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(*b*. Rotterdam, Netherlands, 17 December 1894; *d*. Oegstgeest, Netherlands, 24 April 1952)

theoretical physics.

Kramers, the third of five sons of a physician, received his early schooling at Rotterdam. In 1912 he went to Leiden and studied theoretical physics, mainly with P. Ehrenfest, who in 1912 had succeeded H. A. Lorentz. In 1916, after passing his *doctoraal* (roughly equivalent to obtaining a master’s degree), he taught for a few months in a secondary school and in September set out for Copenhagen, where he became a close collaborator of Niels Bohr. In 1920 Bohr’s Institute of Theoretical Physics was opened; Kramers was first an assistant and in 1924 became a lecturer. In 1926 he accepted the chair of theoretical physics at Utrecht and in 1934 returned to Leiden as a successor to Ehrenfest, who had died in September 1933. From 1934 until his death Kramers taught at Leiden and paid numerous visits to other countries, including the United States.

During his years at Copenhagen, Kramers worked mainly on the further development of the quantum theory of the atom. It was a surprising feature of Bohr’s theory that the frequency of a spectral line determined by the equation

$$h_v = E_n - E_m$$

did not coincide with a kinetic frequency of electrons. The situation was mitigated by Bohr’s correspondence principle: The frequency v is an average of kinetic frequencies of electrons in the initial and in the final states, and in the limit of high quantum numbers these two frequencies and the frequency of the emitted radiation approach each other. Bohr further concluded that polarizations and intensities should, in the limit of high quantum numbers, be given by the Fourier components of the quantized motion and that even at low quantum numbers the Fourier com. ponents in the initial and final states should give an indication of the intensities to be expected. In his doctoral thesis at Leiden in 1919 (published by the Royal Danish Academy of Sciences) Kramers devel. oped the mathematical formalism required to apply these ideas; he also carried out detailed calculations for the case of a hydrogen atom in an external electric field. This led to a satisfactory interpretation of the intensities of Stark components.

Other papers from this period deal with the rela. tivistic theory of the Stark effect in hydrogen (1920), the continuous X-ray spectrum (1923), and the quantization of the rotation of molecules when there is a “built-in flywheel” (an electronic angular momen. tum around an axis fixed with respect to the molecule [1923]). His paper on the helium atom (1923) was of special importance for the development of quantum theory. In this paper Kramers showed that application of the theory of quantization of classical orbits to the fundamental state of helium docs not lead to a stable state and gives far too low a value for the binding energy. He pointed out that this revealed the funda. mental inadequacy of the provisional quantum theory. From then on, the helium atom became a test case for a new theory. Eventually this challenge was successfully met by the new quantum mechanics, as was shown by W. Heisenberg and, with greater numerical precision, by Hylleraas.

Kramers was coauthor of the famous paper by Bohr, Kramers, and J. C. Slater (1924) which suggested that conservation of energy might not hold in elemen. tary processes. Although this idea was not substan. tiated by subsequent experimental and theoretical work, the paper had a profound influence. It empha. sized the notion of virtual oscillators associated with quantum transitions. This concept formed the basis for Kramers’ theory of dispersion. In classical theory an isotropic harmonic oscillator with charge e, mass m, and frequency v_1, in an alternating electric field with amplitude E and frequency v, would acquire an induced polarization.

Kramers showed that a similar formula should hold in quantum theory. To each possible transition there corresponds a virtual oscillator with an effective value $(e^2/ m)^*$ that can be calculated from the transition probabilities. For a transition to a higher level this value is positive but for a transition to a lower level it is negative, an entirely new feature closely related to Einstein’s stimulated emission.

In Kramers’subsequent paper with Heisenberg (1924) the theory was developed in more detail; it was shown that one must expect the scattered radiation also to contain frequencies $v \pm v_1$. This paper thus described quantitatively the effect that was later found experimentally by Raman and that had already been predicted by Smekal on the basis of considerations on light quanta. The notion of virtual oscillators was the starting point of Heisenberg’s quantum mechanics—the virtual oscillators became the matrix elements of the coordinates. In connection with the theory of dispersion, Kramers also wrote two later papers (1927, 1929) in which he established the now well. known relations between the real and the imaginary part of the polarizability (Kramers-Kronig relations).

Kramers' later work, produced after his departure from Copenhagen, may be divided into four groups. There were a number of papers dealing with the mathematical formalism of quantum mechanics. One of his earliest and best-known papers in this field dealt with what became later known as the W(entzel)-K (ramers)-B (rillouin) method (1926). It is a method to obtain approximate solutions of a one-dimensional Schrödinger equation of the form

$$U'' + (\lambda - W(x))U = 0$$

One approximate solution is

$$(\lambda - W)^{-1/4}\exp\int(\lambda - W)^{-1/2}dx$$

but this solution breaks down near the zeros of $\lambda - W$. In this region Kramers replaces $\lambda - W$ with αx. Then the solution becomes a Bessel function of order 1-3, the behavior of which can easily be discussed. A solution can then be "patched" together from the solution shown above and the Bessel function. Kramers showed that this leads to the quantization rule of the older quantum theory but with quantum numbers:

$n + 1/2$ (n =0,1,2,...).

Although this method yielded quite satisfactory approximate wave functions—compare, for example, the paper that Kramers wrote with E. M. van Engers on the ion of molecular hydrogen (1933)—its practical value has diminished since the arrival of modern computers; it nevertheless remains valuable in eluci. dating the relations between quantum mechanics, classical mechanics, and the older methods of quantiza. tion.

Kramers developed a special formalism for dealing with the theory of the multiplet structure of spectra (1930). It was based on Weyl's treatment of the rotation group combined with notations current in the theory of invariants. By this powerful method he derived a general formula for the quantum mechanical analogon of the classical expression P_ι (cos AB), where P_ι is a Legendre polynomial (1931). For $l = 1$ one obtains the well-known Landé cosine; for $l = 2$ and $l = 3$ the expressions for quadrupole and octopole coupling. Later (1943) Kramers also gave a treat. ment of multipole radiation.

With G. P. Ittmann, Kramers studied the Schrödinger equation of the asymmetric top and made several additions to the theory of Lame functions (1933, 1938).In a very elegant paper (1935) he dealt with the solu. tions and eigenvalues of the Schödinger equation for a particle in a one-dimensional periodic force field. Kramers showed in a very general way that there exists an infinite number of zones of allowed energy values separated by forbidden regions. In many of these papers Kramers is as much a mathematician as a physicist. Also his textbook on quantum mechanics (1933, 1938) contains a wealth of mathematical detail not found elsewhere. It is even more valuable, however, because it analyzes very carefully the basic principles and assumptions of quantum mechanics.

A second group of papers dealt with paramagnetism, magneto-optical rotation, and ferromagnetism. Several of these papers were the result of Kramers' collabora. tion with Jean Becquerel, who regularly came to Leiden to perform low-temperature measurements on magneto-optic rotation in crystals of the rare earths. Kramers' calculations of the behavior of magnetic ions were essentially straightforward and concur in many cases with experimental results. Mention should be made of "Kramers' theorem": If an ion containing an odd number of electrons is placed in an arbitrary static electric field, then every state remains 2 p-fold degenerate ($p = 1, 2, 3, \cdots$). In particular the lowest state is always at least doubly degenerate. Kramers was coauthor of the first papers on cooling by adiabatic demagnetization published by the Leiden school (1933, 1934).

Two of Kramers' papers (1934, 1936) dealt with ferromagnetism and the theory of spin waves; they formed the transition to a third group of papers—those dealing with statistical and kinetic theory. With Wannier, Kramers studied the two-dimensional Ising model. He was unable to find a complete analytical solution—that was done later by L. Onsager, who was much influenced by Kramers' work—but he was able to show that the Curie temperature T_c, if it exists, is related to the coupling constant J by the equation $J/kT_c = 0.8814$; and he worked out approx. imate solutions for high and low temperatures.

Kramers and J. Kistemaker made an important contribution to the kinetic theory of gases (1943, 1949).Maxwell had already shown that the aerodynamic boundary condition, according to which the velocity of a gas at the surface of a wall is equal to the velocity of the wall, is not strictly valid when there is a velocity gradient perpendicular to the wall or a temperature gradient along the wall; Maxwell had calculated in 1879 both this viscosity slip and the thermal slip. Kramers noticed that there should also be a diffusion slip which occurs when there is a concentration gradient along the wall. This would lead to a pressure gradient's arising in a stationary state of diffusion through a capillary, and experiments confirmed this prediction.

Mention should also be made of an early contribu. tion to the theory of strong electrolytes (1927), of a paper on the behavior of macromolecules in inhomo. geneous flow (1946), and a very instructive paper on the use of Gibbs's "grand ensemble" (1938).

In his treatment of Brownian motion in a field of force, Kramers dealt specifically with the escape of a particle over the edge of a potential-hole. Although the most important factor in the probability of escape is

$\exp(-Q/kT)$,

where Q is the height of the potential barrier, Kramers found quite different factors in front of the exponential, depending on whether the viscosity was large or small. The model is used to discuss chemical reactions—in 1923 Kramers had already written on chemical reactions with J. A. Christiansen—but it can also be used in connection with the Bohr-Wheeler theory of fission and has many other applications.

Finally there were also a number of papers on relativistic formalisms in particle theory and on the theory of radiation. Kramers' report to the 1948 Solvay Congress, entitled "Nonrelativistic Quantum Electrodynamics and Correspondence Principle" sum. marized ideas that had already been presented in his textbook on quantum mechanics (1933, 1938). His aim was to arrive at structure-independent results, and his method involved a separation between the proper field of the electron and the external field. To a certain extent these considerations have been superseded by later developments of quantum electro. dynamics.

Kramers' work, which covers almost the entire field of theoretical physics, is characterized both by outstanding mathematical skill and by careful analysis of physical principles. It also leaves us with the impression that he tackled problems because he found them challenging, not primarily because they afforded chances of easy success. As a consequence his work is somewhat lacking in spectacular results that can easily be explained to a layman; but among fellow theoreticians he was universally recognized as one of the great masters. He played an important part in the scientific life of his country and in the world of physics.

In 1946 Kramers was elected chairman of the Scientific and Technological Committee of the United Nations Atomic Energy Commission, and he presented a unanimous report on the technological feasibility of control of atomic energy. From 1946 to 1950 he was president of the International Union of Pure and Applied Physics.

Kramers received honorary degrees from the universities of Oslo, Lund, Stockholm, and the Sorbonne and was a member of many learned societies. He is also remembered as a gifted musician and an excellent linguist.

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The best account of Kramers' life and personality was given by his friend J. Romein, in *Jaarboek van de maat. schappij der Nederlandse letterkunde* (1951-1953), 82-91;see also J. A. Wheeler, *Year Book. American Philosophical Society* (1953).

H. B. G. Casimir