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(*b*. Chios; *fl*. Athens, second half of the fifth century b.c.)

mathematics, astronomy.

The name by which Hippocrates the mathematician is distinguished from the contemporary physician of Cos[1] implies that he was born in the Greek island of Chios; but he spent his most productive years in Athens and helped to make it, until the foundation of Alexandria, the leading center of Greek mathematical research. According to the Aristotelian commentator John Philoponus, he was a mercahnt who lost all his property through being captured by pirates.[2] Going to Athens to prosecute them, he ws obliged to stay a long time. He attended lectures and became so proficient in geometry that he tried to square the circle. Aristotle's own account is less flatering[3]. It is well known, he observes, that persons stupid in one respect are by no means so in others. "Thus Hippocrates, though a competent geometer, seems in other respects to have been stupid and lacking in sense; and by his simplicity, they say, he was defrauded of a large sum of money by the customs officials at Byzantium." Plutarch confirms that Hippocrates, like Thales, engaged in commerce[4]. The "Eudemian summary" of the history of geometry reproduced by Proclus states that Oenopides of Chios was somewhat younger than Anaxagoras of Clazomenae; and "after them Hippocrates of Chios, who found out how to square the lune, and Theodore of Cyrene beame distinguished in geometry. Hippocrates is the earliest of those who are recorded as having written Elements."[5] Since Anaxgoras was born about 500 b.c. and Plato went to Cyrene to hear Theodore after the death of Socrates in 399 b.c., the active life of Hippocrates may be placed in the second half of the fifth century b.c. C. A. Bretschneider has pointed out that the accounts of Philoponus and Aristotle could be reconciled by supposing that Hippocrates' ship was captured by Athenian pirates during the Samian War of 440 b.c., in which Byzantium took part[6].

Paul Tannery, who is followed by Maria Timpanaro Cardini, ventures to doubt that Hippocrates needed to learn his mathematics at Athens.[7] He thinks it more likely that Hippocrates taught in Athens what he had already learned in Chios, where the fame of Oenopides suggests that there was already a flourishing school of mathematics. Pointing out the Pythagoras, Timpanaro Cardini makes a strong case for regarding Hippocrate as coming under Pythagorean influence even though he had no Pythagorean teacher in the formal sense. Although lamblichus does not include Hippocrates' name in his catalog of Pythagoreans, he, like Eudemus, links him with Theodore, who was undoubtedly in the brotherhood.[8]

Mathematics, he notes, advanced after it had been published; and these two men were the leaders. He adds that mathematics came to be divulged by the Pythagoreans in the following way: One of their number lost his fortune, and because of this tribulation he was allowed to make money by teaching geometry. Although Hippocrates is not named, it would, as Allman points out, accord with the accounts of Aristotle and Philoponus if he were the Pythagorean in question.[9] The belief that Hippocrates stood in the Pythagorean tradition is supported by what is known of his astronomical theories, which have affinities with those of Pythagoras and his followers. He was, in Timpanaro Cardini's phrase, a para-Pythagorean, or, as we might say, a fellow traveler.[10]

When Hippocrates arrived in Athens, three special problems—the duplication of the cube, the squaring of the circle, and the trisection of an angle—were already engaging the attention of mathematicians, and he addressed himself at least to the first two. In the course of studying the duplication of the cube, he used the method of reduction or analysis. He was the first to compose an *Elements of Geometry* in the manner of Euclid's famous work. In astronomy he propounded theories to account for comets and the galaxy.

Method of Analysis. Hippocrates is said by Proclus to have been the first to effect the geometrical reduction of problems difficult of solution.[11] By reduction (άπαγωγή) Proclus explains that he means"a transition from one problem or theorem to another, which being known or solved, that which is propounded is also manifest."[12] It has sometimes been supposed, on the strength of a passage int he *Republic,* that Plato was the inventor of this method; and this view has been supported by passages from Proclus and Diogenes Laertius.[13] But Plato is writing of philosophical analysis, and what Proclus and Diogenes Laertius say is that Plato "communicated" or "explained" to Leodamas of Thasos the method of analysis (άναλύσις)—the context makes clear that this is geometrical analysis—which takes the thing sought up to an acknowledged first principle. There would not appear to be any difference in meaning between "reduction" and "analysis," and there is no claim that Plato invented the method.

Duplication of the Cube. Proclus gives as an example of the method the reduction of the problem of doubling the cube to the problem of finding two mean proportionals between two straight lines, after which the problem was pursued exclusively in that form.[14] He does not in so many words attribute this reduction to Hippocrates; but a letter purporting to be from Eratosthenes tp Ptolemy Euergetes, which is preserved by Eutocius, does specifically attribute the discovery to him.[15] In modern notation, if

$a:x = x:y = y:b$, then $a^3:x^3 = a:b$; and if $b = 2\,a$, it follows that a cube of side x is double a cube of side a. The problem of finding a cube that is double acube with side a is therefore reduced to finding two mean proportionals, x, y between a and $2\,a$ (The pseudo-Eratosthenes observes with some truth that the problem was thus turned into one no less difficult.)[16] There is no reason to doubt that Hippocrates was the first to effect this reduction; but is does not follow that he, any more than Plato, invented the method. It would be surprising if it were not in use among the Pythagoreans before him.

The suggestion was made by Bretschneider, and has been developed by Loria and Timpanaro Cardini,[17] that since the problem of doubling a square could be reduced to that of finding one mean proportional between two lines,[18] Hipporcrates conceived that the doubling of a cube might require the finding of two mean proportionals. Heath has made the fur ther suggestion that the idea may have come to him from the theory of numbers.[19] In the *Timaeus* Plato states that between two square numbers there is one mean proportional number but that two mean numbers in continued proportion are required to connect two cube numbers.[20] These propositions are proved as Euclid VII.11, 12, and may very well be Pythagorean. If so, Hippocrates had only to give a geometrical adaptation to the second.

Quadrature of Lunes. The "Eudemian summary" notes that Hippocrates squared the lune—so called from its resemblance to a crescent moon—that is, he found a rectilineal figure equal in area to the area of the figure bounded by two intersecting arcs of circles concave in the same direction.[21] This is the achievement on which his fame chiefly rests. The main source for our detailed knowledge of what he did is a long passage in Simplicius' commentary on Aristotle's *Physics*[22] Simplicius acknowledges his debt to Eudemus' *History of Geometry* and says that he will set out word for word what Eudemus wrote, adding for the sake of clarity only a few things taken from Euclid's *Elements* because of Eudemus' summary style. The task of separating what Simplicius added has been attempted by many writers from Allman to van der Waerden. When Simplicius uses such archaic expressions as τὸ σημεῖον ἐφ' ὧ (or ἐφ' οὗ) *A* for the point *A*, with corresponding expressions for the line and line and triangle, it is generally safe to presume that he is quoting; but it is not a sufficient test to distinguish the words of Hippocrates from those of Eudemus, since Aristotle still uses such pre Euclidean forms. Another stylistic test is the earlier form which Eudemus would have used, δυνάμει εἶναι ("to be equal to when square"), for the form δύνασθαι, which Simplicius would have used more naturally. Although there can be no absolute certainty about the attribution, what remains is of great interest as the earliest surviving example of Greek mathematical reasoning; only propositions are assigned to earlier mathematicians, and we have to wait for some 125 years after Hippocrates for the oldest extant Greek mathematical text (Autolycus).

Before giving the Eudemian extract, Simplicius reproduces two quadratures of lunes attributed to Hippocrates by Alexander of Aphrodisias, whose own commentary has not survived. In the first, *AB* is the diameter of a semicircle, *AC, CB* are sides of a square inscribed in the circle, and *AEC* is a semicircle inscribed on *AC*. Alexander shows that the lune *AEC* is equal to the triangle *ACD*.

In the second quadrature *AB* is the diameter of a semicircle; and on *CD*, equal to twice *AB*, a semicircle

is described. *CE*, *EF*, *FD* are sides of a regular hexagon; and *CGE*, *EHF*, *FKD* are semicircles. Alexander proves that the sum of the lunes *CGE*, *EHF*, *FKD* and the semicircle *AB* is equal to the trapezium *CEFD*.

Alexander goes on to say that if the rectilinear figure equal to the three lunes is subtracted ("for a rectilinear figure was proved equal to a lune"), the circle will be squared. There is an obvious fallacy here, for the lune which was squared was one standing on the side of a square and it does not follow that the lune standing on the side of the hexagon can be squared. John Philoponus, as already noted, says that Hippocrates tried to square the circle while at Athens. There is confirmation in Eutocius, who in his commentary on Archimedes' *Measurement of a Circle* notes that Archimedes wished to show that a circle would be equal to a certain rectilinear area, a matter investigated of old by eminent philosophers before him.[23] "For it is clear," he continues, "that the subject of inquiry is that concerning which Hippocrates of Chios and Antiphon, who carefully investigated it, invented the paralogisms which, I think, are accurately known to those who have examined the *History of Geometry* by Eudemus and have studied the *Ceria* of Aristotle." This is probably a reference

to a passage in the *Sophistici Elenchi* where Aristotle says that not all erroneous constructions are objects of controversy, either because they are formally correct or because they are concerned with something true, "such as that of Hippocrates or the quadrature by means of lunes."[24] In the passage in Aristotle's *physics* on which both Alexander and Simplicius are commenting,[25] Aristotle rather more clearly makes the point that it is not the task of the exponent of a subject to refute a fallacy unless it arises from the accepted principles of the subject. "Thus it is the business of the geometer to refute the quadrature of a circle by means of segments but it is not his business to refute that of Antiphon." [26]

The ancient commentators are probably right in identifying the quadrature of a circle by means of segments with Hippocrates' quadrature of lunes; mathematical terms were still fluid in Aristotle's time, and Aristotle may well have thought there was some fallacy in it. We may be confident, though, that a mathematician of the competence of Hippocrates would not have thought that he had squared the circle when in fact he had not done so. It is likely that when Hippocrates took up mathematics, he addressed himself to the problem of squaring the circle, which was much in vogue; it is evident that in the course of his researches he found he could square certain lunes and, if this had not been done before him, probably effected the two easy quadratures described by Alexander as well as the more sophisticated ones attributed to him by Eudemus. He may have hoped that in due course these quadratures would lead to the squaring of the circle; but it must be a mistake on the part of the ancient commentators, probably misled by Aristotle himself, to think that he claimed to have squared the circle. This is better than to suppose, with Heiberg, that in the state of logic at that time Hippocrates may have thought he had done so; or, with Bjö;rnbo, that he deliberately used language calculated to mislead; or, with Heath, that he was trying to put what he had discovered in the most favorable light. [27] Lets us trun to what Hippocrates actually did, according to Eudemus, who, as Simplicius notes, is to be preferred to Alexander as being nearer in date to the Chian geometer.

Hippocrates, says Eudemus, "made his starting point, and laid down as the first of the theorems useful for the discussion of lunes, that similar segments of circles have the same ratio as the squares on their bases; and this he showed from the demonstration that the squares on the diameters are in the same ratio as the circles." (This latter proposition is Euclid XII.2 and is the starting point also of Alexander's quadratures; the signficance of what Eudemus says

is discussed below.) In his first quadrature he takes a right-angled isosceles triangle *ABC*, describes a semicircle about it, and on the base describes a segment of a circle similar to those cut off by the sides. Since $AB^2 = AC^2 + CB^2$, it follows that the segment about the base is equal to the sum of those about the sides; and if the part of the triangle above the segment about the base is added to both, it follows that the lune *ACB* is equal to the triangle.

Hippocrates next squares a lune with an outer circumference greater than a semicircle.*BA, AC, CD* are equal sides of a trapezium; *BD* is the side parallel to *AC* and $BD^2 = 3AB^2$. About the base *BD* there is described a segment similar to those cut off by the equal sides. The segment on *BD* is equal to the sum of the segments on the other three sides; and by adding the portion of the trapezium about the segment about the base, we see that the lune is equal to the trapezium.

Hippocrates next takes a lune with a circumference less than a semicircle, but this requires a preliminary construction of some interest, it being the first known example of the Greek construction known as a "νεύσις, or "verging,"[28] Let *AB* be the diameter of a circle and *K* its center. Let *C* be the midpoint of *KB* and let *CD* bisect *BK* at right angles. Let the straight line *EF* be placed between the bisector *CD* and the circumference "verging toward B" so that the square on *EF* is 1.5 times the square on one of the radii, that is, $EF^2 = 3/2\ KA^2$. If $FB = x$ and $KA = a$, it can easily be shown that $x = a^2$, so that

the problem is tantamount to solving a quadratic equation. (Whether Hippocrates solved this theoretically or empirically is discussed below.)

After this preliminary construction Hippocrates circumscribes a segment of a circle about the trapezium *EKBG* and describes a segment of a circle about the triangle *EFG*. In this way there is formed a lune having its outer circumference less than a semicircle, and its area is easily shown to be equal to the sum of the three triangles *BFG*, *BFK*, *EKF*.

Hippocrates finally squares a lune and a circle together. Let *K* be the center of two circles such that the square on the diameter of the outer is six times the square on the diameter of the inner. *ABCDEF* is a regular hexagon in the inner circle.*GH*, *HI* are sides of a regular hexagon in the outer circle. About GI let there be drawn a segment similar to that cut off by *GH*. Hippocrates shows that the lune *GHI* and the inner circle are together equal to the triangle *GHI* and the inner hexagon.

This last quadrature, rather than that recorded by Alexander, may be the source of the belief that Hippocrates had squared the circle, for the deduction is not so obviously fallacious. It would be easy for someone unskilled in mathematics to suppose that because Hippocrates had squared lunes with outer circumferences equal to, greater than, and less than a semicircle, and because he had squared a lune and a circle together, by subtraction he would be able to

square the circle. The fallacy, of course, is that the lune which is squared along with the circle is not one of the lunes previously squared by Hippocrates; and although Hippocrates squared lunes having outer circumferences equal to, greater than, and less than a semicircle, he did not square all such lunes but only one in each class.

What Hippocrates succeeded in doing in his first three quadratures may best be shown by trigonometry. Let *O*, *C* be the centers of arcs of circles forming the lune *AEBF*, let *r*, *R* be their respective radii and θ, ϕ the halves of the angles subtended by the arcs at their centers.

It is a sufficient condition for the lune to be squarable that sector *OAFB* = sector *CAEB*, for in that case the area will be equal to $\Delta\ CAB - \Delta\ OAB$, that is, the quadrilateral *AOBC*. In trigonometrical notation, if $r^2\theta = R^2\phi$, the area of the lune will be $1/2(R^2 \sin 2\phi - r^2 \sin 2\theta)$. Let $\theta = k\phi$. Then and the area of the lune is $1/2\ r^2\ (k \sin 2\phi - r^2 \sin 2\theta)$. Let $\theta = k\phi$. Now $r \sin\theta = 1/2 AB = R \sin \phi$, so that . This becomes a quadratic equation in sin ϕ, and therefore soluble by plane methods, when $k = 2, 3, 3/2, 5$, or $5/3$. Hippocrates' three solutions correspond to the values 2, 3, 3/2 for *k*.[29]

Elements of Geometry. Proclus explains that in geometry the elements are certain theorems having to those which follow the nature of a leading principle and furnishing proofs of many properties; and in the summary which he has taken over from Eudemus he names Hippocrates, Leon, Theudius of Magnesia, and Hermotimus of Colophon as writers of elements.[30] In realizing the distinction between theorems which are merely interesting in themselves and those which lead to something else, Hippocrates made a significant discovery and started a famous tradition; but so complete was Euclid's success in this field that all the earlier efforts were driven out of circulation. What Proclus says implies that Hippocrates' book had the shortcomings of a pioneering work, for he tells us that Leon was able to make a collection of the elements in which he was more careful, in respect both of the number and of the utility of the things proved.

Although Hippocrates' work is no longer extant, it is possible to get some idea of what it contained. It would have included the substance of Books I and II of Euclid's *Elements*, since the propositions in these books were Pythagorean discoveries. Hippocrates' research into lunes shows that he was aware of the following theorems:

1. In a right-angled triangle, the square on the side opposite the right angle is equal to the sum of the squares on the other two sides (Euclid I.47).

2. In an obtuse-angled triangle, the square on the side subtending the obtuse angle is greater than the sum of the squares on the sides containing it (*cf.* II.12).

3. In any triangle, the square on the side opposite an acute angle is less than the sum of the squares on the sides containing it (*cf.* II.13).

4. In an isosceles triangle whose vertical angle is double the angle of an equilateral triangle (that is, 120°), the square on the base is equal to three times the square on one of the equal sides.

5. In equiangular triangles, the sides about the equal angles are proportional.

Hippocrates' *Elements* would have included the solution of the following problems:

6. To construct a square equal to a given rectilinear figure (II.14).

7. To find a line the square on which shall be equal to three times the square on a given line.

8. To find a line such that twice the square on it shall be equal to three times the square on a given line.

9. To construct a trapezium such that one of the parallel sides shall be equal to the greater of two given lines and each of the three remaining sides equal to the less.

The "verging" encountered in Hippocrates' quadrature of lines suggests that his *Elements* would have included the "geometrial algebra" developed by the Pythagoreans and set out in Euclid I.44, 45 and 11.5, 6, 11. It has been held that Hippocrates may have contented himself with an empirical solution, marking on a ruler a length equal to *KA* in Figure 5 and moving the ruler about until the points marked lay on the circumference and on *CD*, respectively, while the edge of the ruler also passed through *B*. In support, it is pointed out that Hippocrates first places *EF* without producing it to *B* and only later joins *BF*.[31] But it has to be admitted that the complete theoretical solution of the equation , having been developed by the Pythagoreans, was well within the capacity of Hippocrates or any other mathematician of his day. In Pythagorean language it is the problem "to apply to a straight line of length rectangle exceeding by a square figure and equal to a^2 in area," and it would be solved by the use of Euclid II. 6.

Hippocrates was evidently familiar with the geometry of the circle; and since the Pythagoreans made only a limited incursion into this field, he may himself have discovered many of the theorems contained in the third book of Euclid's *Elements* and solved many of the problems posed in the fourth book. He shows that he was aware of the following theorems:

1. Similar segments of a circle contain equal angles. (This implies familiarity with the substance of Euclid III.20–22.)

2. The angle of a semicircle is right, that of a segment greater than a semicircle is acute, and that of a segment less than a semicircle is obtuse. (This is Euclid III.31, although there is some evidence that the earlier proofs were different.)[32]

3. The side of a hexagon inscribed in a circle is equal to the radius (IV. 15, porism). He knew how to solve the following problems: (1) about a given triangle to describe a circle (IV.5); (2) about the trapezium drawn as in problem 9, above, to describe a circle; (3) on a given straight line to describe a segment of a circle similar to a given one (*cf.*III.33).

Hippocrates would not have known the general theory of proportion contained in Euclid's fifth book, since this was the discovery of Eudoxus, nor would he have known the general theory of irrational magnitudes contained in the tenth book, which was due to Theaetetus; but his *Elements* may be presumed to have contained the substance of Euclid VI-IX, which is Pythagorean.

It is likely that Hippocrates' *Elements* contained some of the theorems in solid geometry found in Euclid's eleventh book, for his contribution to the Delian problem (the doubling of the cube) shows his interest in the subject. It would be surprising if it did not to some extent grapple with the problem of the five regular solids and their inscription in a sphere, for this is Pythagorean in origin; but it would fall short of the perfection of Euclid's thirteenth book. The most interesting question raised by Hippocrates' *Elements* is the extent to which he may have touched on the subjects handled in Euclid's twelfth book. As we have seen, his quadrature of lunes is based on the theorem that circles are to one another as the squares on their diameters, with its corollary that similar segments of circles are to each other as the squares on their bases. The former proposition is Euclid

XII.2, where it is proved by inscribing a square in a circle, bisecting the arcs so formed to get an eight-sided polygon, and so on, until the difference between the inscribed polygon and the circle becomes as small as is desired. If similar polygons are inscribed in two circles, their areas can easily be proved to be in the ratio of the sqaures on the diameters; and when the number of the squares on the diameters; and when the number of the sides is increased and the polygons approximate more and more closely to the circles, this suggests that the ares of the two circles are in the ratio of the squares on their diameters.

But this is only suggestion, not proof, for the ancient Greeks never worked out a rigorous procedure for taking the limits. What Euclid does is to say that if the ratio of the squares on the diameters is not equal to the ratio of the circles, let it be equal to the ratio of the first place to be less than the second circle. He then lays down that by continually doubling the number of sides in the inscribed polygon, we shall eventually come to a point where the residual segments of the second circle over S. For this he relies on a lemma, which is in fact the first proposition of Book X: "If two unequal magnitudes be set out, and if from the greater there be subtracted a magnitude greater than its half, and from the remainder a magnitude greater than its half, and so on continually, there will be left some magnitude which is less than the lesser magnitude set out." On this basis Euclid is able to prove rigorously by *reductio ad absurdum* that S cannot be less than the second circle. Similarly, he proves that it cannot be greater. Therefore S must be equal to the second circle, and the two circles stand in the ratio of the squares on their diameters.

Could Hippocrates have proved the proposition in this way? Here we must turn to Archimedes, who in the preface to his *Quadrature of the Parabola*[33] says that in order to find the area of a segment of a parabola, he used a lemma which has accordingly become known as "the lemma of Archimedes" but is equivalent to Euclid X.I; "Of unequal areas the excess by which the greater exceeds the less is capable, when added continually to itself, of exceeding any given finite area." [34] Archimedes goes on to say:

The earlier geometers have also used this lemma. For it is by using this same lemma that they have proved (1) circles are to one another in the same ratio as the squares on their diameters; (2) spheres are to one another as the cubes on their diameters; (3) and further that every pyramid is the third part of the prism having the same base as the pyramid and equal height; and (4) that every cone is a third part of the cylinder having the same base as the cone and equal height they proved by assuming a lemma similar to that above mentioned.

In his *Method* Archimedes states that Eudoxus first discovered the proof of (3) and (4) but that no small part of the credit should be given to Democritus, who first enunciated these theorems without proof.[35]

In the light of what has been known since the discovery of Archimedes' *Method*, it is reasonable to conclude that Hippocrates played the same role with regard to the area of a circle that Democritus played with regard to the volume of the pyramid and cone; that is, he enunciated the proposition, but it was left to Eudoxus to furnish the first rigorous proof. Writing before the discovery of the *Method*, Hermann Hankel thought that Hippocrates must have formulated the lemma and used it in his proof; but without derogating in any way from the genius of Hippocrates, who emerges as a crucial figure in the history of Greek geometry, this is too much to expect of his age.[36] It is not uncommon in mathematics for the probable truth of a proposition to be recognized intuitively before it is proved rigorously. Reflecting on the work of his contemporary Antiphon, who inscribed a square (or, according to another account, an equilateral triangle) in a circle and kept on doubling the number of sides, and the refinement of Bryson in circumscribing as well as inscribing a regular polygon, and realizing with them that the polygons would eventually approximate very closely to the circle, Hippocrates must have taken the further step of postulating that two circles would stand to each other in the same ratio as two similar inscribed polygons, that is, in the ratio of the squares on their diameters.

A question that has been debated is whether Hippocrates' quadrature of lunes was contained in his *Elements* or was a separate work. There is nothing about lunes in Euclid's *Elements*, but the reason is clear: an element is a proposition that leads to something else; but the quadrature of lunes, although interesting enough in itself, proved to be a mathematical dead end. Hippocrates could not have foreseen this when he began his investigations. The most powerful argument for believing the quadratures to have been contained in a separate work is that of Tannery: that Hippocrates' argument started with the theorem that similar segments of circles have the same ratio as the squares on their bases. This depends on the theorem that circles are to one another as the squares on their bases, which, argues Tannery, must have been contained in another book because it was taken for granted.[37]

Astronomy. What is known of Oenopides shows that Chios was a center of astronomical studies even before Hippocrates; and he, like his contemporaries, speculated about the nature of comets and the galaxy. According to Aristotle, [38] certain Italians called Pythagoreans said that the comet—it was apparently believed that there was only one—was a planet which appeared only at long intervals because of its low elevation above the horizon, as was the case with Mercury.[39] The circle of Hippocrates and his pupil Aeschylus[40] expressed themselves in a similar way save in thinking that the comet's tail did not have a real existence of its own; rather, the comet, in its wandering through space, occasionally assumed the appearance of a tail through the deflection of our sight toward the sun by the moisture drawn up by the comet when in the neighborhood of the sun.[41] A second reason for the rare appearance of the comet, in the view of Hippocrates, was that it retrogressedc so slowly in relation to the sun, and therefore took a long time to get clear of the sun. It could get clear of the sun to the north and to the south, but it was only in the north that the conditions for the formation of a tail were favorable; there was little moisture to attract in teh space between the tropics, and although there was plenty of moisture to the south, when the comet was in teh south only a small part of its circuit was visible. Aristotle proceeds to give five fairly cogent objections to these theories.[42]

After recounting the views of two schools of Pythagoreans, and of Anaxagoras and Democritus on the Milky Way, Aristotle adds that there is a third theory, for "some say that the galaxay is a deflection of our sight toward the sun as is the case with the comet." He does not identify the third school with Hippocrates; but the commentators Olympiodorus and Alexander have no hesitation in so doing, the former noting that the deflection is caused by the stars and not by moisture.[43]

NOTES

1. The similarity of the names impressed itself upon at least one ancient commentator, Olympiodorus. *In Aristotelis Meteora*, Stuve ed., 45, 24–25: 'Ιπποκράτης, ούχ ὸ Κώος, άλλ' ό χˆιος

2. John Phioloponus, *In Aristotelis Physica*, Vitelli ed., 31.3–9.

3. Aristotle, *Ethica Eudemia H* 14, 1247a17, Susemihl ed., 113.15–114.1.

4. Plutarch, *Vita Solonis* 2. *Plutarchi vitae parallelae*, Sintenis ed., I, 156.17–20.

5. Proclus, *In primum Euclidis*, Friedlein ed., 65. 21–66.7.

6. C. A. Bretschneider, *Die Geometrie und die Geometer vor Eukleides*, P.98.

7. Pauk Tannery, *La geometrie grecque*, p. 108; Maria Timpanaro Cardini, *Pitagorici*, fasc. 2, pp. 29–31.

8. Iamblichus, *De vita Pthagorica* 36, Deubner ed., 143.19–146.16; and, for the link with Theodore, *De communi mathematica scientia* 25, Festa ed., 77.24–78.1. The same passage, with slight variations, is in *De vita Pythagorica* 18, Deubner ed., 52.2–11, except for the sentence relating to Hippocrates.

9. G. J. Allman, *Greek Geometry From Thales to Euclid*, p. 60.

10. Timpanaro Cardini, *op. cit*., fasc. 2, p. 31.

11. Proclus, *op. cit*., 213.7–11. He adds that Hippocrates also squared the lune and made many other discoveries in geometry, being outstanding beyond all others in his handling of geometrical problems.

12.*Ibid*., 212.25–213.2

13. Plato, *Republic* VI, 510B-511C, Burnet ed.; Proclus, *op. cit*., 211.18–23; Diogenes Laertius, *Vitae philosophorum* III.24, Long ed., 1.131.18–20.

14. Proclus, *op. cit*., 213.2–6.

15.*Archimedis opera omnia*, Heiberg ed., 2nd ed., III, 88.4–96.27.

16.*Ibid*., 88.17–23.

17. Bretschneider, *op. cit*., p. 97; Gino Loria, *Le scienze esatte nell' antica Grecia*, 2nd ed., pp. 77–78; Timpanaro Cardini, *op. cit*., fasc. 2, pp. 34–35.

18. If $a;x = x:2a$, the square with side x is double the square with side a. The problem of doubling a square of side x is thus reduced to finding a mean proportional between a and $2a$.

19. Thomas Heath, *A History of Greek Mathematics*, I, 201.

20. Plato, *Timaeus* 32 a, b, Burnet ed. With the passage should be studied *Epinomis* 990bs-991b4, Burner ed.; and the note by A. C. Lloyd in A. E. Taylor, *Plato: Philebus and Epinomis*, p. 249.

21. Proclus, *op. cit*., 66.4–6, in fact mentions the squaring of the lune as a means of identifying Hippocrates.

22. Simplicius, *In Aristotelis Physica*, Diels ed., 53.28–69.35.

23.*Archimedis opera omnia*, Heiberg ed., 2nd ed., III, 228.11–19.

24. Aristoltle, *Sophistici Elenchi* 11, 171b12–16. Toward the end of the third century Sporus of Nicaea compiled a work known as Κηρία, or Αριστοτελικά κηρία, which was used by Pappus, Simplicius, and Eutocius; but Heiberg sees here a reference to the *Sophistici Elenchi* of Aristotle. Grammatically it is possible that "the quadrature by means of lunes" is to be distinguished from "that of Hippocrates"; but it is more likely that they are to be identified, and Diels and Timpanaro Cardini are probably right in bracketing "the quadrature by menas of lunes" as a (correct) gloss which has crept into the text from 172a2–3, where the phrase is also used.

25. Aristotle, *Physics* A 2, 185a14, Ross ed.

26. Aristotle does an injustice to Antiphon, whose inscription of polygons with an increasing number of sides in a circle was the germ of a fruitful idea, leading to Euclid's method of exhaustion; Aristotle no doubt thought it contrary to the principles of geometry to suppose that the side of the polygon could ever coincide with an arc of the circle.

27. J. L. Heiberg, *Philologus*, **43** , p. 344; A. A. Björnbo, in Pauly Wissowa, VIII, cols. 1787–1799; Heath, *op. cit*., I, 196, note. Montucla, *Histoire des recherches sur la quadrature du cercle*, pp. 21–22, much earlier (1754) had given the correct interpretation: "Hippocrate ne vouloit point proposer un moyen qu'il jugeoit propre à conduire quelque jour à la quardrature du cercle?"

28. There is a full essay on this subject in T. L. Heath, *The Works of Archimedes*, pp. c-cxxii.

29. It was shown by M. J. Wallenius in 1766 that the lune can be squared by plane methods when $x = 5$ or 5/3 (Max Simon, *Geschichte der Mathematik im Altertum*, p. 174). T. Clausen gave the solution of the last four cases in 1840, when it was not known that Hippocrates had solved more than the first. ("Vier neue mondförmige Flachen, deren Inhalt quadrirbar ist," in *Journal für die reine und angewandte Mathematik*, **21** 375–376). E. Landau has investigated the ases where the difference between $r^2\phi$ and $R^2\phi$ is not zero but equal to an area that can be squared, although this does not lead to new sqarable lunes: "Ueber quadrirbare Kreisbogen zweiecke," in *Sitzungsberichte der Berliner mathematischen Gesellschaft*, **2** (1903).

30. Proclus, *op. cit*., 72.3–13, 66.7–8, 66.19–67.1, 67. 12–16, 20–23. Tannery (*Memorires scientifiques*, I, 46) is not supported either in antiquity or by modern commentators in discerning a written Pythagorean collection of *Elements* preceding that of Hippocrates.

31. Heath, *op. cit*., I, 196.

32. See Aristotle, *Posterior Analytics* II 11, 94a28–34; *Metaphysics* Θ and the comments by W. D. Ross, *Aristotle's Metaphysics*, pp. 270–271; and Thomas Heath, *Mathematics in Aristotle*, pp. 37–39, 71–74.

33.*Archimedis opera omnia*, Heiberg ed., 2nd ed., II, 264.1–22.

34. More strictly "the lemma of Archimedes" is equivalent to Euclid V, def. 4—"Magnitudes are said to have a ratio one to another if they are capable, when multiplied, of exceeding one another"—and this is used to prove Euclid X.1. Archimedes not infrequently uses the lemma in Euclid's form.

35.*Archimedis opera omnia*, Heiberg ed., 2nd ed., II, 430.1–9. In the preface to Book I of his treatise *On the Sphere and Cylinder* Archimedes attributes the proofs of these theorems to Eudoxus without mentioning the part played by Democritus.

36. Hermann Hankel, *Zur Geschichte der Mathematik in Alterthum und Mittelalter*, p. 122.

37. Tannery, *op. cit*., I, 354–358. Loria, *op. cit*., p. 91, inclines to the same view; but Timpanaro Cardini, *op. cit*., fasc. 2, p. 37, is not persuaded.

38.*Meteorologica* A6, 342b30–343a20, Forbes ed., 2nd ed.

39. Because, like Mercury, it can be seen with the naked eye only when low on the horizon before dawn or after sunset, since it never sets long after the sun and cannot be seen when the sun is above the horizon.

40. Nothing more is known of Aeschylus. This and references by Aristotle to οί περί 'Ιπποκράτην imply that Hippocrates had a school.

41. It is not clear how Aristltle thought the appearance to be caused, and the commentators and translators—Thomas Heath, *Aristarchus of Samos*, p. 243; E. W. Webster, *The Works of Aristotle*, III, *Meteorologica, loc. cit*.; pp. 40–43; Timpanaro Cardini, *op. cit*., fasc. 2, pp. 66–67—give only limited help. It is clear that Hippocrates, like Alcmaeon and Empedocles before him, believed that rays of light proceeded from the eye to the object; and it seems probable that he thought visual rays were *refracted* in the moisture around the comet toward the sun (the sun then being in a position in which this could happen), and *reflected* from the sun back to the moisture and the observer's eye (hence the choice of the neutral word "deflected").

Hippocrates believed that somehow this would create the appearance of a tail in the vapors around the comet; but since this is not the "correct explanation, it is impossible to know exactly what he thought happened . It is tempting to suppose" that he thought the appearance of the comet's tail to be formed in the moisture in the same way that a stick appears to be formed in the moisture in the same way that a stick appears to be bent when seen partly immersed in water, but the Greek will not bear this simple interpretation.

Olympildorus, *op. cit.,* Stuve ed., 45.29–30, notes that where as Pythagoras maintained that both the comet and the tail were made of the fifth substance, Hippocrates held that the comet was made of the fifth substance but the tail out of the sublunary space. This is anachronistic. It was Aristotle who added the "fifth substance" to the traditional four elements—earth, air, fire, water.

42. Aristotle, *Meteorologica*, A6, 343a21–343b8, Fobes ed., 2nd ed.

43. Oltmpiodorus, *op. cit.*, Stuve ed., 68.30–35; he reckons it a "fourth opinion," presumably counting the two Pythagorean schools separately. Alexander, *In Aristotelis Meteorologica*, Hayduck ed., 38.28–32.

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