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11-13 minutes

(*b*. Stuttgart, Germany, 22 December 1859; *d*. Leipzig, Germany, 29 August 1937)

mathematics.

Hölder came from a Württemberg family of public officials and scholars. His father, Otto Hölder, was professor of French at the Polytechnikum in Stuttgart; his mother was the former Pauline Ströbel. In Stuttgart, Hölder attended one of the first Gymnasiums devoted to science and there he studied engineering for a short time. A colleague of his father's suggested that the best place to study mathematics was Berlin, where Weierstrass, Kronecker, and Kummer were teaching. When Hölder arrived at the University of Berlin in 1877, Weierstrass, lecturing on the theory of functions, had already covered the fundamentals of analysis. Hölder caught up to the class with the aid of other students' notes and was thus led to his first independent studies in mathematics.

Influenced by the rigorous foundation of analysis given by Weierstrass, Hölder developed the continuity condition for volume density that bears his name. It appeared in his dissertation (*Beiträge zur Potentialtheorie*), which he presented at Tübingen in 1882; his referee was Paul du Bois-Reymond. The Hölder continuity is sufficient for the existence of all the second derivatives of the potential and for the validity of the Poisson differential equation. These derivatives, as Arthur Korn later showed, possess exactly the same continuity properties as the density. Hölder's work on potential theory was continued on a larger scale by Leon Lichtenstein, O. D. Kellogg, P. J. Schauder, and C. B. Morrey, Jr.

Next Hölder investigated analytic functions and summation procedures by arithmetic means. He provided the first completely general proof of Weierstrass' theorem that an analytic function comes arbitrarily close to every value in the neighborhood of an essential singular point. He showed that it might be possible to compute, by repetition of arithmetic means (Hölder means), the limit of an analytic function the power series of which diverges at a point of the circle of convergence. This technique is equivalent to the one introduced by Cesàro, as Walter Schnee demonstrated.

In his *Habilitationsschrift* submitted in 1884 at Göttingen, Hölder examined the convergence of the Fourier series of a function that was not assumed to be either continuous or bounded; for such functions the Fourier coefficients had first to be defined in a new fashion as improper integrals. After qualifying as a lecturer, Hölder discovered the inequality named for him. This advance involved and extension of Schwarz's inequality to general exponents as well as to inequalities for convex functions of the type that were later treated by J. L. Jensen. After unsuccessful attempts to find an algebraic differential equation for the gamma function, Hölder inverted the method of posing the question an proved the impossibility of such a differential equation.

Hölder owed his interest in group theory and Galois theory primarily to Kronecker, but also to Felix Klein, in whose seminar at Leipzig Hölder participated soon after receiving his doctorate. To these fields he contributed "Zurückführung einer algebraischen Gleichung auf eine Kette von Gleichungen," in which he reduced an algebraic equation by using simple groups and by introducing the concept of "natural" irrationals. Here Hölder extended C. Jordan's theorem (stated in his *Commentary* on Galois) of the uniqueness of the indexes of such a "composition series," to the uniqueness of the "factor groups" that Hölder had introduced. This new concept and the Jordan-Hölder theorem are today fundamental to group theory.

With the help of these methods Hölder solved the old question of the "irreducible case." A solution of the cubic equation is given by the so-called Cardano formula, in which appear cube roots of a square root $\sqrt{D}$. For three distinct real roots $D < 0$, and therefore the quantities under the cube root sign are imaginary. The real solution is thus obtained as the sum of imaginary cube roots. Hölder showed that in this case it is impossible to solve the general cubic equation through real radicals, except where the equation decomposes over the base field.

Hölder turned his attention first to simple groups. Besides the simple groups of orders 60 and 168 already known at the time, he found no new ones with a composite order less than 200. Nevertheless, he considered his method to be "of some interest so long as we do not possess a better one suitable for handling the problem generally." Such a general method is still lacking, despite the progress and great efforts of recent years.

In further works Hölder treated the structure of composite groups having the following orders: p^3, pq^2, pqr, p^4, where p, q, r are primes, and n, where n is square-free. Finally, he studied the formation of groups constructed from previously given factor groups and normal subgroups.

While an associate professor at Tübingen, Hölder verified (in "Über die Prinzipien von Hamilton und Maupertuis") that the variational principles of Hamilton are valid for nonholonomic motions—their applicability in these cases had been questioned by Heinrich Hertz. Physicists are indebted to Hölder for this confirmation of the Hamiltonian principle, which has often been used since then in deriving differential equations of physics.

The first third of Hölder's career in research was the most fruitful. A period of depression seems to have occurred at Königsberg, where he succeeded Minkowski in 1894. He was happy to leave that city in 1899, when he accepted an offer from Leipzig to succeed Sophus Lie. In the same year he married Helene Lautenschlager, who also came from Stuttgart.

At Leipzig, Hölder turned to geometrical questions, beginning with his inaugural lecture *Anschauungen und Denken in der Geometrie* (1900). He became interested in the geometry of the projective line and undertook investigations published in his paper "Die Axiome der Quantität und die Lehre vom Mass" (1901). The topics covered in this work were, in his view, important for physics. Moreover, in 1911 he published an article on "Streckenrechnung und projektive Geometrie."

Between 1914 and 1923 this work led to the logicophilosophical studies of the foundations of mathematics which are included in *Die mathematische Methode* (1924). These philosophical inquiries attracted less attention than Hilbert's axiomatic method, but Hölder saw connections between Brouwer's intuitionism and Weyl's logical investigations and his own ideas. P. Lorenzen's recent work on logic contains ideas which are in essence similar to those of Hölder's. In his obituary on Hölder, B. L. van der Waerden wrote:

According to Hölder one of the essential features of the mathematical method consists in constructing for given concepts, concepts of higher order in such a way that concepts and methods of proof of one stage are taken as objects of mathematical investigation of the next higher stage. This is done, for example, by first developing a method of proof and afterward counting the steps of the proof or by letting them correspond to other objects, or by combining them by means of relations [p. 161].

On the basis of this conception Hölder concluded—and recent logical investigations of Gödel fully justify his position—that "one can never grasp the whole of mathematics by means of a logical formalism, because the new concepts and syllogism that are applied to the formulas of the formalism necessarily go beyond the formalism and yet also belong to mathematics."

In his last years one of Hölder's favorite topics was elementary number theory—his third great teacher in Berlin had been Kummer. Hölder's contributions in this area appeared mainly in the *Bericht. Sächsische Gesellschafl* (later *Akademie) der Wissenschaften*. From 1899 he was active in the academy and for several years served as president. He was also a member of the Prince Jablonowski Society. In 1927 Hölder became a corresponding member of the Bavarian Academy of Sciences.

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II. Secondary Literature. The main source is the obituary by B. L. van der Waerden, "Nachruf auf Otto Hölder," in *Mathematische Annalen*, **116** (1939), 157–165, with bibliography; it also appeared in *Bericht, Sächsische Akademie der Wissenschaften*, Math.-nat. Klasse, **90** (1938), without the bibliography. See also Poggendorff, IV, 651; V, 547; VI, 1136; VIIa, 509.

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