Al-Khayyām | Encyclopedia.com

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also known as **Omar Khayyam**

(*b*. Nīshāpũr, Khurasan [now Iran], 15 May 1048 [?]; *d*. Nīshāpũr, 4 December 1131 [?])

mathematics, astronomy, philosophy.

As his name states, he was the son of Ibrāhīm the epithet "al-Khayyāmī" would indicate that his father or other forebears followed the trade of making tents. Of his other names, "Umar" is his proper designation, while "Ghiyāth al-Din" ("the help of the faith") is an honorific he received later in life and "al-Nīsābũrī" refers to his birthplace. Arabic sources of the twelfth to the fifteenth centuries[1] contain only infrequent an sometimes contradictory references to al-Khayyāmī differing even on the dates of his birth and death. The earliest birthdate given is approximately 1017, but the most probable date (given above) derives from the historian Abu'l-Hasan al-Bayhaqī (1106-11740, who knew al-Khayyāmī personally and left a record of his horoscope. The most probable death date is founded in part upon the account of Nizāmī 'Aũdī Samarqandī's tomb in A.H. 530 (A.D. 1135/1136), four years after the latter's death.[2] This date is confirmed by the fifteenth-century writer Yār-Ahmed Tabrīzī.[3]

At any rate, al-Khayyāmī was born soon after Khurasan was overrun by the Seljuks, who also conquered Khorezm, Iran, and Azerbaijan, over which they established a great but unstable military empire. Most sources, including al-Bayhaqī, agree that he came from Nīshāpũr, where, according to the thirteenth/fourteenth-century historian Fadlallāh Rashīd al-Din, he received his education. Tabrīzī, on the other hand, stated that al-Khayyāmī spent his boyhood and youth in Balkh (now in Afghanistan), and added that by the time he was seventeen he was well versed in all areas of philosophy.

Wherever he was educated, it is possible that al-Khayyāmī became a tutor, Teaching, however, would not have afforded him enough leisure to pursue science. The lot of the scholar at that time was, at best, precarious, unless he were a wealthy man. He could undertake regular studies only if he were attached to the court of some sovereign or magnate, and his work was thus dependent on the attitude of his master, court politics and the fortunes of war. Al-Khayyāmī gave a lively description of the hazards of such an existence at the beginning of his *Risālafi' l-barāhīn 'alā masā il al-jabr wa'l-muqābala* ("Treatise on Demostration of Problems of Algebra and Almuqabala"):

I was unable t devote myself to the learning of this *al-jabr* and the continued concentration upon it, because of obstacles in the vagaries of Time which hindered me; for we have been deprived of all the people of knowledge save for a group, small in number, with many troubles, whose concern in life is to snatch the opportunity, when Time is asleep, to devote themselves meanwhile to the investigation and perfection of a science; for the majority of people who imitate philosophers confuse the true with the false, and they do nothing but deceive and pretend knowledge, and they do not use what they know of the science except for base and material purposes; and if they see a certain person seeking for the right and preferring the truth, doing his best to refute the false and untrue and leaving aside hypocrisy and deceit, they make a fool of him and mock him. [4]

Al-Khayyāmī was nevertheless able, even under the unfavorable circumstances that he described, to write at this time his still unrecovered treatise *Mushkilāt al-hisāb* ("Problems of Arithmetic") and his first, untitled, algebraical treatise, as well as his short work on the theory of music, *al-Qāwl 'alā ajnās allatī bi'l-arba'a* ("Discussion on Genera Contained in a Fourth").

About 1070 al-Khayyāmī reached Samarkand, where he obtained the support of the chief justice, Abũ Tāhir, under whose patronage he wrote his great algebraical treatise on cubic equations, the *Risāa* quoted above, which he had planned long before. A supplement to this work was written either at the court of Shams al-Mulũk, *khaqan* of Bukhara, or at Isfahan, where al-Khayyami had been invited by the Seljuk sultan, Jalāl al-Dīn Malik-shāh, and his vizier Nizām al-Mulk, to supervise the astronomical observatory there.

Al-Khayyāmi stayed at Isfahan for almost eighteen years, which were probably the most peaceful of his life. The best astronomers of the time were gathered at the observatory and there, under al-Khayyāmī's shaāh Astronomical Tables"). Of this work only a small portion—tables of ecliptic coordinates and of the magnitudes fo the 100 brightest fixed starssurvives . A further important task of the observatory was the reform of the solar calendar then in use in Iran.

Al-Khayyāmī presented a plan for calendar reform about 1079. He later wrote up a history of previous reforms, the *Naurū-nāma*, but his own design is known only Nasīr al-Din al-Tũsī and Ulugh Beg. The new calendar was to be based on a cycle of thirtythree years, name “Maliki era” or “Jalālī era” in honor4 of the sultan. The years 4, 8, 12, 16, 20, 24, 28, and 33 of each period were designated as leap years of 366 days. while the average length of the year was to be 365.2424 days (a deviation of 0.0002 day from the true solar calendar), a difference of one day thus accumulating over a span of 5,000 years. (In the Gregorian calendar, the average year is 365.2425 days long 3,333 years.)

Al-Khayyāmī also served as court astrologer, although he himself, according to Nizāmī Samarqandī, did not believe in judicial astrology. Among his other, less official activities during this time, in 1077 he finished writing his commentaries on Euclid’s theory or parallel lines and theory of ratios; this book, together with his earlier algebraical *Risāla*, is his most important scientific contribution. He also wrote on philosophical subjects during these years, composing in 1080 a *Risāla al-kawn wa’l-taklif* (“Treatise on Being and duty”), to which is appended *Al-Jawab ’an thalāth masāil: daũrat al-tadadd fi’l’ālam wa’l-jabr wa’l baqā* (“An Answer to the Three Questions: On the Necessity of Contradiction in the World, on the Necessity of Determinism, and on Longevity”). At about the same time he wrote, for a son of Mu’ayyid al-Mulk (vizier in 1095-1118), *Risāla fi’l kulliyat al-wujũd* (“Treatise on the Universality of Being”). (His two other philosophical works, *Risāla al-diyā’ al-’aqli fi mawdũ’ al-’ilm al kulli* [“Treatise on Existence”] cannot be dated with any certainty.)

In 1092 al-Khayyāmī fell into disfavor, Malik-shāh having died and his vizier Nizām al-Mulk having been murdered by an Assassin. Following the death of Malik-shāh his second wife, Turkān-Khātũn, for two years ruled as regent, and al-Khātũn, for two years ruled as regent, and al-Khayyāmī fell heir to some of the hostility she had demonstrated toward his patron, Nizām al-Mulk, with whom she had quarreled over the question of royal succession. Financial support was withdrawn from the observatory and its activities came to a halt; the calendar reform was not completed; and orthodox Muslims, who disliked al-Khayyāmī because of the religion was to freethinking evident in his quatrains, became highly influential at court . (His apparent lack be a source of difficulty for al-Khayyāmīthroughout his life, and al-Qiftī [1172-1239] reported that in his later in his later years he even undertook a pilgrimage to Mecca to clear himself of the accusation of atheism.)

Despite his fall from grace al-Khayyāmī remained at the Seljuk court. In an effort to induce Malikshāh’s successors to renew their support of the observatory and of science in general, he embarked on a work of propaganda. This was the *Nauũznāma*, mentioned above, an account of the ancient Iranian solar new year’s festival. In it al-Khayyũmī presented a history of the solar calendar and described the ceremonies connected with the Nauũz festival; in particular, he discussed the ancient Iranian sovereigns, whom he pictured as magnanimous, impartial rulers dedicated to education, building edifices, and supporting scholars.

Al-Khayyāmī left Isfahan in the reign of Malikshāh’s third son. Sanjar, who had ascended the throne in 1118. He lived for some time in Merv (now Mary, Turkmen S.S.R.), the new Seljuk capital, where he probably wrote *Mizān al-hikam* (“Balance of Wisdoms”) and *Fi’l-qustas al-mustaqim* (“On Right Qustas”), which were incorporated by his disciple al-Khāzinī (who also worked in Merv), together with works of al-Khayyāmī’s other disciple, al-Muzaffar al-Isifīzarī, into his own *Mīzān* al-hikam, Among other things, al-Khayyāmī’s *Mizān* gives a purely algebraic solution to the problem (which may be tracked back to Archimedes) of determining the quantities of gold an silver in a given alloy by menas of a preliminary determination of the specific weight of each metal. weight and variable scales.[5]

Arithmetic and the Theory of Music . A collection of manuscripts in the library of the University of Leiden, Cod. or. 199, lists al-Khayyāmīs “Problems of Arithmetic” on its title page, but the treatise itself is not included in the collection—it may be surmised that it was part of the original collection from which the Leiden manuscript was copied. The work is otherwise unknown, although in his algebraic work *Risāla fl’ l-barāhīn ’alā masā il-jabr wa’l-muqābala. al-Khayyāmī* wrote of it that:

The Hindus have their own methods for extracting the sides fo squares and cubes based on the investigation of a small number of case, which is [through] the knowledge of the squares of nine integers, that is, the squares of 1, 2, 3, and so on, and of their products into each other, that is, the product of 2 with 3, and so on. I have written a book to prove the validity of those methods and to show that they lead to the required solutions, and I have supplemented it in kind, that is, finding the sides of the square of the square, and the quadrato-cube, and the cubo-cube, however great they may be; and no one has done this before; and these proofs are only algebraical proofs based on the algebraical parts of the book of Elements.[6]

Al-Khayyāmī may have been familiar with the “Hindu methods” that he cites through two earlier works, *Fī usul hisāb al-hind* (“Principles of Hindu Reckoning”), by Kushyār ibn Labbān al-Jili (971-1029), and *Al-muqnī fl’-hisāb al-hindi* (“Things Sufficient to Understand Hindu Reckoning”), by ’Alīibn Ahmad alNasawī (*fl*. 1025). Both of these authors gave methods for extracting square and cube roots from natural numbers, but their method of extracting cube roots differs from the method given in the Hindu literature and actually coincides more closely with the ancient Chinese method. The later was set out as early as the second/first centuries B.C., in the “Mathematics in Nice Books,” and was used by medieval. Chinese mathematicians to extract roots with arbitrary integer exponents and even to solve numerical algebraic equations (it was rediscovered in Europe by Ruffini and Horner at the beginning of the nineteenth century). Muslim mathematics—at least the case of the extraction of the cube root—would thus seem to have been influenced by Chinese, either directly or indirectly. Al-Jīlī’s and al-Nasawī’s term “Hindu reckoning” must then be understood in the less restrictive sense of reckoning in the decimal positional system by means of ten numbers.

The earliest Arabic account extant of the general method for the extraction of roots with positive integer exponents from natural numbers may be found in the *Jāmi al-hisāb bi’l-takht wal-turāb* (“Collection on Arithemtic by Means of Board and Dust”), compiled by al-Tũsī. Since al-Tũsī made no claims of priority of discovery, and since he was well acquainted with the work of al-Khayyāmī, it seems likely that the method he presented is al-Khayyāmī's own. The method that al-Tũsī gave, then, is applied only to the definition of whole part *a* of the root , where

$N=a^n+r, r<(a+1)^n-a^n,$

To compute the correction necessary if the root is not extracted wholly, al-Tũsī formulated—in words rather than symbols—the rule for binomial expansion

$(a + b)^n =a^n + na^{n-r} + . . . + b^n,$

and gave the approximate value of as , the denominator of the root being reckoned according to the binomial formula. For this purpose al-Tũsī provided a table of binomial coefficients up to n = 12 and noted the property of binomials now expressed as

Al-Khayyāmī applied the arithmetic, particularly the theory of commensurable ratios, in his *al-Qawl alā ajnās allatī bi'l-arba a* ("Discussion on Genera Contained in a Fourth"). In the "Discussion" al-Khayyāmī took up the problem—already set by the Greeks, and particularly by Euclid in the *Sectio canois*—of dividing a fourth into three intervals corresponding to the diatonic, chromatic, and enharmonic tonalities. Assuming that the fourth is an interval with the ration 4:3, the three intervals into which the fourth may be divided are defined by ratios of which the product is equal to 4:3. Al-Khayyāmī listed twenty-two examples of the section of the fourth, of which three were original to him. Of the others, some of which occur in more than one source, eight were drawn from Ptolemy's "Theory of Harmony"; thirteen from al-Fārābīs *Kitāb al-musikā al-Kabīr* ("Great Book of Music"); and fourteen from Ibn Sīnā, either *Kitāb al-Shifā* ("The Book of Healing") or *Dānish-nāmah* ("The Book of Knowledge"). Each example was further evaluated in terms of aesthetics.

Theory of Ratios and the Doctrine of Number . Books II and III of al-Khayyāmī's commentaries on Euclid, the *Sharh ma ashkala min musādarāt kitāb Uqlīdis*, are concerned with the theoretical foundations of arithmetic as manifested in the study of the theory of rations. The general theory of ratios and proportions as expounded in book V of the *Elements* was one of three aspects of Euclid's work with which Muslim mathematicians were particularly concerned. (The others were the theory of parallels contained in book I and the doctrine of quadratic irrationals in book X.) The Muslim mathematicians often attempted to improve on Eulid, and many scholars were not satisfied with the theory of ratios in particular. While they did not dispute the truth of the theory, they questioned its basis on Euclid's definition of identity of two rations, $a/b = c/d$, which definition could be traced back to Eudoxus and derived from the quantitative comparison of the equimultiples of all the terms of a given proportion (*Elements*, book V, definition 5).

The Muslim critics of the Euclid-Eudoxus theory of ratios found its weakness to lie in its failure to express directly the process of measuring a given magnitude (*a* or *c*) by another magnitude (*b* or *d*). This process was based upon the definition of a proportion for a particular case of the commensurable quantities *a*, *b*, and *c*, *d* through the use of the so-called Euclidean algorithm for the determination of the greatest common measure of two numbers (*Elements*, book VII). Beginning with al-Māhānī, in the ninth century, a number of mathematicians suggested replacing definition 5, book V, with some other definition that would, in their opinion, better express the essence of the proportion. The definition may be rendered in modern terms by the continued fraction theory: if $a/b=(q_1, q_2,..., q_n,...)$and $c/d=q_1', q_2',..., q_n',...)$, then a/b = c/d under the condition that $q_k' = q_k$ for all *k* up to infinity (for commensurable rations, *k* is finite). Definitions of inequality of ratios a/b > c/d anda/b < c/d, embracing cases of both commensurable and incommensurable ratios and providing criteria for the quantitative comparison of rational and irrationalo values, are introduced analogously. In the Middle Ages it was known that this "anti-phairetical" theory of ratios existed in Greek mathematics before Eudoxus; that it did was discovered only by Zeuthen and Becker. The proof that his theory was equivalent to that set out in the *Elements* was al-Kayyāmī's greatest contribution to the theory of ratios in general. Al-Khayyāmī's proof lay in establishing the equivalence of the definitions of equality and inequalities in both theories, thereby obviating the need to deduce all the propositions of book V of the *Elements* all over again. He based his demonstration on an important theorem of the existence of the fourth proportional *d* with the three given magnitudes *a*. *b*, and *c*; he tried to prove it by means of the principle of the infinite divisibility of magnitudes, which was, however, insufficient for his purpose. His work marked the first attempt at a general demonstration of the theorem, since the Greeks had not treated it in a general manner. These investigations are described in book II of the *Sharh*.

In book III, al-Khayyāmī took up compound ratios (at that time most widely used in arithmetic, as in the rule of three and its generalizations), geometry (the doctrine of the similitude of figures), the theory of music, and trigonometry (applying proportions rather than equalities). In the terms in which al-Khayyāmī, and other ancient and medieval scholars, worked, the ratio *a*/*b* was compounded from the ratio *a*/*c* and *c*/*b*—what would in modern terms be stated as the first ratio being the product of the two latter. In his analysis of the operation of compounding the ratios, al-Khayyāmī first set out to deduce from the definition of a compound ration given in book VI of the *Elements* (which was, however, introduced into the text by later editors) the theorem that the ratio *a*/*c* is compounded from the ratios *a*/*b* and *b*/*c* and an analogous theorem for ratios *a*/*c*, *b*/*c*, *c*/*d*, and so on. Here, cautiously, al-Khayyāmī had begun to develop a new and broader concept of number, including all positive irrational numbers, departing from Aristotle, whose authority he nonetheless respectfully invoked. Following the Greeks, al-Khayyāmī properly understood number as an aggregate of indivisible units. But the development of his own theory—and the development of the whole of calculation mathematics in its numberous applications—led him to introduce new, "ideal" mathematical objects, including the divisible unit and a generalized concept of number which he distinguished from the "absolute and true" numbers (although he unhesitatingly called it a number).

In proving this theorem for compound ratios al-Khayyāmī first selected a unit and an auxiliary quantity g whereby the ratio l/g is the same as a/b. He here took a and b to be arbitrary homogeneous magnitudes which are generally incommensurable; l/g is consequently also incommensurable. He then described the magnitude g:

Let us not regard the magnitude g as a line, a surface, a body, or time; but let us regard it as a magnitude abstracted by reason from all this and belonging in the realm of numbers, but not to numbers absolute and true, for the ratio of a to b can frequently be non-numerical, that is, it can frequently be impossible to find two numbers whose ratio would be equal to this ratio.[7].

Unlike the Greeks, al-Khayyāmī extended arithmetical language to ratios, writing of the equality of ratios as he had previously discussed their multiplication. Having stated that the magnitude g, incommensurable with a unit, belongs in the realm of numbers, he cited the usual practice of calculators and land surveyors, who frequently employed such expressions as half a unit, a third of a unit, and so on, or who dealt in roots of five, ten, or other divisible units.

Al-Khayyāmī thus was able to express any ratio as a number by using either the old sense of the term or the new, fractional or irrational sense. The compounding of ratios is therefore no different from the multiplication of numbers, and the identity of ratios is similar to their equality. In principle, then, ratios are suitable for measuring numerically any quantities. The Greek mathematicians had studied mathematical ratios, but they had not carried out this function to such an extent. Al-Khayyāmī, by placing irrational quantities and numbers on the same operational scale, began a true revolution in the doctrine of number. His work was taken up in Muslim countries by al-Tũsī and his followers, and European mathematicians of the fifteenth to seventeen centuries took up similar studies on the reform of the general ratios theory of the *Elements*. The concept of number grew to embrace all real and even (at least formally) imaginary numbers; it is, however, difficult to assess the influence of the ideas of al-Khayyāmī and his successors in the East upon the later mathematics of the West.

Algebra . Eastern Muslim algebraists were able to draw upon a mastery of Hellenistic and ancient Eastern mathematics, to which they added adaptations of knowledge that had come to them from India and, to a lesser extent, from China. The first Arabic treatise on algebra was written in about 830 by al-Khwārizmī, who was concerned with linear and quadratic equations and dealt with positive roots only, a practice that his successors followed to the degree that equations that could not possess positive roots were ignored. At a slightly later date, the study of cubic equations began, first with Archimedes' problem of the section by a plane of a given sphere into two segments of which the volumes are in a given ratio. In the second half of the ninth century, al-Māhānī expressed the problem as an equation of the type $x^3 + r = px^2$(which he, of course, stated in words rather than symbols). About a century later, Muslim mathematicians discovered the geometrical solution of this equation whereby the roots were constructed as coordinates of points of intersection of two correspondingly selected conic sections—a method dating back to the Greeks. It was then possible for them to reduce a number of problems, including the trisection of an angle, important to astronomers, to the solution of cubic equations. At the same time devices for numerical approximated solutions were created, and a systematic theory became necessary.

Al-Khayyāmī's construction of such a geometrical theory of cubic equations may be accounted the most successful accomplished by a Muslim scholar. In his first short, untitled algebraic treatise he had already reduced a particular geometrical problem to an equation, $x^3+ 200x = 20x^2 + 2{,}000$, and had solved it by an intersection of circumference $y^2=(x - 10).(20 - x)$ and equilateral hyperbola . He also noted that he had found an approximated numerical solution with an error of less than 1 percent, and he remarked that it is impossible to solve this equation by elementary means, since it requires the use of conic sections. This is perhaps the first statement in surviving mathematical literature that equations of the third degree cannot be generally solved with compass and ruler—that is, in quadratic radicals—and al-Khayyāmī repeated this assertion in his later *Risāla*. (In 1637 Descartes presented the same supposition, which was proved by P. Wantzel in 1837.)

In his earlier algebraic treatise al-Khayyāmī also took up the classification of normal forms of equations (that is, only equations with positive coefficients), listing all twenty-five equations of the first, second, and third degree that might possess positive roots. He included among these fourteen cubic equations that cannot be reduced to linear or quadratic equations that cannot be reduced to linear or quadratic equations by division by x^2 or x, which he subdivided into three groups consisting of one binomial equation ($x^3 = r$), six trinomial equations ($x^3 + px^2 = r$; x^3 HI- $r = qx$; $X^3 + r = pX^2$; $x^3 + qx = r$; $x^3 = pX^2 + r$; and $x^3 = qx + r$), and seven quadrinomial equations ($x^3 =px^2$ $gx+r$;$x^3 +qx + r=px^2$;$x^3 + px^2 +r = qx$; $X^3 +px^2 - q-x = r$; $x^3 +px^2 = qx + r$; $X^3 + qx =px^2 +r$; and $x^3 + r =px^2 + qx$).He added that of these four types had been solved (that is, their roots had been constructed geometrically) at some earlier date, but that "No rumor has reached us of any of the remaining ten types, neither of this classification,"[8] and expressed the hope that he would later be able to give a detailed account of his solution of all fourteen types.

Al-Khayyāmī succeeded in this stated intention in his *Risāla*. In the introduction to this work he gave one of the first definitions of algebra, saying of it that, "The art of *al-jabr* and *al-muqaābala* is a scientific art whose subject is pure number and measurable quantities insofar as they are unknown, added to a known thing with the help of which they may be found; and that [known] thing is either a quantity or a ratio"[9] The "pure number" to which al-Khayyāmī refers is natural number, while by "measurable quantities" he meant lines, surfaces, bodies, and time; the subject matter of algebra is thus discrete, consisting of continuous quantities and their abstract ratios. Al-Khayāmī then went on to write, "Now the extractions of *al-jabr* are effected by equating . . .these powers to each other as is well known."[10] He then took up the consideration of the degree of the unknown quantity, pointing out that degrees higher than third must be understood only metaphorically, since they cannot belong to real quantities.

At this point in the *Risāla* al-Khayyāmī repeated his earlier supposition that cubic equations that cannot be reduced to quadratic equations must be solved by the application of conic sections and that their arithmetical solution is still unknown (such solutions in radicals were, indeed, not discovered until the sixteenth century). He did not, however, despair of such an arithmetical solution, adding, "Perhaps someone else who comes after us may find it out in the case, when there are not only the first three classes of known powers, namely the number, the thing, and the square."[11] He then also repeated his classification of twenty-five equations, adding to it a presentation of the construction of quadratic equations based on Greek geometrical algebra. Other new material here appended includes the corresponding numerical solution of quadratic equations and constructions of all the fourteen types of third-degree equations that he had previously listed.

In giving the constructions of each of the fourteen types of third-degree equation, al-khayyāmī also provided an analysis of its "cases." By considering the conditions of intersection or of contact of corresponding conic sections, he was able to develop what is essentially a geometrical theory of the distribution of (positive) roots of cubic equations. he necessarily dealt only with those parts of conic sections that are located in the first quadrant, employing them to determine under what conditions a problem may exist and whether the given type manifests only one case—or one root (including the case of double roots, but not multiple roots, which were unknown)—or more than one case (that is, one or two roots). Al-Khayyāmī went on to demonstrate that some types of equations are characterized by a diversity of cases, so that they may possess no roots at all, or one root, or two roots, He also investigated the limits of roots.

As far as it is known, al-Khayyāmī was thus the first to demonstrate that a cubic equation might have two roots. He was unable to realize, however, that an equation of the type $x^3 + qx = px^2 + r$ may, under certain conditions, possess three (positive) roots; this constitutes a disappointing deficiency in his work. As F. Woepcke, the first editor of the *Risāla*, has shown, al-Khayyāmī followed a definite system in selecting the curves upon which he based the construction of the roots of all fourteen types of third-degree equations; the conic sections that he preferred were circumferences, equilateral hyperbolas of which the axes, or asymptotes, run parallel to coordinate exes; and parabolas of which the axes parallel one of the coordinate axes. His general geometrical theory of distribution of the roots was also applied to the analysis of equations with numerical coefficients, as is evident in the supplement to the *Risāla*, in which al-Khayyāmī analyzed an error of Abũ'I-Kũd Muhammad ibn Layth, an algebraist who had lived some time earlier and whose work al-Khayyāmī had read a few years after writing the main text of his treatise.

His studies on the geometrical theory of third degree equations mark al-Khayyāmī's most successful work. Although they were continued in oriental Muslim countries, and known by hearsay in Moorish countries, Europeans began to learn of them only after Descartes and his successors independently arrived at a method of the geometrical construction of roots and a doctrine of their distribution. Al-Khayyāmī did further research on equations containing degrees of a quantity inverse to the unknown ("part of the thing," "part of the square," and so on) including, for example, such equations as $1/x^3 + 3\ 1/x^2 + 5\ 1/x = 33/8$, which he reduced by substituting $x = 1/z$ in the equations that he had already studied. He also considered such cases as $x^2 + 2x = 2$ H- $2\ 1/x^2$, which led to equations of the fourth degree, and here he realized the upper limit of his accomplishment, writing, "If it [the series of consecutive powers] extends to five classes, or six classes, or seven, it cannot be extracted by any method."[12]

The Theory of Parallels. Muslim commentators on the *Elements* as early as the ninth century began to elaborate on the theory of parallels and to attempt to establish on the theory of parallels and to attempt to establish it on a basis different from that set out by Euclid in his fifth postulate. Thābit ibn Qurra and Ibn al-Haytham had both been attracted to the problem, while al-Haytham had both been attracted to the problem, while al-Khayyāmī devoted the first book of his commentaries to the *Sharah* to it. Al-Khayyāmī took as the point of departure for his theory of parallels a principle derived, according to him, from "the philosopher," that is, Aristotle, namely that "two convergent straight lines should diverge in the direction of convergence."[13] Such a principle consists of two statements, each equivalent to Euclid's fifth postulate. (It must be noted that nothing similar to al-Khayyāmī's principle is to be found in any of the known writings of Aristotle.)

Al-Khayyāmī first proved that two perpendiculars to one straight line cannot intersect because they must intersect symmetrically at two points on both sides of the straight line; therefore they cannot converge. From the second statement the principle follows that two perpendiculars drawn to one straight line cannot diverge because, if they did, they would have to diverge on both sides of the straight line. Therefore, two perpendiculars to the same straight line neither converge nor diverge, being in fact equidistant from each other.

Al-Khayyāmī then went on to prove eight propositions, which, in his opinion, should be added to book I of the *Elements* in place of the proposition 29 with which Euclid began the theory of parallel lines based on the fifth postulate of book I (the preceding twenty-eight propositions are not based on the fifth postulate). He constructed a quadrilateral by drawing two perpendicular lines of equal length at the ends of a given line segment *AB*. Calling the perpendiculars *AC* and *BD*, the figure was thus bounded by the segments *AB*, *AC*, *CD*, and *BD*, a birectangle often called "Saccheri's quadrilateral," in honor of the eightteenth-century geometrician who used it in his own theory of parallels.

In his first three propositions, al-Khayyāmī proved that the upper angles *C* and *D* of this quadrilateral are right angles. To establish this theorem, he (as Saccheri did after him) considered three hypotheses whereby these angles might be right, acute, or obtuse; were they acute, the upper line *CD* of the figure must be longer than the base *AB*, and were they obtuse, *CD* must be shorter than *AB*—that is, extensions of sides *AC* and *BD* would diverge or converge on both ends of *AB*. The hypothetical acute or obtuse angles are therefore proved to be contradictory to the given equidistance of the two perpendiculars to one straight line, and the figure is proved to be a rectangle.

In the fourth proposition al-Khayyāmī demonstrated that the opposite sides of the rectangle are of equal length, and in the fifth, that it is the property of any two perpendiculars to the same straight line that any perpendicular to one of them is also the perpendicular to the other. The sixth proposition states that if two straight lines are parallel in Euclid's sense—that is, if they do not intersect—they are both perpendicular to one straight line. The seventh proposition adds that if two parallel straight lines are intersected by a third straight line, alternate and corresponding angles are equal, and the interior angles of one side are two right angles, a proposition coinciding with Euclid's book I, proposition 29, but one that al-Khayyāmī reached by his own, non coincident methods.

Al-Khayyāmī eighth proposition proves Euclid's fifth postulate of book I: two straight lines intersect if a third intersects them at angles which are together less than two right angles. The two lines are extended and a straight line, parallel to one of them, is passed through one of the points of intersection. According to the sixth proposition, these two straight lines—being one of the original lines and line drawn parallel to it—are equidistant, and consequently the two original lines must approach each other. According to al-Khayyāmī's general principle, such straight lines are bound to intersect.

Al-Khayyāmī's demonstration of Euclid's fifth postulate differs from those of his Muslim predecessors because he avoids the logical mistake of *petitio principle*, and deduces the fifth postulate from his own explicitly the same as the first theorems of the non-Euclidean geometries of Lobachevski and Riemann. Like his theory of ratios, al-Khayyāmī's theory of parallesls influenced the work of later Muslim scholars to a considerable degree. A work sometimes attributed to his follower al-Tūsī influenced the development of the theory of parallels in Europe in the seventeenth and eighteenth centuries, as was particularly reflected in the work of Wallis and Saccheri.

Philosophical and Poetical Writings. Although al-Khayyāmī wrote five specifically philosophical treatises, and although much of his poetry is of a philosophical nature, it remains difficult to ascertain what his world view might have been. Many investigators have dealt with this problems, and have reached many different conclusions, depending in large part on their own views. The problem is complicated by the consideration that the religious and philosophical tracts differ from the quatrains, while analysis of the quatrains themselves is complicated by questions of their individual authenticity. Nor is it possible to be sure of what in the philosophical treatises actually reflects al-Khayyāmī's own mind, since they were written under official patronage.

His first treatise, *Risālat al-kawn wa'taklif* ("Treatise on Being and Duty"), was written in 1080, in response to a letter from a high official who wished al-Khayyāmī to give his views on "the Divine Wisdom in the Creation of the World and especially of Man and on man's duty to pray."[14] The second treatise, *Al-Jawab 'an thalāth masā'il* ("An Answer to the Three Questions"), closely adheres to the formula set out in the first. *Risāla fi'l kulliyat al-wujūd* ("Treatise on the Universality of Being") was written at the request of Mu'ayyid al-Mulk, and, while it is not possible to date or know the circumstances under which the remaining two works, *Risālat al-diyā' al-'aqli fi mawdū' al-'ilm al-kulli* ("The Light of Reason on the Subject of Universal Science") and *Risāla fi'l wujūd* ("Treatise on Existence"), were written, it would seem not unlikely that they had been similarly commissioned. Politics may therefore have dictated the contents of the religious tracts, and it must be noted that the texts occasio9nally strike a cautious and impersonal note, presenting the opinions of a number of other authors, without criticism or evaluation.

It might also be speculated that al-Khayyāmī wrote his formal religious and philosophical works to clear his name of the accusation of freethinking. Certainly strife between religious sects and their common aversion to agnosticism were parat of the climate of the time, and it is within the realm of possibility that al-Khayyāmī's quatrains had become known to the religious orthodoxy and had cast suspicion upon him. (The quatrains now associated with his name contain an extremely wide range of ideas, ranging from religious mysticism to materialism and almost atheism; certainly writers of the thirteenth century thought al-Khayyāmī a freethinker, al-Qiftāi calling the poetry "a stinging serpent to the Sharia" and the theologian Abū Bakr Najm al-Dīn al-Rāxi characterizing the poet as "an unhappy philosopher, materialist, and naturalist.")[15]

Insofar as may be generalized, in his philosophical works al-Khayyāmī wrote as an adherent of the sort of eastern Aristotelianism propagated by Ibn Sīlnā—that is, of an Aristotelianism containing considerable amounts of Platonism, and adjusted to fit Muslim religious doctrine. Al-Bayhaqī called al-Khayyāmī "a successor of Abū Ali [Ibn Sīnā] in different domains of philosophical sciences,"[16] but from the orthodox point of view such a rationalistic approach to the dogmas of faith was heresy. At any rate, al-Khayyāmī's philosophy is scarcely original, his most interesting works being those concerned with the analysis of the problem of existence of general concepts. Here al-Khayyāmī—unlike Ibn Sānā, who held views close to Plato's realism—developed a position similar to that which was stated simultaneously in Europe by Abailard, and was later called conceptualism.

As for al-Khayyāmī's poetical works, more than 1,000 quatrains, written in Persian, are now published under his name. (Govinda counted 1,069.) The poems were preserved orally for a long time, so that many of them are now known in several variants. V. A. Zhukovsky, a Russian investigator of the poems, wrote of al-Khayyāmī in 1897:

He has been regarded variously as a freethinker, a subverter of Faith, an atheist and materialist; a pantheist and a scoffer at mysticism; an orthodox Musulman; a true philosopher, a keen observer, a man of learning; a bon vivant, a profligate, a dissembler, and a hypocrite; a blasphemer—nay, more, an incarnate negation of positive religion and and of all moral beliefs; a gentle nature, more given to the contemplation of things diving than the wordly enjoyments; an epicurean septic; the Persian Abū'l-'Alā, Voltaire, and Heine. One asks oneself whether it is possible to conceive, not a philosopher, but merely an intelligent man (provided he be not a moral deformity) in whom were conmmingled and embodied such a diversity of convictions, paradoxical inclinations and tendencies, of high moral courage and ignoble passions, of torturing doubts and vacillations?[17]

The inconsistencies noted by Zhukovsky are certainly present in the corpus of the poems now attributed to al-Khayyāmī, and here again questions of authenticity arise. A. Christensen, for example, thought that only about a dozen of the quatrains might with any certainty be considered genuine, although later he increased this n umber to 121. At any rate, the poems generally known as al-khayyāmī's are one of the summits of philosophical poetry, displaying an unatheistic freethought and love of freedom, humanism and aspirations for justice, irony and skepticism, and above all an epicurean spirit that verges upon hedonism.

Al-Khayyāmī's poetic genius was always celebrated in the Arabic East, but his fame in European countries is of rather recent origin. In 1859, a few years after Woepcke's edition had made al-Khayyāmī' algebra—previously almost unknown—available to Western scholars, the English poet Edward FitzGerald published translations of seventy-five of the quatrains, an edition that remains popular. Since then, many more of the poems have been published in a number of European languages.

The poems—and the poet—have not lost their power to attract. In 1934 a monument to al-Khayyāmī was erected at his tomb in Nishāpūr, paid for by contributions from a number of countries.

NOTES

1. V. A. Zhukovsky, *Omar Khayyam i "stranstvuyushchie" chetverostishia*; Swami Govinda Tirtha, *The Nectar of Grace*; and Nizāmī 'Arumacr;di Samarqandi, *Sobranie redkostei ili chetyre besedy*.

2. Samarqandī, *op. cit*., p. 97; in the Browne trans., p. 806, based on the later MSS, "four years" is "some years."

3. Govinda, *op. cit*., pp. 70-71.

4. *Risāla fi'l-barāhin 'alā masū' il al-janr wa' l-muqābala*, Winter 'Arafat trans., pp. 29-30.

5. I. S. Levinova, "Teoria vesov v traktatakh Omara Khayyama i ego uchenika Abu Hatima al-Muzaffara ibn Ismila al-Asfizari."

6. *Risāla*, Winter-'Arafat trans., pp. 34 (with correction), 71.

7. *Omar Khayyam, Trajtaty*, pp. 71, 145.

8. First algebraic treatise, Krasnova and Rosenfeld trans., p. 455; omitted from Amir-Moéz trans.

9. *Risāla*, Winter-'Arafat trans., p. 30 (with correction).

10. *Ibid*., p. 31.

11. *Ibid*., p. 31 (with correction).

12. *Ibid*., p. 70.

13. *Omar Khayyam, Traktaty*, pp. 120-121; omitted from *Sharh mā ashkala min musādarāt kitāb Uqlidis*. Amir-Moéztrans.

14. *Omar Khayyam, Traktaty*, p. 152.

15. Zhukovsky, *op. cit*., pp. 334, 342.

16. Govinda, *op. cit*., pp. 32-33.

17. Zhukovsky, *op. cit*., p. 325.

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