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(*b* Baden-Baden, Germany, 26 August 1899; *d*. Bonn, Federal Republic of Germany, 12 April 1971)

mathematics.

Krull was the son of Helmuth Krull, a dentist in Baden-Baden, and Adele Siefert Krull. After graduating from high school in 1919, he studied at the University of Freiburg and the University of Rostock. In 1920 and 1921 he studied at Göttingen, where he became acquainted with Felix Klein, whom he greatly admired. It was Emmy Noether, however, who awakened in Krull an enthusiasm for modern algebra, which at that time was making rapid advances. On his return to Freiburg in 1921. Krull earned his doctor’s degree with a dissertation on the theory of elementary divisors.

On 1 October 1922 Krull became an instructor at Freiburg, and in 1926 he was appointed unsalaried associate professor. In 1928 he went to Erlangen as full professor. His early publications were about the theory of rings and the theory of algebraic extensions of fields. In 1925 he had proved a theorem concerning the decomposition of an abelian group of operators as the direct sum of indecomposable groups, the Krull-Schmidt theorem. In a paper published in 1928 Krull applied the fundamental ideas of the Galois theory, at first valid only for finite extensions, to infinite normal separable extensions. In this way the Galois group becomes a topological group with Krull topology (a linear topology), and the classical theorems of Galois theory carry over verbatim to the general case provided we replace the term “subgroup” by “closed subgroup.” Later Krull again examined the topological groups he had introduced, especially the compact abelian groups with linear topology and countable bases. The same fundamental thought (introduction of a “natural” topological structure in algebraic systems) is to be found again in a paper published in 1955 with applications to the arithmetic of infinite algebraic number fields and the construction of a generalized multiplicative ideal theory. The theory of groups with subgroup topology, published in 1965, belongs to the same sphere of work.

Theyears Krull spent as full professor in Erlangen were the high point of his creative life. About thirtyfive publications of fundamental importance for the development of commutative algebra and algebraic geometry date from this period. In 1921 Emmy Noether had recognized the importance of the rings in which the maximal condition for ideals is satisfied, rings that are now termed Noetherian.

In 1928 Krull introduced the important concept of the Krull dimension of a commutative Noetherian ring. Krull’s basic results on dimension (*e.g*., Krull’s principal ideal theorem) mark a turning point in the development of the general theory of Noetherian rings. Previously a Noetherian ring had been a kind of pale shadow of a polynomial ring, but after the publication of Krull’s results (1928–1929) the way was open for the introduction of a surprising amount of interesting detail, as shown in the work of D. G. Northcott.

In 1932 Krull introduced the theory of additive valuations. The ideas of this theory are used in the theory of integrally closed rings and in algebraic geometry. Krull also defined rings that today are called Krull rings. The importance of Krull rings lies in the fact that the integral closure of a Noetherian integral domain is not necessarily a Noetherian ring, but it is always a Krull ring. In 1937 Krull proved the main part of the Krull-Akizuki theorem, and in 1938 the Krull-Amazuya lemma, a classical line of reasoning in algebra.

Let p be a proper prime ideal of a Noetherian ring R and put S = $R - p$ then we can form the ring $R_p \subset R$ of quotients of R with respect to S. and R_p has precisely one maximal prime ideal: R_p is a local ring. Thename “local ring” has been given to these rings because they are used to study the local properties of algebraic varieties. The German name is *Stellenring*, and the algebraic study of local rings began with Krull’s famous investigation of *Stellenringe* (1938). Let R be a Noetherian local ring with maximal ideal m; then

This is Krull’s intersection theorem (1938), the basis of the Krull topology of the ring R in which R is a metric space. The Cauchy completion $\bar{R}$ of R is a Noetherian local ring and dim $\bar{R}$R= dim R (1938). Krull posed the problem of determining the structure of all complete local rings, which I. S. Cohen solved in 1946.

While at Erlangen, Krull became chairman of the Faculty of Science. In 1929 he married Gret Meyer; they had two daughters. In 1939 he accepted an appointment to the University of Bonn. During the war he was called up into the naval meteorological service. In 1946 he resumed his work in Bonn. From this time until his death over fifty further publications appeared. These were in part a continuation of his earlier studies, but they also dealt with other fields of mathematics: group theory-calculus of variations, differential equations. Hilbert spaces.

As a mathematician of high international standing he received many invitations and honors. In 1962 the University of Erlangen conferred on him the degree of honorary doctor; he was the only mathematician given this honor. Krull had close professional and human contact with his many students. He directed thirty-five doctoral theses.

Krull described his attitude toward mathematics as that of an aesthete: “For the mathematician it is not merely a matter of finding theorems and proving them. He wants to arrange and group these theorems together in such a way that they appear not only as correct but also as imperative and self-evident. To my mind such an aspiration is an aesthetic one and not one based on theoretical cognition.” Krull ascribed great importance to the mathematical imagination of the mathematician and said that it is the possession of this imagination that distinguishes the great scientist from gifted average people.

BIBLIOGRAPHY

1. Original Works. Important works include “Über verallgemeinerte endliche abelsche Gruppen,” in *Mathematische Zeitschrift***23** (1925), 161–196;” Glaois sche Theorie der undendlichen algebraischen Erweiterungen,” in Mathematishe Annalen.**100** (1928). 687–698; “Prim idealketten in allgemeinen Ringbereichen,” in *Sitzungs berichte der Heidelberger Akademie der Wissenschaften, Mathematische-Naturwissenschaftliche Klasse* (1928); “Über einen Hauptsatz der allgemeinen Idealthoeorie, ’ ibid., (1929);’ Uber die asthetische Betrachtungsweise in der Mathemtik,” in *Semesterberichte Erlangen*, **61** (1930), 207–220; “Allgemeine Bewertungstheorie,” in *Journal für die reine und angerwandte Mathematik*, **167** (1932), 160–196; “Galossche Theorie der ganz abgeschlossenen Stellenringe,” in *Semesterberichte Erlangen*. **67/68** (1937), 324–328; “Beitrage zur Arithemtik kommutativer Intergritatsbereiche, III Zum Dumensionsbegriff der Idealtheorie,” in *Mathematische Zeitschrift*, **42** (1937), 745–766

“Dimensionstheorie in Stellenringen.”in *Journal für die reine und angewandte Mathematik***179** (1938), 204– 226; “Allgemeine ModulRingund Idealtheorie,” in *Enzyklopädie der Mathematischen Wissenschaften*, 2nd ed., **I** (Leipzig, 1939); “Beiträge zur Arithmetik kommutativer Integritätsbereiche, VI Der allgemeine Diskriminantensatz: Unverzweigte Ringerweiterungen.”in *Mathematische Zeitschrift*. **45** (1939), 1–19; “Über Separable. insbesondere kompakte separable Gruppen.“in *Journal für die lreine und angewandte Mathematik*. **184** (1942), 19–48; “Jacobsonsche Ringe. Hilbertscher Nullstellensatz, Dimensionstheorie.”in *Mathematische Zeitschrift*, **54** (1951), 354–387; “Jacobsonsche Radikal und Hilbertscher Nullsltellensatz.”in *Proceedings of the International Congress of Mathematicians. 1950*, **II** (Cambridge. Mass., 1952); “Charakterentopologie. Isomorphismentopologie, Bewertungstopologie.”*Memorias de matematica del instituto Jorge Juan* (Spain). no.16 (1955);“Zur Theorie der Gruppen mit Untergruppentopologie,” in *Abhandlungen aus dem Mathematischen Seminar Universität Hamburg*, **28** (1965), 50–97; Idealtheorie (New York. 1968)

II. Secondary Literature. H. J.Nastold,” Wolfgang Krull’s Arbeiten zur kommutativen Algerbra und ihre Bedeutung fÜr die algebraische Geometrie.’*Jahresberichte der Deutschen Mathematiker-Vereinigung*, **82** (1980), 63– 76; H. Schoeneborn,” In Memoriam Wolfgang Krull, ’ ibid., 51–62;, and completel bibiography of Krull’s works, ibid., 77–80; D. G.Northcott *Ideal Theory* (Cambridge, England. 1953).

Heinz Schoenebkorn