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(*b* Hildesheim, Germany, 26 March 1859; *d* Zurich, Switzerland, 18 November 1919)

mathematics.

Hurwitz, the son of a manufacturer, attended the Gymnasium in Hildsheim. His mathematics teacher, H. C. H. Schubert, was known as the inventor of a dazzling calculus for enumerative geometry. He discovered Hurwitz, gave him private lessons on Sundays, and finally persuaded Adolf's father, who was not wealthy, to have his son study mathematics at the university, financially supported by a friend. Before leaving the Gymnasium, Hurwitz published his first paper, jointly with Schubert, on Chasles's theorem (*Werke*, paper no. 90).

In the spring term of 1877 he enrolled at the Munich Technical University, recommended to Felix Klein by Schubert. From the fall term of 1877 through the spring term of 1879 he was at Berlin University, where he attended courses given by Kummer, Weierstrass, and Kronecker. Then he returned to Munich, only to follow Klein in the fall of 1880 to Leipzig, where he took his Ph.D. with a thesis on modular functions. In 1881–1882, according to Meissner, he turned anew to Berlin to study with Weierstrass and Kronecker. (Hilbert did not know of a second stay in Berlin.) In the spring of 1882 he qualified as *Privatdozent* at Göttingen University, where he came into close contact with the mathematician M. A. Stern and the physicist Wilhelm Weber. In 1884 Hurwitz accepted Lindemann's invitation to fill an extraordinary professorship at Königsberg University, which was then a good place for mathematics. Among its students were Hilbert and Minkowski. Hurwitz, a few years their elder, became their guide to all mathematics and their life-long friend. Hilbert always acknowledged his indebtedness to Hurwitz. In 1892 Hurwitz was offered Frobenius' chair at the Zurich Polytechnical University and H. A. Schwarz's at Göttingen University. He had already accepted the first offer when the second arrived. He went to Zurich and remained there for the rest of his life. He married the daughter of Professor Samuel, who taught medicine at Königsberg.

Hurwitz' health was always poor. Twice he contracted typhoid fever, and he often suffered from migraine. In 1905 one kidney had to be removed; and the second did not function normally. Although seriously ill, he continued his research.

Hurwitz' papers reveal a lucid spirit and a love of good style and perspicuous composition. Hilbert depicted him as a harmonious spirit; a wise philosopher; a modest, unambitious man; a lover of music and an amateur pianist; a friendly, unassuming man whose vivid eyes revealed his spirit.

His papers were collected by his Zurich colleagues, particularly G. Polya. Although entitled *Werke*, the edition does not include his book on the arithmetic of quaternions and his posthumous function theory. The *Werke* lists his twenty-one Ph.D. students and contains an obituary written by Hilbert in 1919 and Ernst Meissner's eulogy. All present biographical data were extracted from these contributions. Hilbert's obituary is rather disappointing—even more so if it is compared with Hilbert's commemoration of Minkowski, which rings of high enthusiasm and deep regret. Certainly Hilbert had esteemed Hurwitz as a kind man, an erudite scholar, a good mathematician, and a faithful guide. But one may wonder whether he appreciated Hurwitz' mathematics as sincerely as he appreciated its creator. of course it is easier to write a brilliant biography if the subject is as brilliant as was Minkowski. Hurwitz was anything but brilliant, although he was as good a mathematician as Minkowski. Or, if that was not the reason, was it perhaps because Hilbert himself had changed in the ten years since he wrote Minkowski's biography, and his own productivity had come to a virtual standstill. anyhow, because Hilbert wrote his biography, Hurwitz never got the one he deserved.

In a large part of Hurwitz' work the influence of Klein is overwhelming. Among Klein's numerous Ph.D. students Hurwitz was second to none except, perhaps, Furtwängler. Much of Klein's intuitiveness is found again in Hurwitz, although the latter was superior in the rationalization of intuitive ideas. Klein was at the peak of his creativity when Hurwitz studied with him and Klein's best work was that in which Hurwitz took a share. Klein's new view on modular functions, uniting geometrical aspects such as the fundamental domain with group theory tools such as the congruence subgroups and with topological notions such as the genus of the Riemann surface, was fully exploited by Hurwitz. In his thesis he worked out Klein's ideas to reach an independent reconstruction of the theory of modular functions and, in particular, of multiplier equations by Eisenstein principles (*Werke*, paper no. 2). Modular functions were applied by Hurwitz to a classical subject of number theory—realtions between the class numbers of binary quadratic forms with negative discriminant—which had been tackled long before by Kronecker and Hermite, and afterward by J. Gierster, another student of Klein's.

The problem of how to derive class number relations from modular equations and correspondences was put in general form by Hurwitz, although the actual execution was restricted to particular cases (*Werke*, papers no. 46, 47). The problem has long remained in the state in which Hurwitz had left it; but in the last few years it has been revived in C. L. Siegel's school although, strangely enough, no attention whatsoever has been paid to Hurwitz' other, unorthodox approach to class numbers (*Werke,* papers no. 56, 62, 69, 77). It is, first, a reduction of quadratic forms by means of Farey fractions and so-called Farey polygons: on the conic defined by x:y:z = 1:-λ:$λ^2$, a pair of points λ = p/q/, r/s (p, q, r, s are integers) is called an elementary chord if $ps - qr = 1$, and such chords are taken to form elementary triangles; the reduction is carried by a systematic transition from one triangle to the next. The splitting of the conic surface into such triangles led Hurwitz in 1905 to a curious nonarithmetic infinite sum forclass numbers, genralized in 1918 to ternary forms Hurwitz also refashioned the classical expressions for class numbers into fast-converging infinite series, which, together with congruence arguments, provide easy means of computation (*Werke*, paper no. 59).

More direct products of Hurwitz' collaboration with Klein were his remarkable investigations on the most general correspondences on Riemann surfaces (*Werke,* paper no. 10), in particular Chasle's correspondence principle, and his work on elliptic π products and their behavior under the transformation of the periods (Klein's elliptic normal curves, *Werke*, paper no. 11). For Dirichlet series occurring in class number formulas. Hurwitz derived transforamtions like those of the ζ function (*Werke*, paper no. 3). By means of complex multiplication he studied the development coefficients of the lemniscatic function, which look much like the Bernoulli numbers (*Werke*, paper no. 67). He also investigated the automorphic groups of algebraic Riemann surfaces of genus > 1; showed that they were finite; estimated the maximal order of automorphism as $\leqq 10\,(p-1)$, the best value, according to A. Wiman, being $2\,(2p+1)$; estimated the group order as $\leqq 84(p-1)$; and constructed Riemann surfaces from group theory or branching data (*Werke*, papers no. 12, 21, 22, 23, 30). Hurwitz' formula $p' - 1 = w/2 + n(p-1)$ for the genus p′ of a surface w times branched over a surface of genus p is found in *Werke*, paper no. 21. p. 376. Automorphic functions of several variables were also among Hurtwitz' subvjects (*Werke*, paper no. 36).

In general complex-function theory Hurwitz studied arithmetic properties of transcendents which generalize those of the exponential function (*Werke*, papers no. 6, 13), the roots of Beesl functions and other transcendents (*Werke*, papers no. 14, 17), and difference equations (*Werke*, paper no. 26). Giving a solution of the isoperimetric problem, he became interested in Fourier series, to which he devoted several papers (*Werke*, papers no. 29, 31, 32, 33). Hurwitz was the author of a condition, very useful in stability theory, on a polynomial having all its roots in the left half-plane, expressed by the positivity of a sequence of determinants (see also I. Schur, "Über algebraische Gleichungen"). He gave a proof of Weierstrass' theorem that an everywhere locally rational function of n variables should be globally rational (*Werke*, paper no. 8). He was much interested in continuous fractions, to which he devoted several papers (*Werke*, paper no. 49, 50, 52, 53, 63). He also gave remarkable proof of Minkowski's theorem on linear forms (*Werke*, paper no. 65).

In algebraic number theory Hurwitz devised new proofs for the fundamental theorem on ideals (*Werke*, papers no. 57, 58, 60, 66). He studied the binary unimodular groups of algebraic number fields of finite degree and proved that they were finitely generated. (A survey on modern extensions of this result is found in Borel's "Arithmetic Properties of Linear Algebraic Groups.") He discovered the "correct" definition of integrity in quaternions (*Werke*, paper no. 64). In the theory of invariants he wrote several papers, among them a new proof for Franz Mertens' theorems on the resultant of n forms in n forms in n variables, in which he introduced the notion of the inertia form (*Werke*, paper no. 86). To obtain orthogonal invariants he devised the invariant volume and integration in the orthogonal groups (*Werke*, paper no. 81), which, generalized to compact groups by I. Schur and H. Weyl and complemented by the invention of Haar's measure, have become extremely powerful tools in modern mathematics.

This was one of the fundamental discoveries for which Hurwitz' name will be remembered. The other is the theorem on the composition of quadratic forms (*Werke*, papers no. 82, 89), which concerns the search for algebras over the reals with a nondegenerate quadratic rorm Q such that $Q(xy) = Q(x)Q(y)$. The complex numbers had ben known as an example of dimension 2 for centuries; in 1843 W. R. Hamilton had discovered the quaternions, of dimension 4; and in 1845 Cayley and J. T. Graves independently hit upon the octaves, of dimension 8. Attempts to go further failed. In 1898 Hurwitz proved that the classical examples exhausted the algebras over the reals with a quadratic norm. With the increasing importance of quaternions and octaves in the theory of algebras, in foundations of geometry, in topology, and in exceptional Lie groups, Hurwitz' theorem has become of fundamental importance. Many new proofs have been given; and it has been extended several times, with the final result by J. W. Milnor that algebras over the reals without zero divisors eixt in dimensions 1, 2, 4, and 8 only.

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