I**. 12. Manitius ed., 1.44.
60. *ibid.*, **VII**. 1. Heiberg ed., **II**. **II**. **II** 23-24: *ταῖς κατατόντων Ππαρον τησσεας σφαίρας αστερωμόνδιαν Πωσεωσ.*
61. Strabo. *Geography* 7. H.L. Jones, ed., **I**. 22-24.
62. Hephaestion, *Apotelesmatica*. D. Pingree. ed., I (Leipzig, 1973), 4.22.
63. *Doxographi Graeci* H. Diels, ed. (Berlin. 1879), 404.
64. Simplicius, *Commentary on Aristotle’s De caelo*. J. L. Heiberg. ed. (Berlin. 1894). which is *Commentaria in Aristotelem Graeca* **VII**, 264-265.

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