Sonya Kovalevsky | Encyclopedia.com

Complete Dictionary of Scientific Biography COPYRIGHT 2008 Charles Scribner's Sons
13-16 minutes

(*b*. Moscow, Russia, 15 January “1850; *d*. Stockholm, Sweden, 10 February 1891)

mathematics.

Sonya Kovalevsky was the greatest woman mathe. matician prior to the twentieth century. She was the daughter of Vasily Korvin-Krukovsky, an artillery general, and Yelizaveta Shubert, both well-educated members of the Russian nobility. The general was said to have been a direct descendant of Mathias Korvin, king of Hungary; Soviet writers believe that Krukovsky’s immediate background was Ukrainian and that his family coat of arms resembled the emblem of the Polish Korwin-Krukowskis.

In *Recollections of Childhood* (and the fictionalized version, *The Sisters Rajevsky*), Sonya Kovalevsky vividly described her early life: her education by a governess of English extraction; the life at Palabino (the Krukovsky country estate); the subsequent move to St. Petersburg; the family social circle, which included Dostoevsky; and the general’s dissatisfaction with the “new” ideas of his daughters. The story ends with her fourteenth year. At that time the temporary wallpaper in one of the children’s rooms at Palabino consisted of the pages of a text from her father’s schooldays, namely, Ostrogradsky’s lithographed lec. ture notes on differential and integral calculus. Study of that novel wall-covering provided Sonya with her introduction to the calculus. In 1867 she took a more rigorous course under the tutelage of Aleksandr N. Strannolyubsky, mathematics professor at the naval academy in St. Petersburg, who immediately recognized her great potential as a mathematician.

Sonya and her sister Anyuta were part of a young people’s movement to promote the emancipation of women in Russia. A favorite method of escaping from bondage was to arrange a marriage of conven. ience which would make it possible to study at a foreign university. Thus, at age eighteen, Sonya con. tracted such a nominal marriage with Vladimir Kovalevsky, a young paleontologist, whose brother Aleksandr was already a renowned zoologist at the University of Odessa. In 1869 the couple went to Heidelberg, where Vladimir studied geology and Sonya took courses with Kirchhoff, Helmholtz, Koenigsberger, and du Bois-Reymond. In 1871 she left for Berlin, where she studied with Weierstrass, and Vladimir went to Jena to obtain his doctorate. As a woman, she could not be admitted to university lectures; consequently Weierstrass tutored her pri. vately during the next four years. By 1874 she had completed three research papers on partial differential equations, Abelian integrals, and Saturn’s rings. The first of these was a remarkable contribution, and all three qualified her for the doctorate in *absentia* from the University of Göttingen.

In spite of Kovalevsky’s doctorate and strong letters of recommendation from Weierstrass. she was unable to obtain an academic position anywhere in Europe. Hence she returned to Russia where she was reunited with her husband. The couple’s only child, a daughter, “Foufie,” was born in 1878, When Vladimir’s lectureship at Moscow University failed to materialize, he and Sonya worked at odd jobs, then engaged in business and real estate ventures. An unscrupulous company involved Vladimir in shady speculations that led to his disgrace and suicide in 1883. His widow turned to Weierstrass for assistance and, through the efforts of the Swedish analyst Gösta Mittag-Leffler. one of Weierstrass’ most dis. tinguished disciples, Sonya Kovalevsky vas appointed to a lectureship in mathematics at the University of Stockholm. In 1889 Mittag-Leffler secured a life professorship for her.

During Kovalevsky’s years at Stockholm she carried on her most important research and taught courses (in the spirit of Weierstrass) on the newest and most advanced topics in analysis. She completed research already begun on the subject of the propaga. tion of light in a crystalline medium. Her memoir, On the *Rotation of a Solid Body About a Fixed Point* (1888), won the Prix Bordin of the French Academy of Sciences. The judges considered the paper so exceptional that they raised the prize from 3,000 to 5,000 francs. Her subsequent research on the same subject won the prize from the Swedish Academy of Sciences in 1889. At the end of that year she was elected to membership in the Russian Academy of Sciences. Less than two years later, at the height of her career.she died of influenza complicated by pneumonia.

In mathematics her name is mentioned most frequently in connection with the Cauchy-Kovalevsky theorem, which is basic in the theory of partial differential equations. Cauchy had examined a funda. mental issue in connection with the existence of solu. tions, but Sonya Kovalevsky pointed to cases that neither he nor anyone else had considered. Thus she was able to give his results a more polished and general form. In short, Cauchy, and later Kovalevsky, sought necessary and sufficient conditions for the solution of a partial differential equation to exist and to be unique, In the case of an ordinary differential equation the general solution contains arbitrary constants and therefore yields an infinity of formulas (curves); in the general solution of a partial differential equation, arbitrary functions occur and the plethora of formulas (surfaces or hypersurfaces) is even greater than in the ordinary case. Hence additional data in the form of “initial” or “boundary” conditions are needed if a unique particular solution is required.

The simplest form of the Cauchy-Kovalevsky theorem states that any equation of the form

$p = f(x, y, z, q)$

where $p=\partial z/\partial x$, $q=\partial z/\partial y$, and the function *f* is analytic (has convergent power series development. in its arguments for values near (x_0, y_0, z_0, q_0), pos. sesses one and only one solution $z\ (x, y)$ which is analytic near (x_0, y_0) and for which

$z(x_0, y)=g(y)$

where g (y) is analytic at y_0 with

where $g\ (y_0) = z_0$ and $g'(y_0) = q_0$

In the general theorem, the simple case illustrated is generalized to functions of more than two independent variables, to derivatives of order higher than the first, and to systems of equations.

To place Sonya Kovalevsky’s second doctoral paper and some of her later research in a proper setting, one must examine analytic concepts developed grad. ually in the work of Legendre, Abel, Jacobi, and Weierstrass. It is a familiar fact of elementary calculus that the integral,

$\int f(x, y)\, dx,$

can be expressed in terms of elementary functions (algebraic, trigonometric, inverse trigonometric, expo. nential, logarithmic) if y^2 is a polynomial of degree 1 or 2 in x, and (x, y) is a rational function of x and y.If the degree of the polynomial for y^2 is greater than 2, elementary expression is not generally possible. If the degree is 3 or 4, the integral is described as *elliptic* because a special case of such an integral occurs in the problem of finding the length of an arc of an ellipse. If the degree is greater than 4, the integral is called *hyperelliptic*. Finally, one comes to the general type that includes the others as special cases. If y is an algebraic function of x, that is, if y is a root of $P\ (x, y) = 0$, where P is a polynomial in x and y, the above integral is described as *Abelian*, after Abel, who carried out the first important research with such integrals. Abel’s brilliant inspiration also clarified and simplified the theory of elliptic integrals (just after Legendre had given some forty years to investi. gating their properties).

If the integral

is “inverted,” one obtains $x = \sin u$, which elementary trigonometry indicates to be easier to manipulate than its inverse, $u = \sin^{-1}x$. Therefore it occurred to Abel (and subsequently to Jacobi) that the inverses of elliptic integrals might have a simpler theory than that of the integrals themselves. The conjecture proved to be correct, for the inverses, namely the *elliptic functions*, lend themselves to a sort of higher trigo. nometry of doubly periodic functions. For example, while the period of sin x is $2\ \pi$, the corresponding elliptic function, sn z, has two periods whose ratio is a complex number, a fact indicating that the theory of elliptic functions belongs to complex (rather than real) analysis. Inversion of Abelian integrals leads to *Abelian functions* which, in the first generalization beyond the elliptic functions, have two independent complex variables and four periods.

Abel died within a year of the research he started in that area, and there was left to Weierstrass and his pupils the stupendous task of developing the theory of general Abelian functions having k complex variables and $2k$ periods and of considering the implications for the inverses, the corresponding Abelian integrals. Kovalevsky’s doctoral research contributed to that theory by showing how to express a certain species of Abelian integral in terms of the relatively simpler elliptic integrals.

Complex analysis and nonelementary integrals were also a feature of the Kovalevsky paper which won the Bordin Prize. In her paper site generalized work of Euler, Poisson, and Lagrange, who had considered two elementary cases of the rotation of a rigid body about a fixed point. Her predecessors had treated two symmetric forms of the top or the gyro. scope, whereas she solved the problem for an asym. metric body. This case is an exceedingly difficult one and she was able to solve the differential equations of motion by the use of hyperelliptic integrals. Her solution was so general that no new case of rotatory motion about a fixed point has been researched to date.

In her study of the form of Saturn’s rings, as in her other research, she had great predecessors—Laplace, in particular, whose work she generalized. Whereas, for example, he thought certain cross sections to be elliptical, she proved that they were merely egg. shaped ovals symmetric with respect to a single axis. Although Maxwell had proved that Saturn’s rings could not possibly be continuous bodies—either solid or molten and hence must be composed of a myriad of discrete particles, Kovalevsky considered the general problem of the stability of motion of liquid ring-shaped bodies; that is, the question of whether such bodies tend to revert to their primary motion after disturbance by external forces or whether deviation from that motion increases with time. Other researchers completed her task by establishing the instability of such motion.

Her concern for Saturn's rings caused the British algebraist Sylvester to write a sonnet (1886) in which he named her the" Muse of the Heavens. "Later, Fritz Leffler, the mathematician's brother, stated in a poetic obituary,

While Saturn's rings still shine,

While mortals breathe,

The world will ever remember your name.

She was remembered by the eminent Russian historian Maxim Kovalevsky (who was unrelated to her husband) who dedicated several works to her. She had met him when he came to lecture at Stock. holm University in 1888 after he had been discharged from Moscow University for criticizing Russian constitutional law. It was believed that they were engaged to be married but that she hesitated because his new permanent position was in Paris, and joining him there would have meant sacrificing the life professorship for which she had worked so long and hard.

She was remembered, too, by her daughter who, at the age of seventy-two, was guest speaker when the centenary of her mother's birth was celebrated in the Soviet Union. After her mother's death, Foufie had returned to Russia to live at the estate of her godmother Julia Lermontov, a research chemist and agronomist, and a good friend from Sonya's Heidel. berg days. Foufie studied medicine and translated major foreign literary works into Russian.

An unusual aspect of Sonya Kovalevsky's life was that, along with her scientific work, she attempted a simultaneous career in literature. The titles of some of her novels arc indicative of their subject matter: *The University Lecturer, The Nihilist* (unfinished), The *Woman Nihilist*, and, finally, *A Story of the Riviera*.In 1887 she collaborated with her good friend and biographer, Mittag-Leffier's sister, Anne Charlotte Leffler-Edgren (later Duchess of Cajanello), in writing a drama, *The Struggle for Happiness*, which was favorably received when it was produced at the Korsh Theater in Moscow, She also wrote a critical com. mentary on George Eliot, whom she and her husband had visited on a holiday trip to England in 1869.

BIBLIOGRAPHY

I. Original Works. Among Kovalevsky's papers are"Zur Theorie der partiellen Differential-gleichungen," in *Journal für die reine und angewandte Mathematik*, **80** (1875), 1-32; "Zusätze und Bemerkungen zu Laplaccs Unter. suchungen über die Gestalt der Saturnsringe," in *Astrono. mische Nachrichten*, **3** (1883), 37-48; "Über die Reduction eincr bestimmten Klasse Abelscher Integrale dritten Ranges auf elliptische Integrate," in *Acta Mathematica*, **4** (1884), 393-414; and "Sur le probléme de la rotation d'un corps solide autour d'un point fixe," in *Acta Mathematica*, **12** (1889), 177-232.

II. Secondary Literature. See E. T, Bell, *Men of Mathematics* (New York, 1937). 423-429; J. L Geronimus, *Sofja Wasilyvena Kowalewskaja—Mathematische Berech. nung der Kreiselbewegung* (Berlin, 1954); E. E. Kramer, *The Main Stream of Mathematics* (New York, 1951), 189-196, and *The Nature and Growth of Modern Mathematics* (New York, 1970), 547-549; A. C. Leffler-Edgren, duchessa di Cajanello, *Sonia Kovalevsky*, *biography and Autobio. xruphy*, English trans., L. von Cossel (New York, 1895);O. Manville, "Sophie Kovalevsky," in *Mélanges scientifigures offers à M, Luc Picart* (Bordeaux, 1938); G. Mittag. Leffler, "Sophie Kovalevsky, notice biographique, "in Acta Mathematica, **16** (1593), 385-390; and P. Polubari. nova-Kochina. *Sophia Vasilyevna Kovalevskaya*, *Her Life and Work*, English trans., P. Ludwick (Moscow, 1957).

Edna E. Kramer